19-6-02 Blois

Matching the Electromagnetic Penguins Q_7 and Q_8

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1. Introduction

2. Technique

   - Matching Short-Distances

   - Matching Long–Short Distances

3. Results

   - Numerical Analysis

★

1. Introduction

$K \to \pi\pi$ Amplitudes

Isospin Symmetry Limit Decomposition

$$A[K^0 \to \pi^0\pi^0] \equiv \frac{1}{\sqrt{3}} A_0 - \sqrt{\frac{2}{3}} A_2$$

$$A[K^0 \to \pi^+\pi^-] \equiv \frac{1}{\sqrt{3}} A_0 + \frac{1}{\sqrt{6}} A_2$$

$$A_I \equiv -i\, a_I\ \exp(i\delta_I)$$

δ_I are strong FSI phases •

$$|\varepsilon'|_{\mathrm{FSI}} = \frac{1}{\sqrt{2}} \frac{\mathrm{Re}\,(a_2)}{\mathrm{Re}\,(a_0)} \left[\frac{\mathrm{Im}\,(a_0)}{\mathrm{Re}\,(a_0)} - \frac{\mathrm{Im}\,(a_2)}{\mathrm{Re}\,(a_2)}\right]$$

$\Rightarrow$ Direct CP Violation •

CHPT to $O(e^2 p^0)$ E. de Rafael

$$\mathcal{L}_{\Delta S=1} = C\, \underline{F_0^6 e^2 G_E} \left(U^\dagger \mathcal{Q} U\right)_{23} + \underset{O(p^2)}{\cdots}$$

$$(G_8, G_{27}, \cdots)$$

$$\begin{cases} C \equiv -\frac{3}{5}\frac{G_F}{\sqrt{2}} V_{ud}V_{us}^* \ ; \\ U \equiv \exp\left(i\frac{\sqrt{2}\,\boldsymbol{\Phi}}{F_0}\right) \ ; \\ (\pi, K, \eta) \to \boldsymbol{\Phi} \ ; \\ \mathcal{Q} \equiv \frac{1}{3}\mathrm{diag}(2,-1,-1) \ \bullet \end{cases}$$

$$\left.\frac{\mathrm{Im}\,(a_2)}{\mathrm{Re}\,(a_2)}\right|_{\substack{m_u = m_d \\ \alpha^2 = 0}} \propto \frac{\mathrm{Im}\,(e^2 G_E)}{G_{27}}$$

Including FSI to <u>all</u> orders in CHPT !

★

In the Standard Model, just

$$Q_7 \equiv (\bar{s}_\alpha \, \gamma_\mu (1-\gamma_5) d_\alpha) \sum_q \frac{3}{2} e_q \left(\bar{q}_\beta \gamma^\mu (1+\gamma_5) q_\beta \right) ;$$

$$Q_8 \equiv \left(\bar{s}_\alpha \, \gamma_\mu (1-\gamma_5) d_\beta\right) \, \sum_q \frac{3}{2} e_q \left(\bar{q}_\beta \gamma^\mu (1+\gamma_5) q_\alpha \right)$$

can contribute to $\mathrm{Im}(e^2 G_E)$!

And,

they only mix between themselves if $\alpha_{\mathrm{EM}} = 0$; i.e.

$$\nu \frac{\mathrm{d}}{\mathrm{d}\nu} Q_i(\nu) = - \sum_{j=7,8} \gamma^{ij} \, Q_j(\nu) \quad \bullet$$

$$(i = 7, 8)$$

$\Rightarrow$ Calculate $\mathbf{Im}\,(e^2 G_E)$ $\Leftarrow$

A lot of recent work :

▶ Analytically

◇ V. Cirigliano, J. Donoghue, E. Golowich, K. Maltman

◇ M. Knecht, S. Peris, E. de Rafael

◇ S. Narison

▶ Lattice

- Wilson Fermions

◇ A. Donini et al

◇ D. Becirevic et al

} Roma

◇ SPQR (Ph. Boucaud et al)

- Domain Wall Fermions

◇ CP – PACS (S. Aoki et al)

◇ RBC (T. Blum et al)

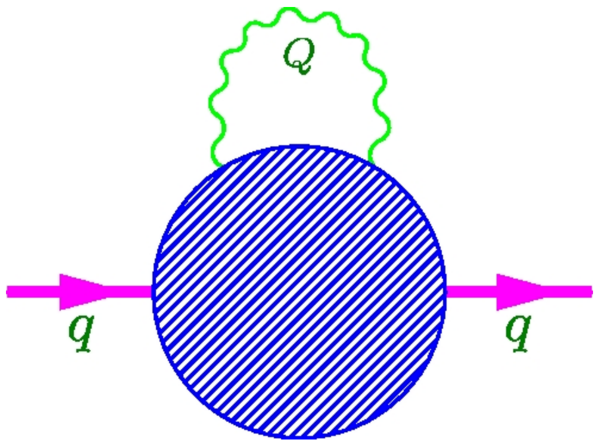
- Staggered Fermions

◇ T. Bhattacharya et al.

2. Technique

$\Gamma_{\Delta S=1}$ is generated by W-exchange •

CHPT couplings are coefficients of the Taylor expansion of



$$\Rightarrow \quad \text{Im}\,(e^2\,G_E) \sim \int_0^\infty \mathrm{d}Q^2\,Q^2\,\mathbf{\Pi_{LRPP}}(Q^2, q^2 = 0)$$

Involves QCD at all scales !

Perturbative

Renormalization

Group techniques

& matching

at heavy quark

mass thresholds

★ Various choices of schemes, regulators, operator basis, $\cdots$

$$\underline{\mathbf{\Gamma}_{\Delta S=1}(C_i, \nu, \cdots)} = \sum_{i=7,8} \mathrm{Im}\,(C_i)\; Q_i$$

★ NLO in NDR, HV

- A. Buras et al.
- M. Ciuchini et al.

M_W, M_b, M_c, μ

⇒ Matching $\langle 2|\mathbf{\Gamma}_{\Delta S=1}|1\rangle = \langle 2|\mathbf{\Gamma}_{\mathrm{LD}}|1\rangle$ ⇐

Non–

–Perturbative

Long – Distance

Part

★ Different degrees of freedom, operator basis

★ Regulator schemes $\cdots$

(D = 4, cut – off)

$$\underline{\mathbf{\Gamma}_{\mathrm{LD}}(g_i, \mu, \cdots)}$$

μ, 0

Matching Γ_{LD} and $\Gamma_{\Delta S=1}$ fixes short-distance behaviour of $g_i(\mu_C,\cdots)$ •

$$|g_i(\mu_C)|^2 = \mathcal{F}\left(C_i(\nu), \alpha_s, \cdots\right)$$

⇛ Several advantages

√ Resummed large logs $\alpha_s\left(\alpha_s \ln\left(\frac{M_W}{\nu}\right)\right)^n$

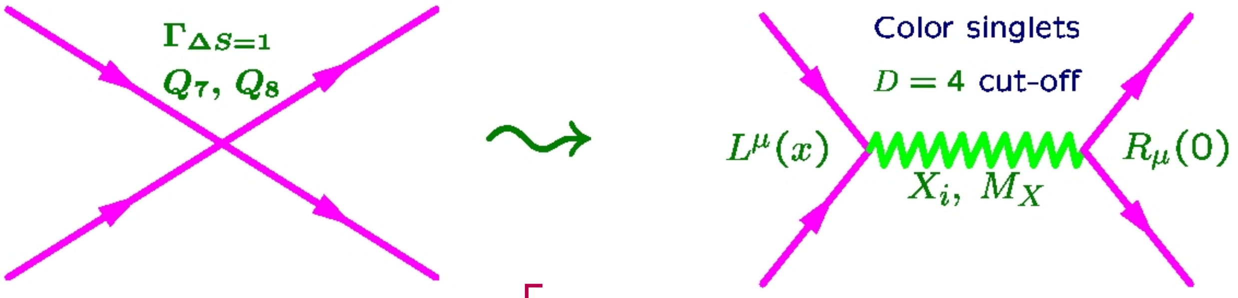
√ Short-distance scheme dependence (NDR, HV, $\cdots$) taken into account analytically !

(Use $D = 4$ for long-distance, for instance)

Γ_{LD} completely fixed •

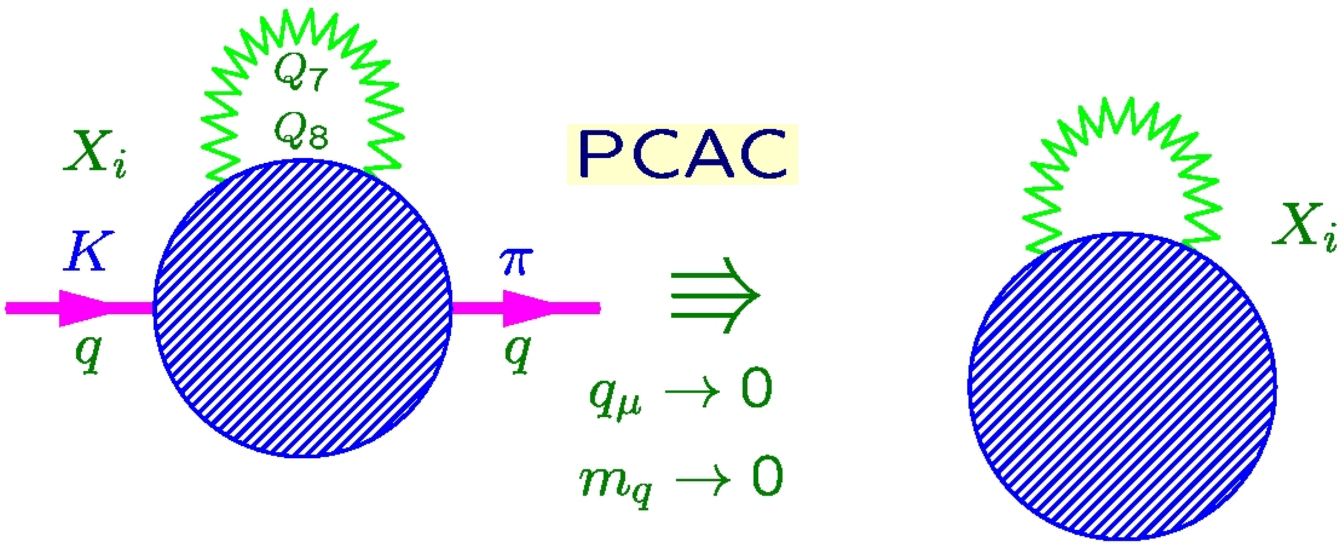
Ready to calculate the Long-Distance part !

Our choices:



$$\Gamma_{\rm LD} = \underline{g_7(\mu_C)}\, X_7^\mu \left[(\bar{s}\gamma_\mu d)_L + \frac{3}{2}\sum_q e_q\ (\bar{q}\gamma_\mu q)_R \right]$$

$$+ \quad \underline{g_8(\mu_C)} \sum_q X_{q,8} \left[(\bar{q} d)_L + (-2)\,\frac{3}{2}\, e_q\ (\bar{s} q)_R \right]$$



Exact expression !

$$-\frac{3}{5} F_0^6\, \text{Im}(e^2\, G_E) = -|g_7|^2\, 3\, i \int \frac{\text{d}^4 p_X}{(2\pi)^4}\, \frac{g_{\mu\nu}}{p_X^2 - M_X^2}\, \mathbf{\Pi}_{LR}^{\mu\nu}$$

$$+|g_8|^2\, i \int \frac{\text{d}^4 p_X}{(2\pi)^4}\, \frac{1}{p_X^2 - M_X^2} \left[\mathbf{\Pi}^{(0)}_{SS+PP} - \mathbf{\Pi}^{(3)}_{SS+PP} \right]$$

$$\frac{|g_7(\mu_C)|^2}{M_X^2} = \text{Im}\, C_7(\mu_R)\,[1 + \frac{\alpha_S}{\pi}\,(\underline{\gamma_{77}^{(1)}}\,\ln\frac{M_X}{\mu_R} + \underline{\Delta\, r_{77}})]$$

$$+\,\text{Im}\, C_8(\mu_R)\,[\frac{\alpha_S}{\pi}\,\underline{\Delta\, r_{78}}] + \mathcal{O}(\alpha_S^2)$$

$$\frac{|g_8(\mu_C)|^2}{M_X^2} = \text{Im}\, C_8(\mu_R)\,[1 + \frac{\alpha_S}{\pi}\,(\underline{\gamma_{88}^{(1)}}\,\ln\frac{M_X}{\mu_R}$$

$$+\underline{\tilde{\gamma}_{88}^{(1)}}\,\ln\frac{\mu_C}{M_X} + \underline{\Delta\, r_{88}})]$$

$$+\,\text{Im}\, C_7(\mu_R)\,[\frac{\alpha_S}{\pi}\,(\underline{\gamma_{87}^{(1)}}\,\ln\frac{M_X}{\mu_R} + \underline{\Delta\, r_{87}})] + \mathcal{O}(\alpha_S^2)$$

Term multiplying $|g_7(\mu_C)|^2$

$$-\frac{9}{16\pi^2}\int_0^\infty \mathrm{d}\,Q^2\,\frac{Q^4}{Q^2+M_X^2}\,\mathbf{\Pi}_{LR}^{T}(Q^2)$$

$$\int_0^\infty \mathrm{d}\,Q^2 = \underbrace{\int_0^{\mu^2}\mathrm{d}\,Q^2}_{\text{LD}} + \underbrace{\int_{\mu^2}^{\infty}\mathrm{d}\,Q^2}_{\text{SD}}$$

Long-Distance Part

Using dispersion relations

$$-\frac{9}{16\pi^2}\int_0^\infty \mathrm{d}\,t\,\frac{t^2}{M_X^2}\ln\left(1+\frac{\mu^2}{t}\right)\underbrace{\frac{1}{\pi}\,\mathrm{Im}\,\mathbf{\Pi}_{LR}^T(t)}_{Data}$$

Short-Distance Part

In QCD (χ-limit) at $Q^2 \gg 1\mathrm{GeV}^2$

$$\mathbf{\Pi}_{LR}^T(Q^2) \longrightarrow \sum_{n=0}\frac{D_{2(n+3)}^{(i)}(\nu)\,\langle 0|Q_{(2(n+3))}^{(i)}|0\rangle(\nu)}{Q^{2(n+3)}}$$

$\Rightarrow$ Dimension six OPE

$$\frac{1}{M_X^2}\frac{\alpha_S}{\pi}\ln\frac{\mu}{M_X}\,i\int\frac{\mathrm{d}^4\tilde{q}}{(2\pi)^4}$$

$$\times\left[\gamma_{77}^{(1)}\,g_{\mu\nu}\mathbf{\Pi}_{LR}^{\mu\nu}(\tilde{q})-\gamma_{87}^{(1)}\left(\mathbf{\Pi}_{SS+PP}^{(0)}-\mathbf{\Pi}_{SS+PP}^{(3)}\right)(\tilde{q})\right]$$

$\Rightarrow$ Dimension eight and all higher dimensional OPE !

$$\frac{9}{16\pi^2}\int_0^{s_0} \mathrm{d}\,t\,\frac{t^2}{M_X^2}\ln\left(1+\frac{t}{\mu^2}\right)\underbrace{\frac{1}{\pi}\,\mathrm{Im}\,\mathbf{\Pi}_{LR}^T(t)}_{Data}$$

Exact resummation of all higher dimensional operators

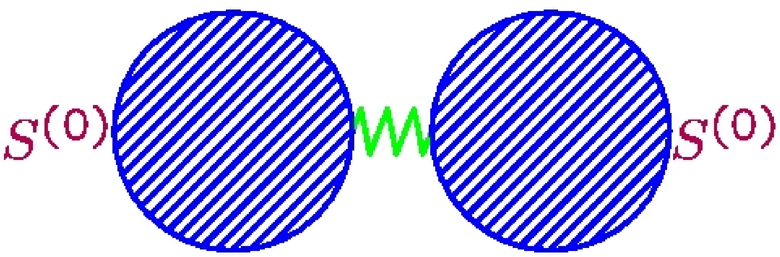
$$\int_0^{s_0} \mathrm{d}\,t\; t^2 \left[\ln\left(1+\frac{\mu^2}{t}\right) - \ln\left(1+\frac{t}{\mu^2}\right)\right] \underbrace{\frac{1}{\pi}\,\mathrm{Im}\,\mathbf{\Pi}^T_{LR}(t)}_{Data}$$

The exact needed $\ln\left(\frac{\mu^2}{t}\right)$ •

Term multiplying $|g_8(\mu_C)|^2$

$$\frac{1}{16\pi^2}\int_0^\infty \mathrm{d}\,Q^2\,\frac{Q^2}{Q^2+M_X^2}\left[\mathbf{\Pi}^{(0)}_{SS+PP} - \mathbf{\Pi}^{(3)}_{SS+PP}\right](Q^2)$$

$\Rightarrow$ Disconnected contribution



$$3\,\langle 0|\overline{q}q|0\rangle^2\Big|_{\overline{\mathrm{MS}}}(\mu_C)$$

$\Rightarrow$ Connected contribution

Long-Distance Part

$$-\frac{1}{16\pi^2}\int_0^\infty \mathrm{d}t\,\frac{t}{M_X^2}\ln\left(1+\frac{\mu^2}{t}\right)\underbrace{\frac{1}{\pi}\,\mathrm{Im}\,\mathbf{\Pi}^{(0-3)}_{SS+PP}(t)}_{Data}$$

Short-Distance Part

Analogous to $\mathbf{\Pi}^T_{LR}(Q^2)$

$$\int_{\mu^2}^\infty \mathrm{d}Q^2\,\frac{1}{16\pi^2}\,\frac{Q^2}{Q^2+M_X^2}\,\mathbf{\Pi}^{(0-3)\,C}_{SS+PP}(Q^2)$$

$\Rightarrow$ Dimension six OPE

$$-\frac{1}{M_X^2}\,\frac{\alpha_S}{\pi}\,\ln\frac{\mu}{M_X}\,\gamma^{(1)}_{88}\,i\int\frac{\mathrm{d}^4\widetilde{q}}{(2\pi)^4}\,\mathbf{\Pi}^{(0-3)\,C}_{SS+PP}(\widetilde{q})$$

$\Rightarrow$ Dimension eight and higher !

$$\frac{1}{16\pi^2}\int_0^{\widehat{s}_0} \mathrm{d}\,t\,\frac{t}{M_X^2}\ln\left(1+\frac{t}{\mu^2}\right)\underbrace{\frac{1}{\pi}\,\mathrm{Im}\,\mathbf{\Pi}^{(0-3)}_{SS+PP}(t)}_{Data}$$

3. Results

$$-\frac{3}{5} F_0^6 \,\mathrm{Im}(e^2 G_E) = -6\,\mathrm{Im}C_7(\mu_R)\,\langle 0|Q_7|0\rangle^{\chi}(\mu_R) + \mathrm{Im}C_8(\mu_R)\,\langle 0|Q_8|0\rangle^{\chi}(\mu_R)\,;$$

$$\langle 0|Q_7|0\rangle_{\mathrm{NDR}} \equiv \langle 0|(\overline{u}\gamma^{\mu}(1+\gamma_5)d)\,(\overline{d}\gamma_{\mu}(1-\gamma_5)u)|0\rangle_{\mathrm{NDR}}$$

$$= \frac{3}{32\pi^2}\left(1+\frac{1}{24}\frac{\alpha_S}{\pi}\right)\mathcal{A}_{\mathbf{LR}}(\mu_{\mathbf{R}}) + \frac{1}{48}\frac{\alpha_S}{\pi}\mathcal{A}_{\mathbf{SP}}(\mu_{\mathbf{R}})$$

- Donoghue, Golowich
- Knecht, Peris, de Rafael

$$\langle 0|Q_8|0\rangle_{\mathrm{NDR}} \equiv \langle 0|(\overline{d}(1+\gamma_5)d)\,(\overline{s}(1-\gamma_5)s)|0\rangle_{\mathrm{NDR}}$$

$$= \left(1+\frac{23}{12}\frac{\alpha_S}{\pi}\right)\mathcal{A}_{\mathbf{SP}}(\mu_{\mathbf{R}}) + \frac{3}{32\pi^2}\frac{9}{2}\frac{\alpha_S}{\pi}\mathcal{A}_{\mathbf{LR}}(\mu_{\mathbf{R}})$$

$\Rrightarrow$ Both also in HV

$$\mathcal{A}_{\mathbf{LR}}(\mu_{\mathbf{R}}) \equiv \int_0^{s_0} \mathrm{d}t \; t^2 \ln\left(\frac{\mu_R^2}{t}\right) \underbrace{\frac{1}{\pi} \mathrm{Im}\, \mathbf{\Pi}_{LR}^T(t)}_{Data}$$

$\Rightarrow$ Excellent V-A Tau data up to $t \sim M_\tau^2$ (ALEPH, OPAL)

★ Assign s_0 to each data distribution requiring 1^{st} and 2^{nd} WSRs

★ $\ln\left(\frac{\mu_R^2}{t}\right)$ kills data points for $t \simeq \mu_R^2 \simeq s_0$

$\Rightarrow$ Good **!**

$$\mathcal{A}_{\mathbf{LR}}^{\mathrm{ALEPH}}(\mathbf{2}\mathrm{GeV}) = (4.5 \pm 0.5) \cdot 10^{-3}\ \mathrm{GeV}^6\,;$$

$$\mathcal{A}_{\mathbf{LR}}^{\mathrm{OPAL}}(\mathbf{2}\mathrm{GeV}) = (4.2 \pm 0.4) \cdot 10^{-3}\ \mathrm{GeV}^6\,;$$

$\Rightarrow$ Average

$$\mathcal{A}_{\mathbf{LR}}(\mathbf{2}\mathrm{GeV}) = (4.35 \pm 0.50) \cdot 10^{-3}\ \mathrm{GeV}^6$$

$$\mathcal{A}_{\mathbf{SP}}(\mu_{\mathbf{R}}) \equiv \underbrace{3\langle 0|\overline{q}q|0\rangle^2_{\overline{MS}}(\mu_R)}_{\mathcal{O}(N_c^2)} +$$

$$\int_0^{\widehat{s}_0} dt\ t\ \ln\left(\frac{\mu_R^2}{t}\right)\ \underbrace{\frac{1}{\pi}\,\mathrm{Im}\,\mathbf{\Pi}^{(0-3)}_{SS+PP}(t)}_{\mathcal{O}(1)}$$

★ There are WSRs-like for $\langle SS+PP\rangle^{(0-3)}$, $\langle SS\rangle^{(a)}$ & $\langle PP\rangle^{(a)}$ •

$\Rightarrow$ Required to fix $\widehat{s}_0$

———— • ————

1^{st}) Estimate of $\mathcal{A}_{\mathbf{SP}}$

★ Use phenomenological models for $\frac{1}{\pi}\mathrm{Im}\mathbf{\Pi}^{(0-3)}_{SS}(t)$

- B. Moussallam (L_6)

- R. Kaminski et al

(agrees with naîve Lowest Scalar Dominance)

★ Use Lowest Pseudo-Scalar Dominance and narrow width for π and η_1 $\surd$

★ QCD $\Rightarrow$ $\int_0^{\hat{s}_0} \mathrm{d}t \left[t \, \frac{1}{\pi} \, \mathrm{Im} \mathbf{\Pi}^{(0-3)}_{SS+PP}(t) \right] \simeq 0 \, ;$

Total $\Rrightarrow \sim -[30 \sim 40]\%$ very stable in μ_R •

2^{nd}) Calculation of $\langle 0|Q_8|0\rangle$

Notice

$$M_2 \equiv \int_0^{s_0} \mathrm{d}t \left[t^2 \, \frac{1}{\pi} \, \mathrm{Im}\, \mathbf{\Pi}^T_{LR}(t) \right] \simeq$$

$$-\frac{4\pi}{3} \alpha_S(s_0) \left[1 + \frac{25}{8} \frac{\alpha_S}{\pi} \right] \langle 0|Q_8|0\rangle^{\mathrm{NDR}}_{\chi}(s_0)$$

- Donoghue, Golowich

- Knecht, Peris, de Rafael

★ ALEPH and OPAL Data

$$M_2 = -\,[1.9 \pm 1.0] \cdot 10^{-3}\,\text{GeV}^6$$

Very compatible with neglecting

$$\frac{1}{\pi}\text{Im}\,\mathbf{\Pi}^{(0-3)}_{SS+PP}$$

$$\left(M_2 = -\,[2.0 \pm 0.9] \cdot 10^{-3}\,\text{GeV}^6\right)$$

$\Rightarrow$ Small non-factorizable corrections in $\langle 0|Q_8|0\rangle$ $\surd$

We used $\langle 0|\overline{q}q|0\rangle_{\overline{MS}} = -\,[0.018 \pm 0.004]\ \text{GeV}^3$

- Bijnens, J.P., de Rafael

(very similar to latest Lattice and QCD Sum Rules determinations)

Comparison

$-10^5 \langle 0|Q_7|0\rangle_\chi$ (2GeV) GeV^6

	NDR	HV
This Work Data & Duality FESR	4.0 ± 0.5	6.2 ± 1.0
Knecht et al $N_c \to \infty$, LMD	1.9 ± 0.6	11.0 ± 2.0
Cirigliano et al Weighted Data	2.7 ± 1.7	8.2 ± 0.9
Donini et al Lattice (Wilson)	2.6 ± 0.7	4.3 ± 1.1
RBC Coll. Lattice (Chiral)	4.5 ± 0.5 (Stat. Err.)	—
CP-PACS Coll. Lattice (Chiral)	4.0 ± 0.5 (Stat. Err.)	—
T. Bhattacharya et al. Lattice (Staggered)	4.5 ± 1.1	—

Comparison

$10^3\,\langle 0|Q_8|0\rangle_\chi$ (2GeV) GeV^6

	NDR	HV
Factorization	1.2 ± 0.5	1.3 ± 0.6
This Work Data & Duality FESR	1.2 ± 0.8	1.3 ± 0.8
Knecht et al $N_c \to \infty$, LMD	2.3 ± 0.7	2.5 ± 0.8
Cirigliano et al Weighted Data	2.2 ± 0.7	2.4 ± 0.7
Donini et al Lattice (Wilson)	0.7 ± 0.2	0.8 ± 0.2
RBC Coll. Lattice (Chiral)	1.1 ± 0.2 (Stat. Err.)	—
CP-PACS Coll. Lattice (Chiral)	1.0 ± 0.2 (Stat. Err.)	—
T. Bhattacharya et al. Lattice (Staggered)	1.1 ± 0.2 (Stat. Err.)	—

Comments

Q_7 ✓

Better with more accurate $\mathrm{Im}\,\mathbf{\Pi}_{LR}(t)$
data around 2 GeV^2

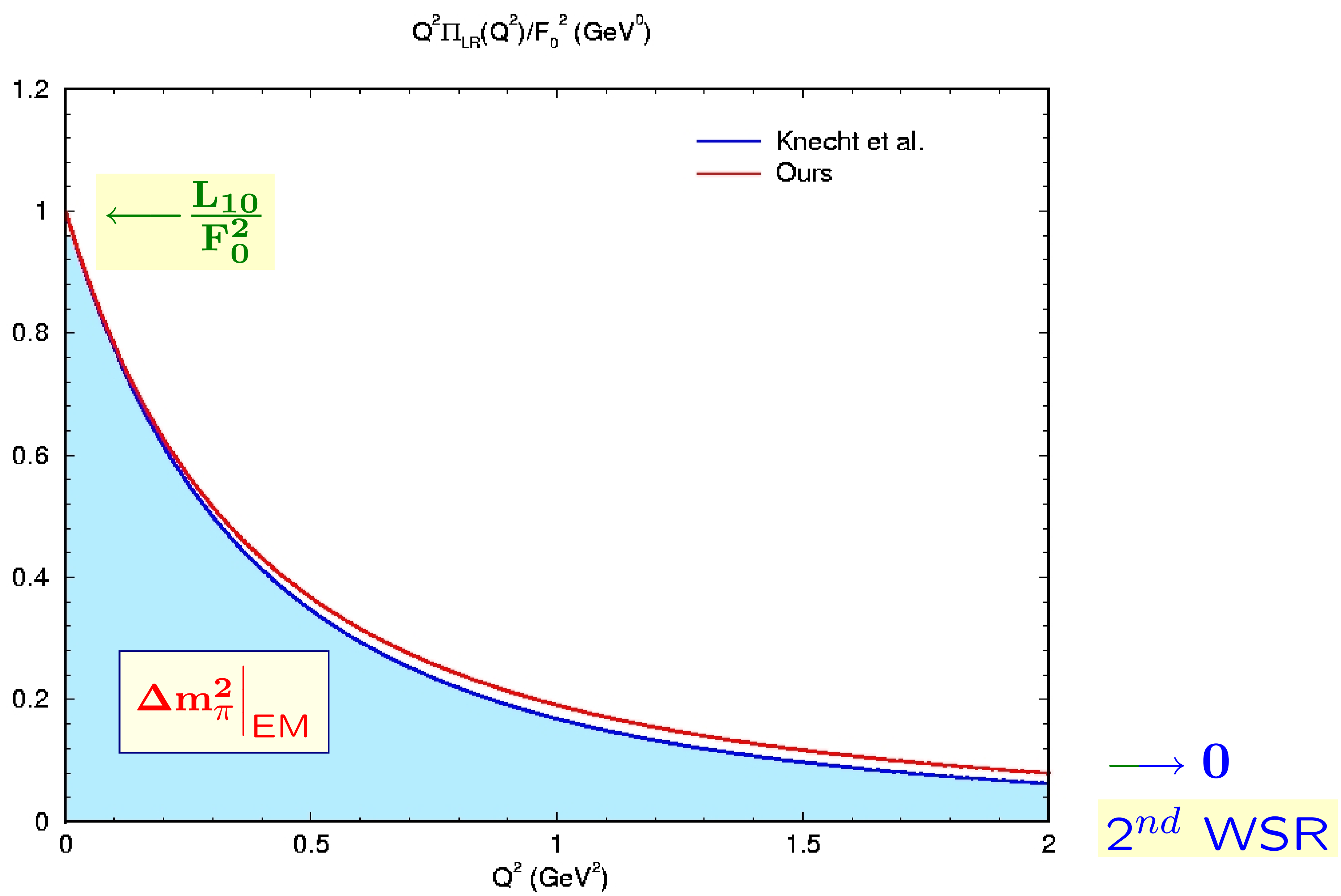
Q_8 More problematic

Better with (again) accurate $\mathrm{Im}\,\mathbf{\Pi}_{LR}(t)$
data around 2 GeV^2

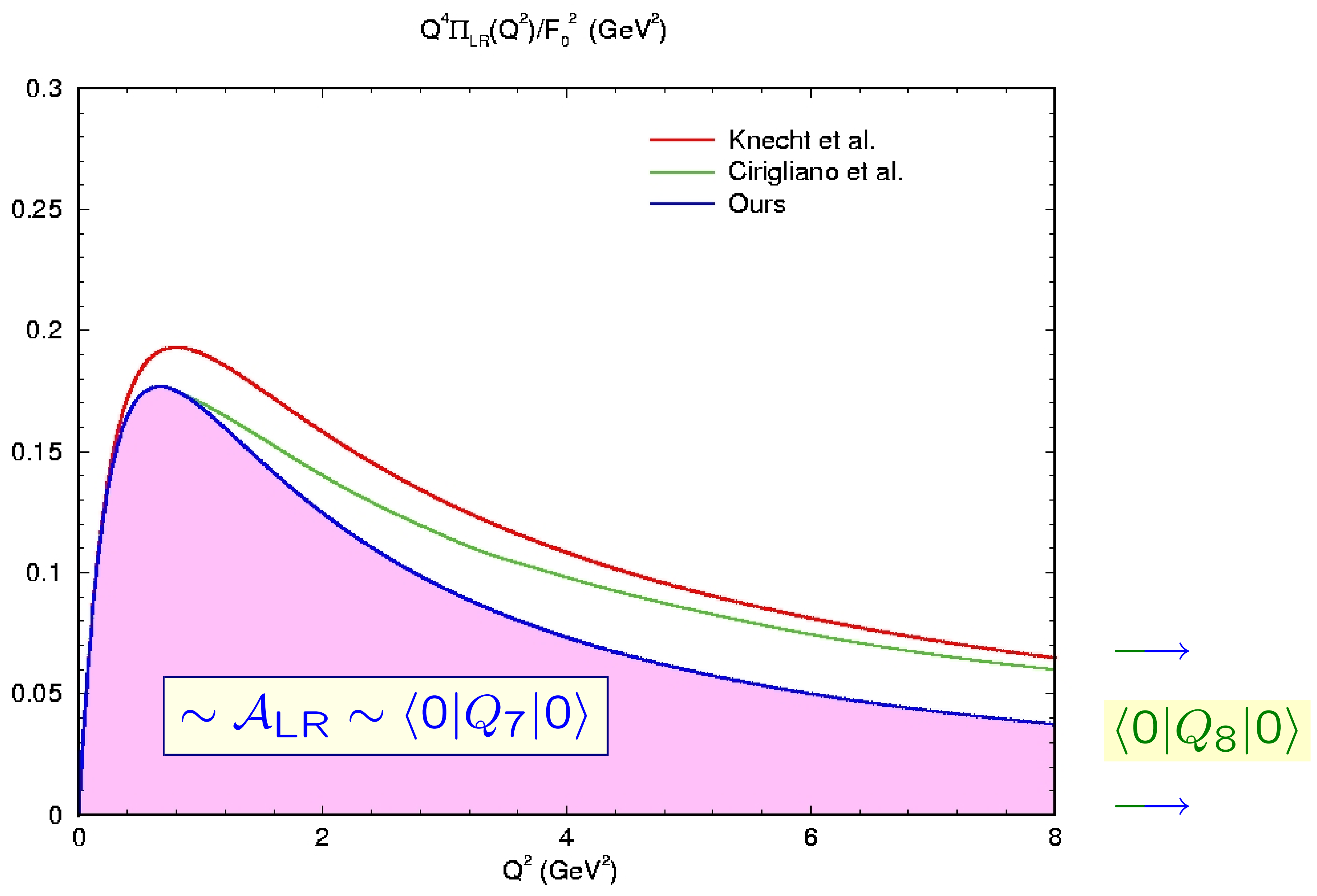
$\Rightarrow$ M_2 with smaller uncertainty

$\mathrm{Im}\,\mathbf{\Pi}_{VV}(t)$ is already good !

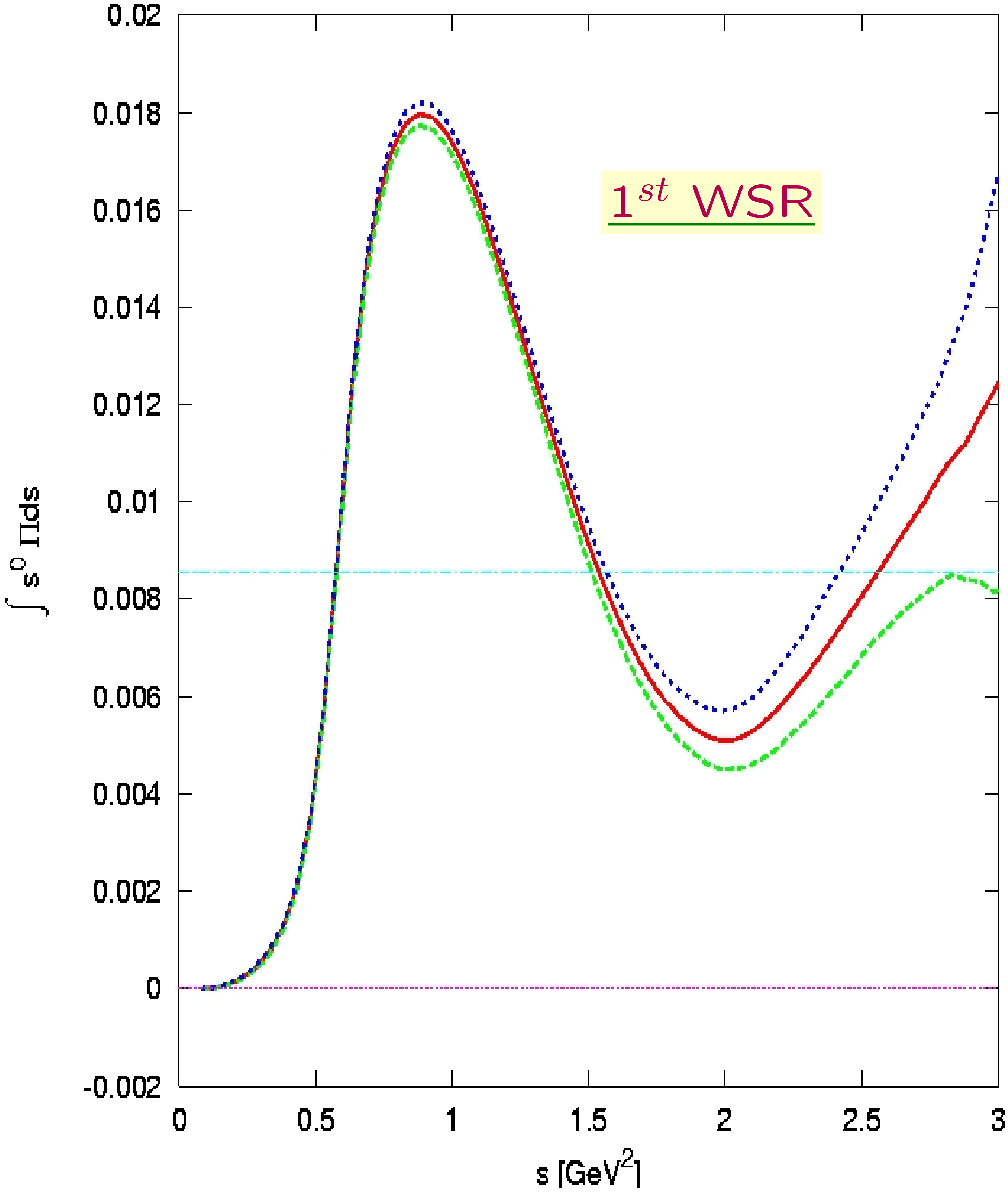
(ρ' peak $\sim$ 2 GeV^2)



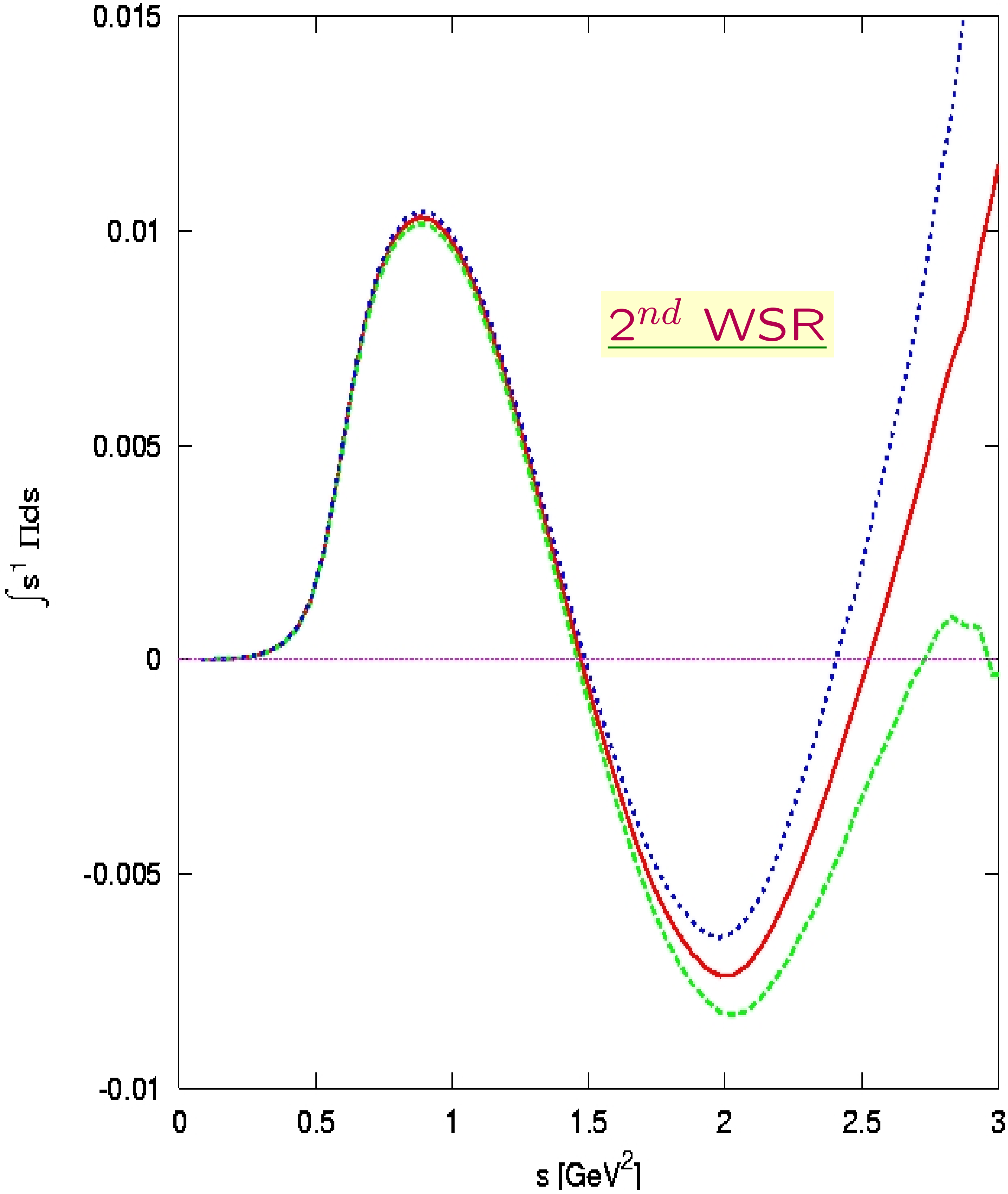
$Q^2\Pi_{LR}(Q^2)/F_0^{\,2}$ (GeV^0)
Knecht et al.
Ours
$\longleftarrow \frac{L_{10}}{F_0^2}$
$\Delta m_\pi^2\big|_{EM}$
$\longrightarrow 0$
2^{nd} WSR
Q^2 (GeV^2)



$Q^4\Pi_{LR}(Q^2)/F_0^2$ (GeV^2)
Knecht et al.
Cirigliano et al.
Ours
0.3
0.25
0.2
0.15
0.1
0.05
0
0
2
4
6
8
Q^2 (GeV^2)
$\sim \mathcal{A}_{LR} \sim \langle 0|Q_7|0\rangle$
$\langle 0|Q_8|0\rangle$



ALEPH Tau Data



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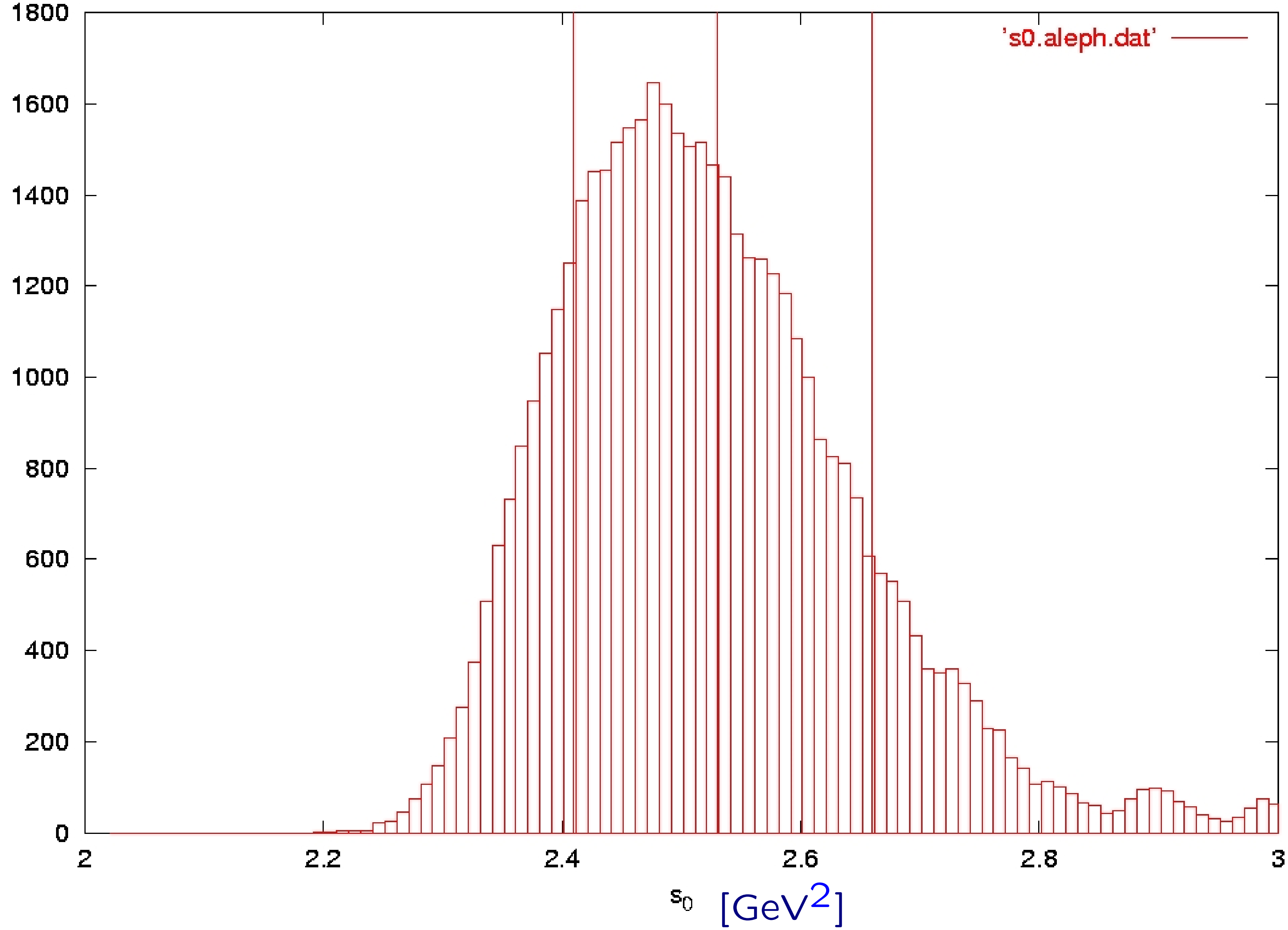
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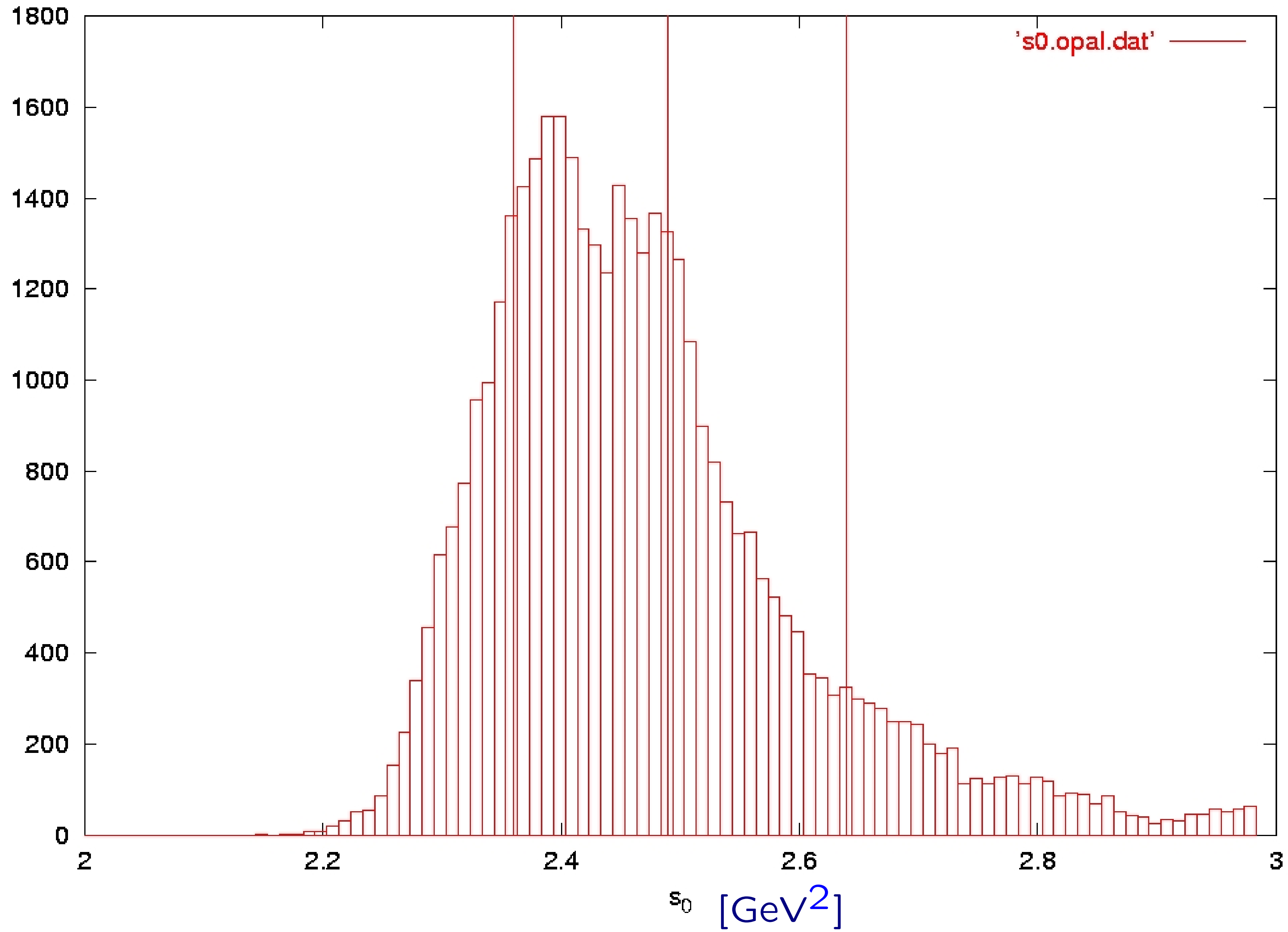
3
s0 [GeV2]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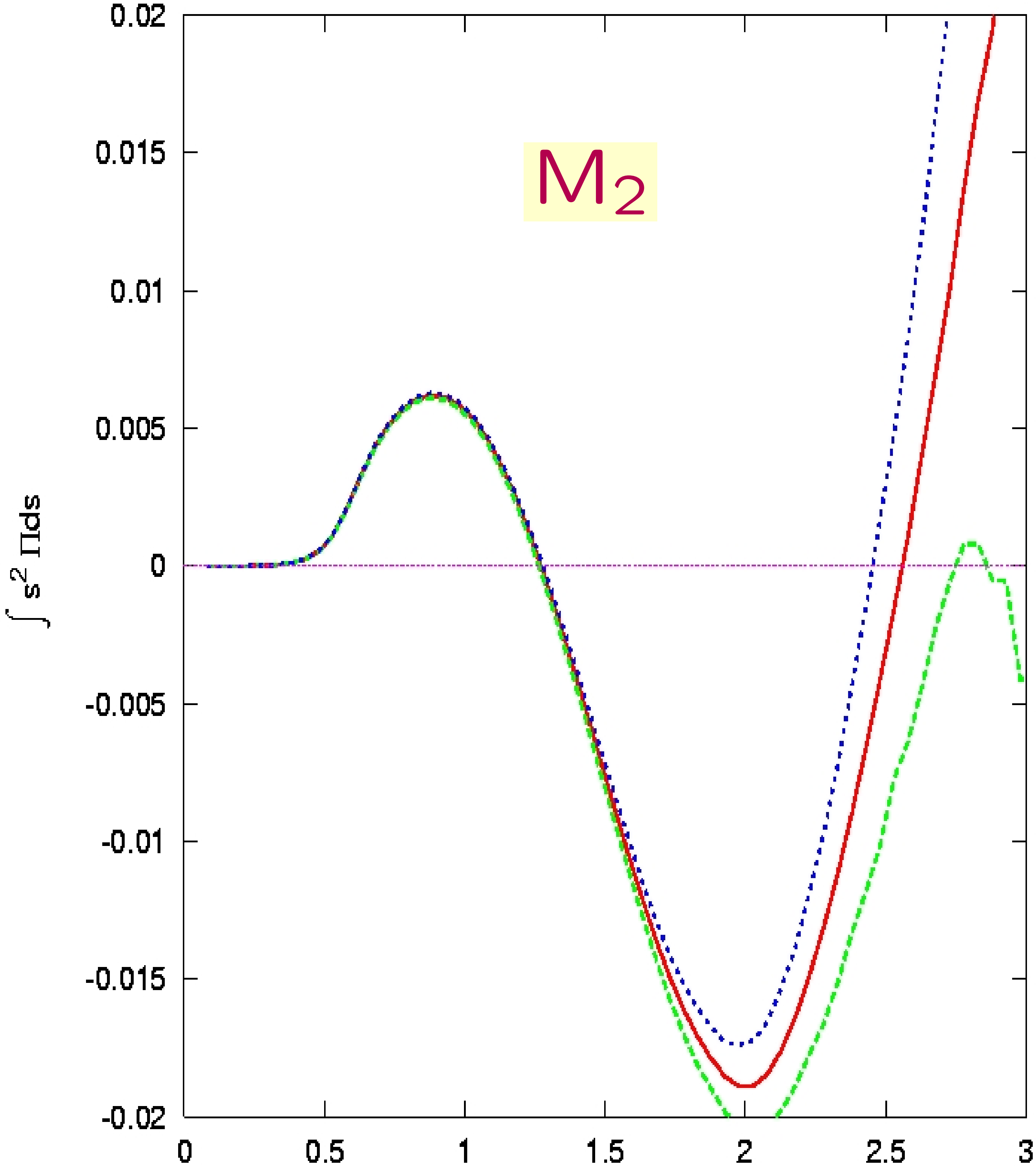
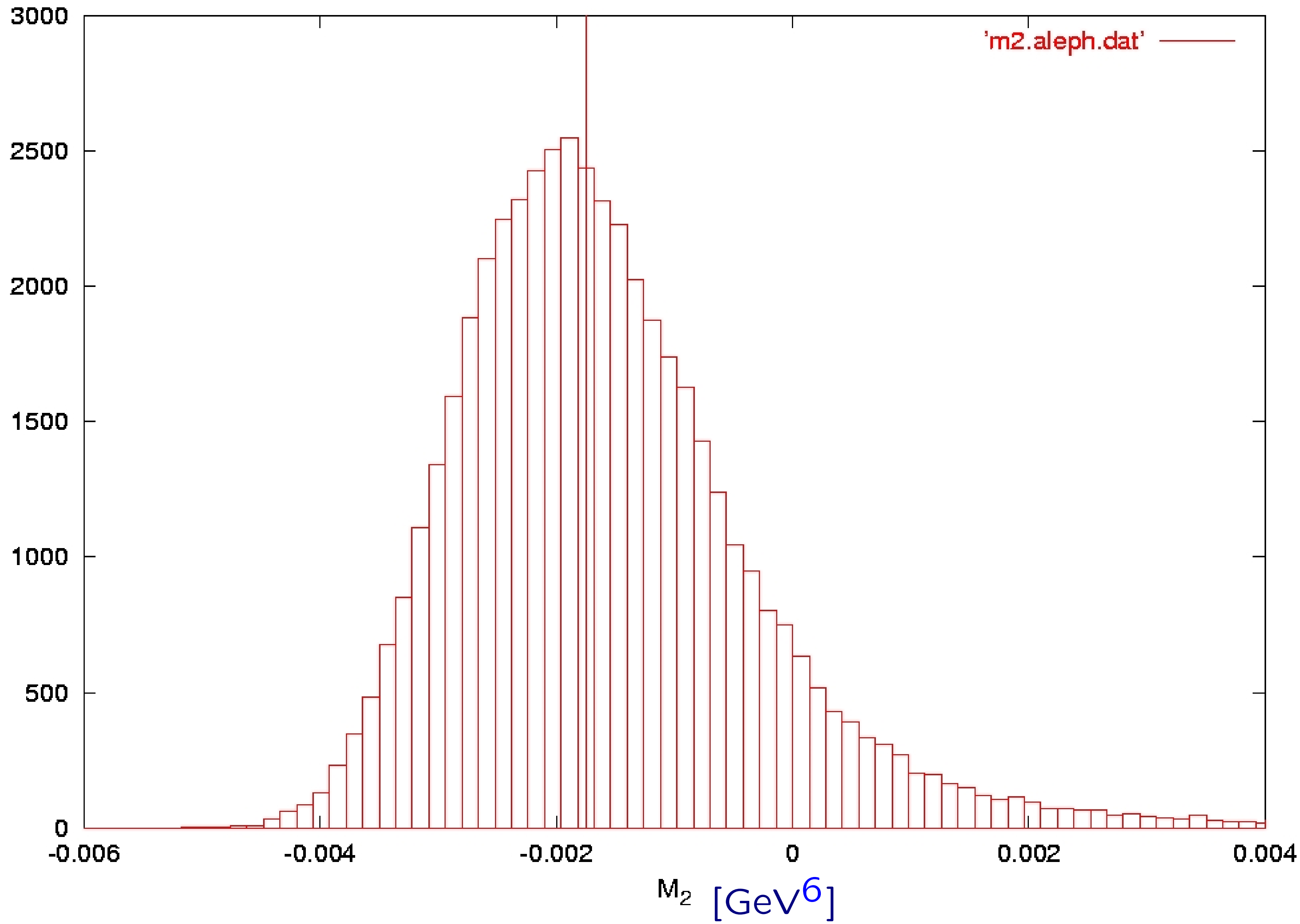


ALEPH Tau Data



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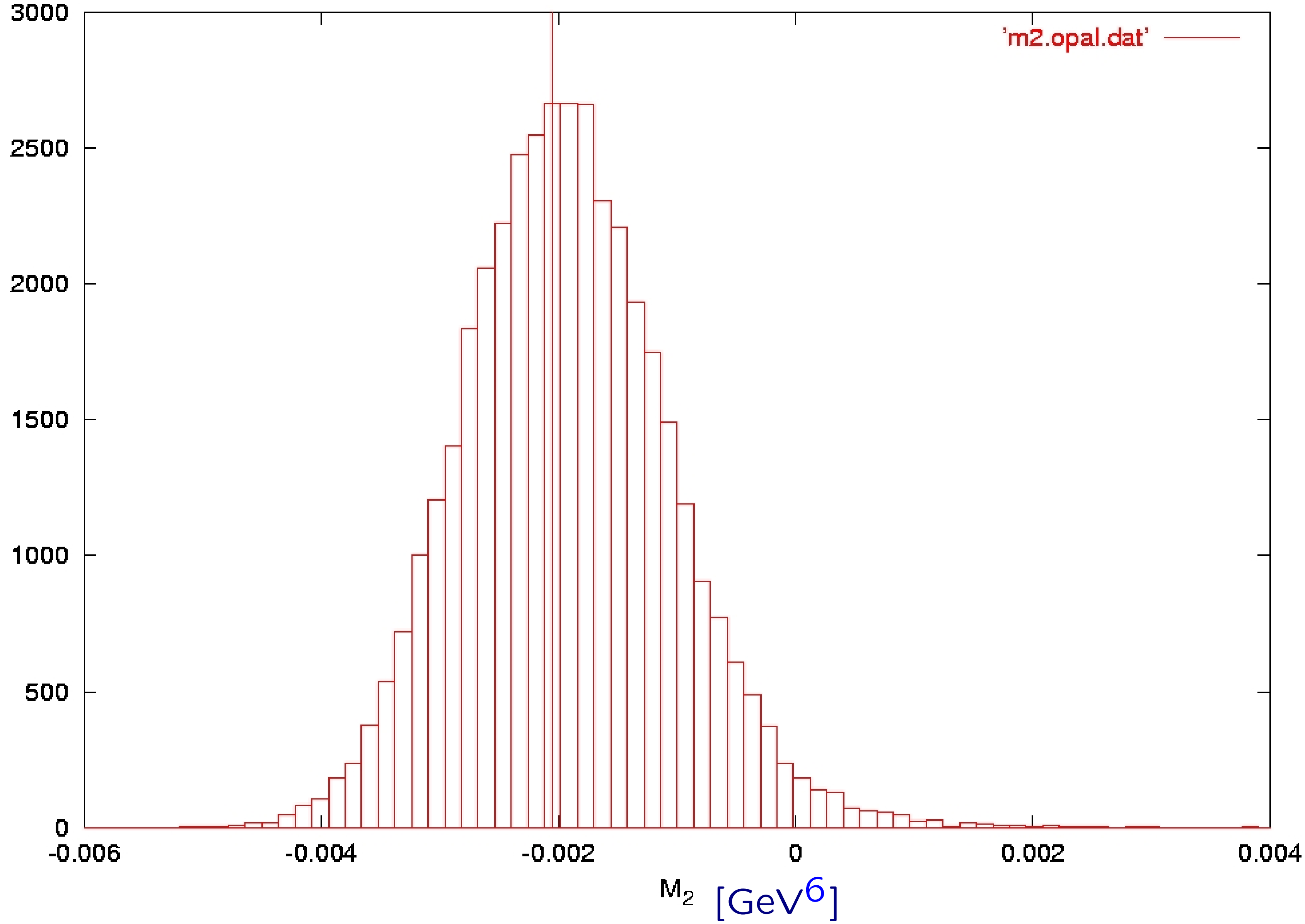
0.004
M2 [GeV6]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M2 [GeV6]