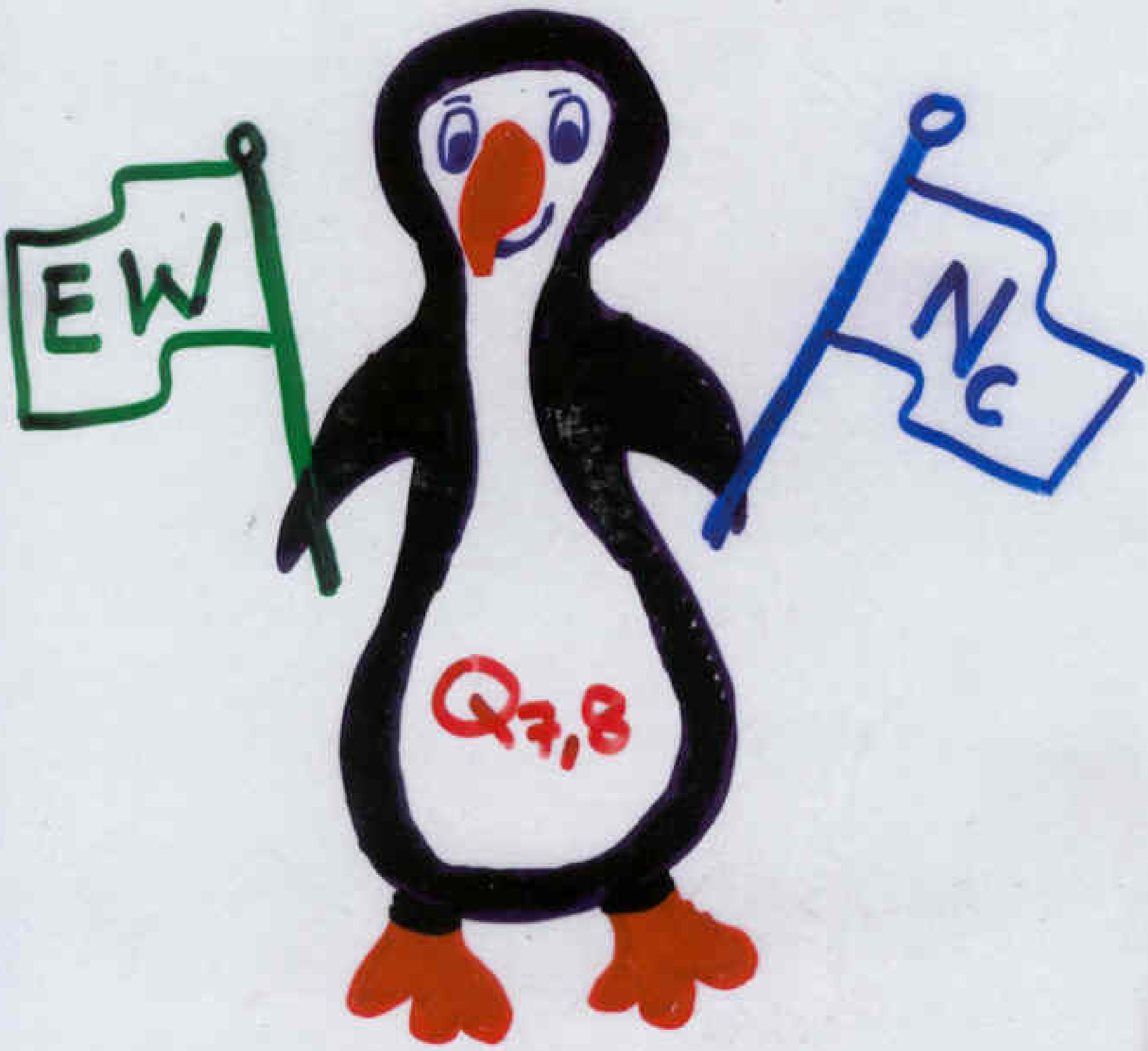

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Work done in collaboration
with M. KNECHT
E. de RAFAEL

Battle field:

$$\frac{\epsilon'}{\epsilon} \sim C_6 \langle Q_6 \rangle - C_8 \langle Q_8 \rangle$$

$\alpha_s^{\overline{MS}}(\mu)$, γ_5
evanescent op's.
etc...

Meson mass
g_8, g_{27}, f_M, etc...

DICTIONARY ?

(The large N_c expansion)

$\pi^\pm - \pi^0$ mass difference

χ limit: $\mathcal{L}_{eff} = e^2 C \, \mathrm{Tr}(Q_R U Q_L U^\dagger)$

$$m^2_{\pi^+} - \underbrace{m^2_{\pi^0}}_{=0} = \frac{2e^2}{f_\pi^2} C$$

$C \sim$

$A_\mu L^\mu Q_L$

$A_\mu R^\mu Q_R$

$$C = \frac{3}{32\pi^2} \int_0^\infty dQ^2 \left(-Q^2 \, \Pi_{LR}(Q^2) \right)$$

Π_{LR} ?

Minimal Hadronic Approx. to large-N_c QCD

$$\Pi_{LR}(Q^2) = \sum_V^{\infty} \frac{f_V^2 M_V^2}{Q^2+M_V^2} - \sum_A^{\infty} \frac{f_A^2 M_A^2}{Q^2+M_A^2} - \frac{f_\pi^2}{Q^2} \quad \text{, large-}N_c \text{ QCD}$$

$$\sim \frac{C_1}{Q^2} + \frac{C_2}{Q^4} + \frac{C_3}{Q^6} + \cdots \quad \text{, OPE high } Q^2$$

$$\sim -\frac{f_\pi^2}{Q^2} - 4L_{10} + \text{"}L\text{"}\, Q^2 + \cdots \quad \text{, XPT low } Q^2$$

$$\sim \sum_V^{N} \frac{f_V^2 M_V^2}{Q^2+M_V^2} - \sum_A^{N'} \frac{f_A^2 M_A^2}{Q^2+M_A^2} - \frac{f_\pi^2}{Q^2} \quad \text{, MHA interpolating fnc}$$

Matching

low Q^2) $\quad -4L_{10} = \sum f_V^2 - \sum f_A^2$

⋮

high Q^2)

$$\left.\begin{aligned} C_1 = 0 &= f_\pi^2 + \sum_A f_A^2 M_A^2 - \sum_V f_V^2 M_V^2 \\ C_2 = 0 &= \sum_A f_A^2 M_A^4 - \sum_V f_V^2 M_V^4 \end{aligned}\right\} \text{Weinberg '67}$$

$$\underbrace{\alpha_s \langle \bar\psi\psi\bar\psi\psi\rangle}_{\text{Quarks \& Gluons}} \sim C_3 = \underbrace{\sum_A f_A^2 M_A^6 - \sum_V f_V^2 M_V^6}_{\text{Mesons}}$$

Knecht de Rafael '98

⋮

⇒ A DICTIONARY !

Numbers

$$\frac{1}{\pi} \mathrm{Im}\Pi_{LR}^{MHA}(t) = f_V^2 M_V^2 \delta(t-M_V^2) - f_A^2 M_A^2 \delta(t-M_A^2) - f_\pi^2 \delta(t)$$

FESR's (π-less):

$$\int dt\, t^n\, \mathrm{Im}\Pi_{LR}^{MHA}(t) = \int_0^{s_0} dt\, t^n\, \mathrm{Im}\Pi_{LR}^{ALEPH}(t)$$

χ corrected

$$n = 0, \pm 1, \pm 2, 3, 4.$$

(overconstrained)
Solution!

$f_\pi = 87 \pm 3.5$ MeV ; $M_V = 748 \pm 29$ MeV

$\frac{M_V^2}{M_A^2} = 0.50 \pm 0.06$; $f_V = 0.16 \pm 0.01$

$f_A = 0.08 \pm 0.02$

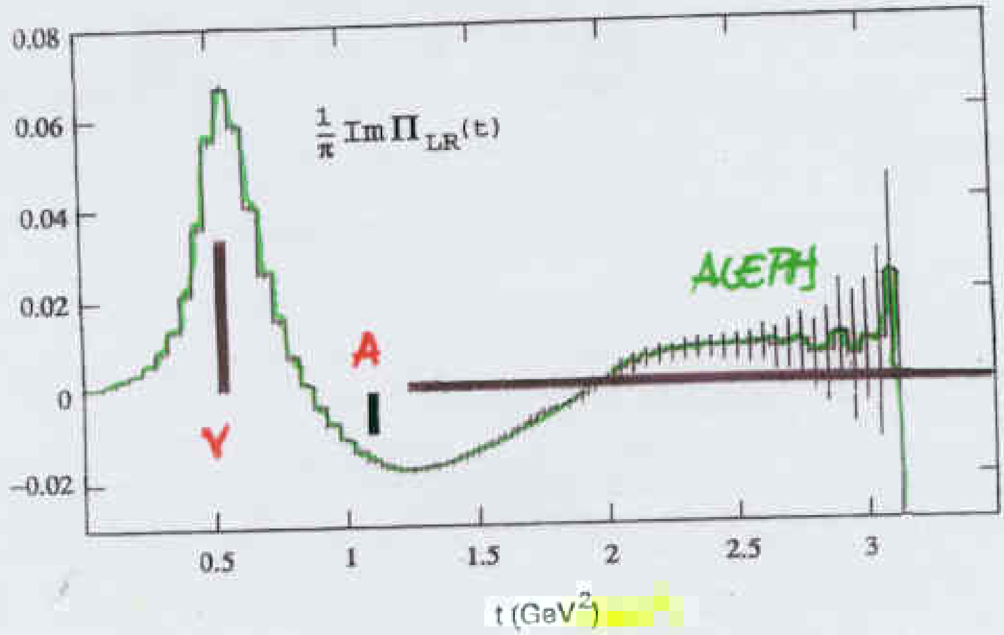
$s_0 = 1.4 \pm 0.2$ GeV^2

Phily, deRafael
S.P. '01

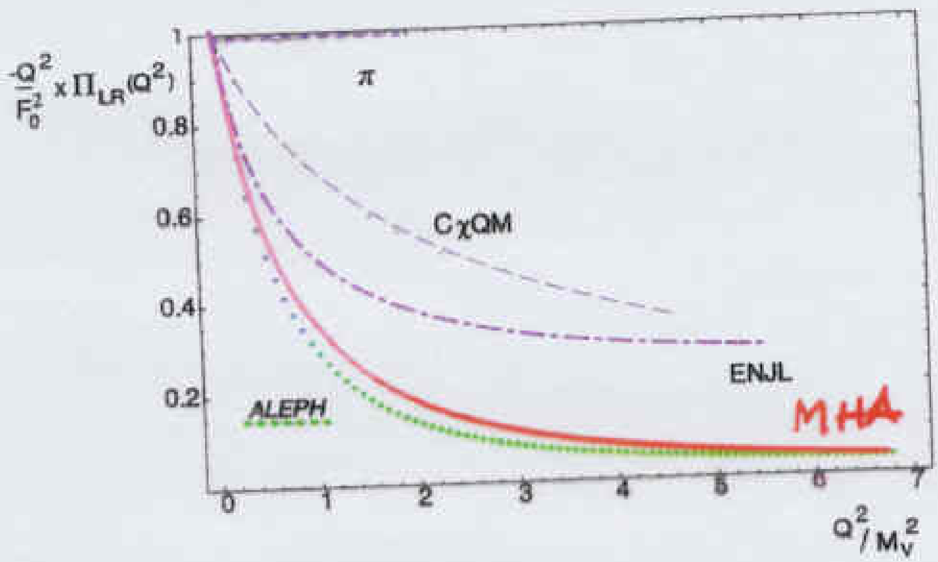
$$\Rightarrow \Pi_{LR}^{MHA}(Q^2) = -\frac{f_\pi^2}{Q^2} \frac{M_V^2 M_A^2}{(Q^2+M_V^2)(Q^2+M_A^2)}$$

$\Rightarrow$ $m_{\pi^+} - m_{\pi^0} = 4.9\ (1 \pm 30\%)$ MeV (MHA)

4.5936(5) MeV (EXP).



$\frac{1}{\pi}\,\mathrm{Im}\,\Pi_{LR}(t)$
ALEPH
V
A
t (GeV^2)



$\frac{-Q^2}{F_0^2} \times \Pi_{LR}(Q^2)$
π
CχQM
ENJL
ALEPH
MHA
Q^2/M_V^2

The Moments Problem

You want to compute $\int_0^\infty dt\, t^2 \rho(t)$, but $\rho(t)$ is unknown, you only know

$$\int_0^\infty dt\ \rho(t) = A$$

$$\int_0^\infty dt\, t\rho(t) = B$$

Construct $\rho(t) = w_1\, \delta(t - M_1^2) + w_2\, \delta(t - M_2^2)$

with $w_{1,2}$ / $\int_0^\infty dt\ \rho(t) = A$, $\int_0^\infty dt\, t\, \rho(t) = B$

Then

$$\int_0^\infty dt\, t^2 \rho(t) = \left(w_1 M_1^4 + w_2 M_2^4\right) +$$

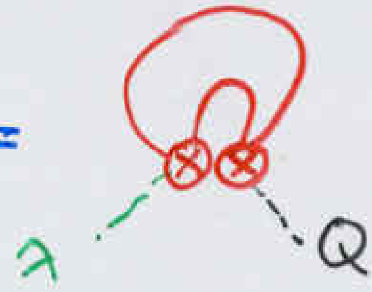
$$+ \underbrace{\int_0^\infty dt\ (t - M_1^2)(t - M_2^2)\left(\rho(t) - \cancel{\rho(t)}\right)}_{\text{Rel} \sim \frac{\Gamma}{M}}$$

(a+b)

$$Q_7 = 6\, \bar{q}_L \gamma^\mu \lambda_{32} q_L \; \bar{q}_R \gamma_\mu Q q_R$$

$$\underset{O(p^0)}{=} g_7 \, \mathrm{tr}\left(U \lambda_{32} U^\dagger Q \right)$$

=



$$\sim \langle \bar{s}_L \gamma^\mu d_L \, \bar{d}_R \gamma_\mu s_R \rangle$$

$$= -3i \left\{ g_{\mu\nu} \int \frac{d^D q}{(2\pi)^D} \Pi^{\mu\nu}_{LR}(q) \right\}_{\overline{MS}}$$

evanescent op's, etc...

MHA

(OPE Matching)

$$\left(f_V^2 M_V^6 - f_A^2 M_A^6 \right)_{D=4-\epsilon} = \underbrace{\left(f_V^2 M_V^6 - f_A^2 M_A^6 \right)_4}_{\sim \alpha_s \langle \bar{\psi}\psi\bar{\psi}\psi \rangle} \left(1 \begin{matrix} -1/2 \;(\mathrm{NDR}) \\ +3/2 \;(\mathrm{HV}) \end{matrix} \frac{\epsilon}{2} \right)$$

Then

$$\langle \bar{s}_L \gamma^\mu d_L \, \bar{d}_R \gamma_\mu s_R \rangle_{\overline{MS}}(\mu) = -\frac{3}{32\pi^2} \left[f_A^2 M_A^6 \log \frac{\mu^2 e^{\frac{1}{3} \begin{smallmatrix} -1/2 \\ +3/2 \end{smallmatrix}}}{M_A^2} - V \right]$$

$$Q_8 = -12 \sum_q e_q \, \bar{s}_L q_R \, \bar{q}_R d_L$$

$$\underset{\mathcal{O}(p^0)}{=} g_8 \, \mathrm{tr}\left(U \lambda_{32} U^\dagger Q\right)$$

Similarly:

$$g_8 \sim \langle \bar{s}_L s_R \, \bar{d}_R d_L \rangle$$

$\sim$  $+$

$\bar{s}_L s_R$ $\bar{d}_R d_L$ $\bar{s}_L s_R$ $\bar{d}_R d_L$

$\longleftrightarrow$ $\longleftrightarrow$

$\langle \bar{\psi}\psi \rangle^2$ $\sigma, \eta', \text{glueballs}, \ldots$

Tough!

Cirigliano et al '01
Bijnens et al '01

$\therefore$ Different strategy:

NDR $\downarrow$

$$-Q^6 \Pi_{LR}(Q^2) \underset{Q^2 \to \infty}{\sim} 16\pi\alpha_s \left(1 + \begin{matrix} 25/8 \\ 21/8 \end{matrix} \frac{\alpha_s}{\pi}\right) \langle \bar{s}_L s_R \bar{d}_R d_L \rangle + \ldots$$

HV $\uparrow$

$$\downarrow Q^2 \to \infty \quad \text{MHA}$$

$$f_A^2 M_A^6 - f_V^2 M_V^6$$

$$\therefore \quad \langle \bar{s}_L s_R \, \bar{d}_R d_L \rangle_{\overline{MS}}(\mu) \simeq \frac{f_A^2 M_A^6 - f_V^2 M_V^6}{16\pi\alpha_s(\mu)} \left(1 - \begin{matrix} 25/8 \\ 21/8 \end{matrix} \frac{\alpha_s}{\pi}\right)$$

with correct scheme dependence

i.e.

$$\begin{pmatrix} Q_7 \\ Q_8 \end{pmatrix}_{HV} = \begin{pmatrix} 1-\frac{\alpha_s}{2\pi} & \frac{3\alpha_s}{2\pi} \\ \frac{\alpha_s}{\pi} & 1+\frac{\alpha_s}{\pi} \end{pmatrix} \begin{pmatrix} Q_7 \\ Q_8 \end{pmatrix}_{NDR}$$

Summary of results for $M_{7,8} \equiv < (\pi\pi)_{I=2}|Q_{7,8}|K^0 > (2\ GeV)$, in units of GeV^3.

This work

Refs.	M_7(NDR)	M_7(HV)	M_8(NDR)	M_8(HV)
Knecht et al.[15]	0.11 ± 0.03	0.67 ± 0.20	2.34 ± 0.73	2.52 ± 0.79
Narison[19]	0.35 ± 0.10		2.7 ± 0.6	
Cirigliano et al.[17]	0.16 ± 0.10	0.49 ± 0.07	2.22 ± 0.67	2.46 ± 0.70
Bijnens et al.[18]	0.24 ± 0.03	0.37 ± 0.08	1.2 ± 0.9	1.3 ± 0.9
Battacharya et al.[20]	0.32 ± 0.06		1.2 ± 0.2	
Donini et al.[21]	0.11 ± 0.04	0.18 ± 0.06	0.51 ± 0.10	0.62 ± 0.12

SUMMARY & OUTLOOK

- I presented the MHA to large-N_c QCD: $Q_{7,8}$.

  Exs: $\cancel{B_K}$, $\pi^0 \to e^+e^-$, $g-2$

  $\eta \to \mu^+\mu^-$

- $Q_6 \to \epsilon'/\epsilon$ (with Thomas Hambye).

  $\Delta I = 1/2$

  $\vdots$