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ε'/ε in the **Standard Model**

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En. scale	Fields	Eff. Theory
M_z	$\boxed{\begin{array}{c}\mathrm{W},\mathrm{Z},\gamma,g,\\ \tau,\mu,e,\nu_i,\\ t,b,c,s..\end{array}}$	SM + ...
	$\Big\downarrow$ OPE	
$\lesssim m_c$	$\boxed{\begin{array}{c}\gamma,g,\mu,e,\\ \nu_i,s,d,u,\end{array}}$	$\mathcal{L}_{QCD}^{(n_f=3)}$
	$\Big\downarrow$ $\mathrm{N}_C \to \infty$	
M_K	$\boxed{\begin{array}{c}\gamma,\mu,e,\nu_i,\\ \pi,K,\eta,..\end{array}}$	ChPT

$$\mathcal{L}_{eff}^{\Delta S=1} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^{*} \sum_i C_i(\mu) Q_i(\mu)$$

The Message

Large $\mathbb{N}_C + CHPT$ give a precise result for ε'/ε once all large logs are included :

- Scale difference $M_Z \leftrightarrow M_{K^0}$:

  √ $Q_{6,8}$ receive main corrections and are well estimated in large-N_C as
  √ Matching S.–L. D. is exact

- Chiral logs $\leftrightarrow$ FSI !!

  ⋇ π-πinteraction with CHPT (1–loop) and Omnès (h.o.)

Initial conditions

Tree level, penguin and electropenguin diagrams at the M_Z scale..

$$
\begin{aligned}
Q_1 &= (\bar{s}_\alpha u_\beta)_{\mathrm{V-A}}\,(\bar{u}_\beta d_\alpha)_{\mathrm{V-A}}\;, \\
Q_2 &= (\bar{s}u)_{\mathrm{V-A}}\,(\bar{u}d)_{\mathrm{V-A}}\;, \\
Q_{3,5} &= (\bar{s}d)_{\mathrm{V-A}}\textstyle\sum_q (\bar{q}q)_{\mathrm{V\mp A}}\;, \\
Q_4 &= \textstyle\sum_q (\bar{s}q)_{\mathrm{V-A}}\,(\bar{q}d)_{\mathrm{V-A}}\,, \\
Q_6 &= -2\textstyle\sum_q (\bar{s}q)_{\mathrm{S+P}}\,(\bar{q}d)_{\mathrm{S-P}}\,, \\
Q_{7,9} &= \tfrac{3}{2}(\bar{s}d)_{\mathrm{V-A}}\textstyle\sum_q e_q\,(\bar{q}q)_{\mathrm{V\pm A}}\;, \\
Q_8 &= -3\textstyle\sum_q e_q\,(\bar{s}q)_{\mathrm{S+P}}\,(\bar{q}d)_{\mathrm{S-P}}\,, \\
Q_{10} &= -3\textstyle\sum_q e_q\,(\bar{s}q)_{\mathrm{V-A}}\,(\bar{q}d)_{\mathrm{V-A}}\,.
\end{aligned}
$$

Renormalization group eq.

Large UV logs are generated in the passage $M_Z \to M_{K^0}$.

$$\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g}\vec{C}(M_Z^2/\mu^2, g^2) \;=\; \hat{\gamma}^T\vec{C}(M_Z^2/\mu^2, g^2)$$

$$\vec{C}(\mu) = \hat{U}(\mu, M_Z)\vec{C}(M_Z)$$

...looking at the ADM, in the limit $N_c \to \infty$

$$(\hat{\gamma}_s^{(0)})_{66,88} \;=\; -6N_c$$

...mmmmmmmm...

Large $\mathbb{N}_C$ Expansion

At leading (non–trivial) order

- $\circledast$ All ingredients are computed $\surd$
- $\circledast$ Matching S.–L. D. is exact $\surd$

... but care of large logs!!

- ⁜ *Scale difference* $M_Z \leftrightarrow M_{K^0}$
- ⁜ *Chiral logs* $\leftrightarrow$ *FSI !!*

At lowest order ..

$$\mathcal{L}_2^{\Delta S=1} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \Big\{ g_8 \, \langle \lambda L_\mu L^\mu \rangle +$$

$$g_{27} \, \left(L_{\mu 23} L_{11}^\mu + \frac{2}{3} L_{\mu 21} L_{13}^\mu \right) +$$

$$e^2 f^6 g_8 g_{ew} \, \langle \lambda U^\dagger \mathcal{Q} U \rangle \Big\} + \text{h.c.}$$

where $L_\mu = -i f^2 U^\dagger D_\mu U$ and then ..

$$\mathcal{L}_4^{\Delta S=1} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \, f^2 \Big(g_8 \sum_i E_i \, O_i^8$$

$$+ g_{27} \sum_i D_i \, O_i^{27}$$

$$+ g_8 \, e^2 f^2 \sum_i Z_i \, O_i^{EW} \Big)$$

Matching with large $\mathbb{N}_C$

Factorization (J=colour singlet):

$$\langle J \cdot J \rangle \;=\; \langle J \rangle \langle J \rangle \left\{ 1 + \mathcal{O}(1/\mathbb{N}_C) \right\}$$

.. *in practice* ..

- For $i \neq 6,\; 8$,
  $Q_i \sim (V - A) \otimes (V \pm A)$
  $\rightsquigarrow$ $\langle J \rangle$ is physical observable

- $Q_{6,8} \sim (S + P) \otimes (S - P)$
  $\rightsquigarrow$ Now $\langle mJ \rangle$ is physical observable!!
  $\Rrightarrow \;\; J \sim m^{-1}$ and $C \sim m^2$

Matching with large $\mathbb{N}_C$

$$g_8{}^{\infty} = \left[\frac{3}{5}C_2 - \frac{2}{5}C_1 + C_4\right] - 16\frac{L_5}{f_\pi^2}B^2C_6$$

$$g_{27}{}^{\infty} = \frac{3}{5}\left(C_2 + C_1\right)$$

$$e^2(g_8 g_{EM})^{\infty} = -3\frac{B^2}{f_\pi^2}C_8$$

Does all this work well?

$$\mathrm{Re}\,\mathrm{g}_8{}^{\infty} \sim \frac{3}{5}C_2$$

$$\mathrm{Im}\,\mathrm{g}_8{}^{\infty} \sim -16\frac{L_5}{f_\pi^2}B^2C_6$$

Amplitudes at large $\mathbb{N}_C$

Including $\mathcal{O}(p^4)$ in χPT,

$$g_8{}^{\infty}\ \left[1+\Delta_C\mathcal{A}_0{}^{(8)}\right]^{\infty} = g_8{}^{\infty}\, f_0^{K\pi}(M_\pi^2)$$

$$e^2\, g_8{}^{\infty}\ \left[g_{ew}+\Delta_C\mathcal{A}_2{}^{(ew)}\right]^{\infty} = (e^2\, g_8 g_{ew})^{\infty}$$

$$\times\left[1+\frac{4L_5}{f_\pi^2}M_\pi^2\right]+\ldots$$

where $f_0^{K\pi}(M_\pi^2)$ is a L.C. of $f_\pm^{K\pi}(M_\pi^2)$,

$$\langle\pi|\bar{s}\gamma^\mu q|K\rangle = C_{K\pi}\ \left\{P_+^\mu\ f_+^{K\pi}(t)+P_-^\mu\ f_-^{K\pi}(t)\right\}.$$

All these leads to Re $\varepsilon'/\varepsilon \approx (0.5-0.8)10^{-3}$

Isospin Amplitudes

$$A[K^0 \to \pi^+\pi^-] \;\equiv\; \mathcal{A}_0 + \frac{1}{\sqrt{2}}\,\mathcal{A}_2\,,$$

$$A[K^0 \to \pi^0\pi^0] \;\equiv\; \mathcal{A}_0 - \sqrt{2}\,\mathcal{A}_2$$

where $\mathcal{A}_I \equiv A_I \; \exp\left\{i\delta_0^I\right\}$ and

$$\left(\delta_0^0 - \delta_0^2\right)\left(M_K^2\right) = 45^\circ \pm 6^\circ$$

Phases are born at 1–loop in χPT

Big phases $\leftrightarrow$ Big 1–loop corrections!!

$$\frac{\varepsilon'}{\varepsilon} = \; e^{i\Phi}\,\frac{\omega}{\sqrt{2}|\varepsilon|}\;\left[\frac{\mathrm{Im} A_2}{\mathrm{Re} A_2} - \frac{\mathrm{Im} A_0}{\mathrm{Re} A_0}\right]\;.$$

Chiral loop corrections

- At tree level no phase is present!

- Loop effects are NLO in $1/N_c$

$$\boxed{\mathcal{A}_I^{(R)} = \mathcal{A}_I^{(R)\infty} \times \mathcal{C}_I^{(R)}}$$

$$\mathcal{C}_I^{(R)} \approx 1 + \Delta_L \mathcal{A}_I^{(R)}$$

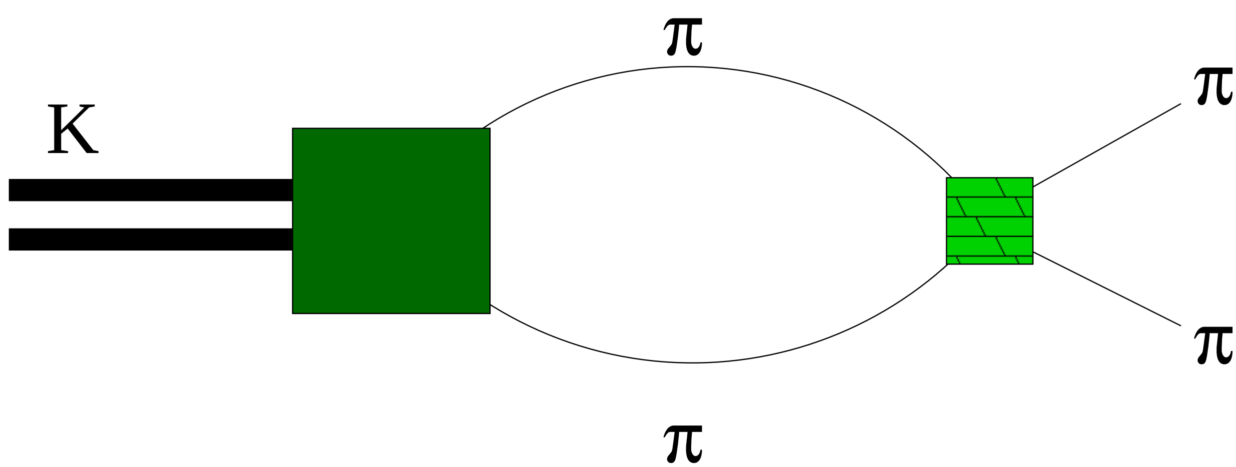
$$\mathcal{C}_0^{(8)} = 1.27 \pm 0.05 + 0.46\ i$$

$$\mathcal{C}_2^{(27)} = 0.96 \pm 0.05 - 0.20\ i$$

$$\mathcal{C}_2^{(ew)} = 0.50 \pm 0.24 - 0.20\ i$$

- All absorptive contributions are from $\pi\pi$ scattering, i.e. the depend only on the final isospin of the outgoing pions.

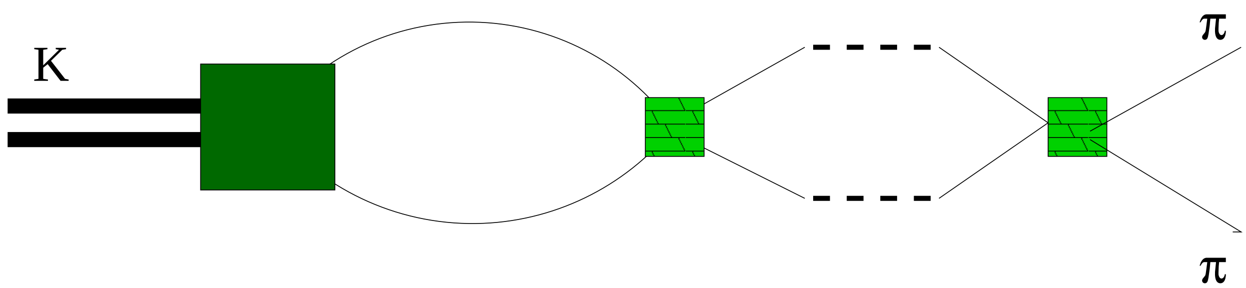
Final State Interactions



Rescattering of π's with $I = 0$ provide a $\sim$ 40% correction already at one loop!! because of $\ln m_\pi^2$.

What about higher orders? Omnés approach allows a partial resummation of all these effects

Final State Interactions



$A_I(s)$ analitic, with a cut $[4m_\pi^2, \infty)$ for $s > 0$ and real, using Cauchy and Watson theorems...etc....

$$A_I(s) = A_I(s_0)\Omega_I(s, s_0) = A_I(s_0)\Re_I(s, s_0)e^{i\delta_I(s)}$$

$$\Omega_I(s, s_0) = \exp\left\{\frac{s - s_0}{\pi}\int_{4m_\pi^2} \frac{dz}{z - s_0}\frac{\delta_I(s)}{z - s - i\epsilon}\right\}$$

$\Rightarrow$ Dependence on s_0 cancels in the product

$\Rightarrow \lim_{s_0 \to s} \Omega_I(s, s_0) = 1$

Final State Interactions

The case of Kaons..

$$\mathcal{C}_I^{(R)} \equiv \mathcal{C}_I^{(R)}(M_K^2) = \Omega_I(M_K^2, s_0)\, \mathcal{C}_I^{(R)}(s_0)$$

To reproduce the $\mathcal{O}(p^4)$ χPT results

$$\begin{aligned} \mathcal{C}_I^{(R)}(s_0) &= \mathcal{C}_I^{(R)} \left[1 - \delta\Omega_I(M_K^2, s_0)\right] \\ &\approx 1 + \Delta_L \mathcal{A}_I^{(R)} - \delta\Omega_I(M_K^2, s_0) \end{aligned}$$

Table 1: Resummed loop corrections with one subtraction and $\bar{z} = 1$ GeV^2.

s_0	$\lvert\mathcal{C}_0^{(8)}\rvert$	$\lvert\mathcal{C}_0^{(27)}\rvert$	$\lvert\mathcal{C}_0^{(ew)}\rvert$	$\lvert\mathcal{C}_2^{(27)}\rvert$	$\lvert\mathcal{C}_2^{(ew)}\rvert$
0	1.37	2.47	1.38	1.06	0.62
M_π^2	1.36	2.42	1.37	1.05	0.61
$2M_\pi^2$	1.35	2.36	1.36	1.05	0.60
$3M_\pi^2$	1.33	2.28	1.34	1.04	0.59

The Omnès analalysis confirms the one–loop results!!

$$\left|\mathcal{C}_0^{(8)}\right| = \Re_0(M_K^2, s_0)\, \mathcal{C}_0^{(8)}(s_0) = 1.31 \pm 0.06\,,$$

$$\left|\mathcal{C}_0^{(27)}\right| = \Re_0(M_K^2, s_0)\, \mathcal{C}_0^{(27)}(s_0) = 2.4 \pm 0.1\,,$$

$$\left|\mathcal{C}_0^{(ew)}\right| = \Re_0(M_K^2, s_0)\, \mathcal{C}_0^{(ew)}(s_0) = 1.31 \pm 0.07\,,$$

$$\left|\mathcal{C}_2^{(27)}\right| = \Re_2(M_K^2, s_0)\, \mathcal{C}_2^{(27)}(s_0) = 1.05 \pm 0.05\,.$$

$$\left|\mathcal{C}_2^{(ew)}\right| = \Re_2(M_K^2, s_0)\, \mathcal{C}_2^{(ew)}(s_0) = 0.62 \pm 0.05\,.$$

$$\mathrm{Re}\frac{\varepsilon'}{\varepsilon} = \frac{e^{i\pi/4}}{\sqrt{2}}\frac{\omega}{\varepsilon\ \mathrm{Re}A_0}\ \left[\frac{\mathrm{Im}A_2}{\omega} - \left(1 - \Omega_{IB}^{\pi_0\eta}\left|\frac{\mathcal{C}_2^{(27)}}{\mathcal{C}_0^{(8)}}\right|\right)\mathrm{Im}A_0\right]$$

Further improvements

★ $\alpha_s(m_\tau) = 0.345 \pm 0.020$

★ $\mathrm{Im}A_{0,2}$, $\varepsilon \propto \mathrm{Im}V_{ts}^* V_{td}$. Using $\varepsilon_{th} \to \hat{B}_K$ explicit and poor dependence on UT $\varepsilon_{th} = \hat{B}_K \mathrm{Im}V_{ts}^* V_{td}\ (21.3 \pm 1.9)$

$$\hat{B}_K|_{\mathbb{N}_c} = 3/4$$

★ $\Omega_{IB}^{\pi_0\eta} = 0.16 \pm 0.03.$

Experimental Status for Re ε'/ε

♢ W.A. $\rightsquigarrow$ $(1.72 \pm 0.18)\ 10^{-3}$

Theoretical Status for Re ε'/ε....

↬ $1/\mathbb{N}_C$+1–loop $\rightsquigarrow$

$$(1.5 \cdot 10^{-3})\ \frac{\mathrm{Im} V_{ts}^* V_{td}}{(1.2 \cdot 10^{-4})} = (1.8 \cdot 10^{-3})$$

↬ $1/\mathbb{N}_C$+FSI $\rightsquigarrow$

$$(1.4 \cdot 10^{-3})\ \frac{\mathrm{Im} V_{ts}^* V_{td}}{(1.2 \cdot 10^{-4})} = (1.6 \cdot 10^{-3})$$

♡ Final result $\rightsquigarrow (1.7 \pm 0.2^{+0.8}_{-0.5} \pm 0.5_{1/\mathbb{N}_c})\ 10^{-3}$

$$\boxed{\mathrm{Re}\ \varepsilon'/\varepsilon = 1.7 \pm 0.9}$$

Conclusions

$\rtimes$ SD + RGE to the 3–flavour theory ($\mu \lesssim m_c$)

$\sharp$ χPT matching+χ–loop corrections

$\varkappa$ Omnès resummation for higher order contributions

$\curlyvee$ SM can well account for the observed value of ε'/ε

$$\boxed{\mathrm{Re}\ \varepsilon'/\varepsilon = 1.7 \pm 0.9}$$