Non-factorizable contributions to $B^0-\bar{B}^0$ mixing.

($B^0-\bar{B}^0$ mixing beyond vacuum saturation)

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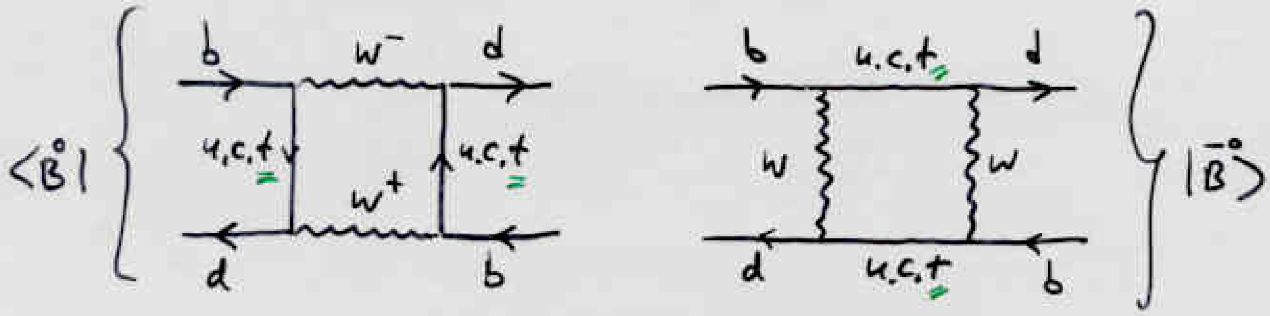
I will discuss nonperturbative QCD effects and corrections to factorization due to these effects.

- Correction to factorization $\sim \langle \alpha_s G G \rangle$ is strictly negative

- give numerical estimate for this correction in the dispersion approach

Based on work with N. Nikitin (Moscow)

- $B-\bar{B}$ mixing in SM:



- box diagrams are responsible for $\Delta B=2$ transition in SM
- all types of QCD radiative corrections
- many different scales involved $(M_W,\ m_B,\ \Lambda_{QCD})$

$$\hat{H}_{eff}^{\Delta B=2} = \frac{G_F^2 M_W^2}{\sqrt{2}} \left(V_{tb}^* V_{td}\right)^2 C(\mu) \cdot \underbrace{\overbrace{\bar{d}\gamma_\sigma(1-\gamma_5)b}^{O_\sigma} \cdot \bar{d}\gamma_\sigma(1-\gamma_5)b \Big|_\mu}_{\text{local 4-quark operator}}$$

Wilson coefficient

- MIXING AMPLITUDE

$$\underline{\langle \bar{B}^\circ | \hat{H}_{eff}^{\Delta B=2} | B^\circ \rangle}$$

$$\langle \bar{B}^\circ | \bar{d}\gamma_\sigma(1-\gamma_5)b \cdot \bar{d}\gamma_\sigma(1-\gamma_5)b | B^\circ \rangle \Big|_\mu =$$

$$= 2 \langle \bar{B}^\circ | O_\sigma \cdot O_\sigma | B^\circ \rangle + 2 \langle \bar{B}^\circ | \overset{O_\sigma}{\underset{O_\sigma}{\quad}} | B^\circ \rangle + O(\alpha_s)$$

$$= \frac{8}{3} \langle \bar{B}^\circ | O_\sigma \cdot O_\sigma | B^\circ \rangle + 4 \langle \bar{B}^\circ | O_\sigma \frac{\lambda^a}{2} \cdot O_\sigma \frac{\lambda^a}{2} | B^\circ \rangle + O(\alpha_s)$$

- vacuum saturation

vacuum saturation, or factorization
$\Rightarrow$

$$\langle \bar{B}^0 | H_{eff} | B^0 \rangle \Big|_{fact} = \frac{8}{3} \frac{G_F^2 M_W^2 M_B^2}{\sqrt{2}} \left(V_{tb}^* V_{td}\right)^2 C(\mu) f_B^2$$

$\langle 0 | \bar{b} \gamma_\mu \gamma_5 d | B \rangle = i f_B p_\mu$

μ-dependent

Definition of B-factor

$$\langle \bar{B}^0 | H_{eff} | B^0 \rangle = \frac{8}{3} \frac{G_F^2 M_W^2 M_B^2}{\sqrt{2}} \left(V_{tb}^* V_{td}\right)^2 C(\mu) f_B^2 \underbrace{B_B(\mu)}$$

$\boxed{1 + \Delta B(\mu)}$

In language of quarks and gluons – factorization = "zero" α_s-order

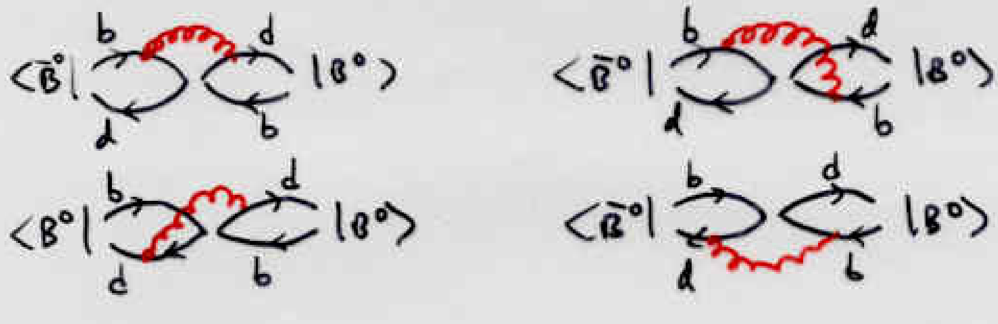
Include α_s-corrections:

'Hard' gluons above μ were taken into account in H_{eff}.
So we need to take into account gluons below μ.

$O(\alpha_s)$: $\langle \bar{B}^0 |$  $| B^0 \rangle + \ldots$

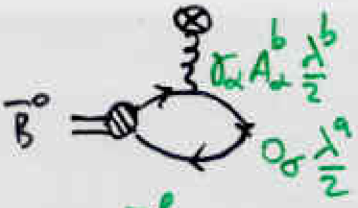
radiative correction to f_B

Corrections to factorization from following diagrams:

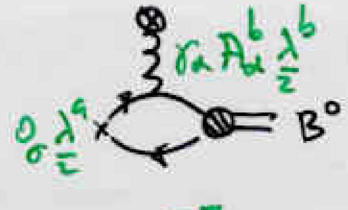


$$A^{(1)} \sim g^2 \int dq\, dq' \, T^{\ell a}_{\sigma\lambda}(p,q)\, T^{r a'}_{\sigma\lambda}(p,q')\, dx\, dx'\, e^{-iqx + iq'x'} \langle A^a_\alpha(x) A^{a'}_\alpha(x') \rangle$$

$\Rightarrow$ $\boxed{\text{Local gluon condensate dominates}}$



T^ℓ



T^r

Important note: For a local condensate

$$A^{(1)} \sim \langle \alpha_s G_{\alpha\beta} G_{\alpha'\beta'} \rangle \, T^{(\ell)}_{\sigma\alpha\beta}(p) \, T^{(r)}_{\sigma\alpha\beta}(p)$$

where we've used fixed-point gauge $x_\mu A^a_\mu = 0$, $\Rightarrow A^a_\mu = \frac{1}{2} x_\beta G^a_{\beta\mu}$

$$T^{(\ell)}_{\sigma\alpha\beta}(p) = F_1 \epsilon_{\alpha\beta\sigma\nu} p^\nu + i F_2 (p_\alpha g_{\sigma\beta} - p_\beta g_{\sigma\alpha})$$

Because of $\hat{C}$-invariance of strong interactions we obtain

$$\boxed{T^{(r)}_{\sigma\alpha\beta}(p) = \overset{*}{T}{}^{(\ell)}_{\sigma\alpha\beta}(p)}$$

We have to be more careful: indeed T^ℓ contains contributions of subprocesses when B and J interact. So as T^r.

$$A^0 \cong \frac{8}{3} M_B^2 f_B^2$$

$$A^{(1)} \simeq -8 C_F M_B^2 \cdot \langle \frac{\alpha_s}{\pi} G G \rangle \pi^2 \cdot \underbrace{\left\{ (F_1^b + F_1^d)^2 + (F_2^b - F_2^d)^2 \right\}}_{\text{positive nonperturb. QCD contribution}} (p)$$

Effect is negative!

$$\boxed{\text{Conclusion: correction to factorization, proportional to } \langle \alpha_s G G \rangle \text{ is NEGATIVE}}$$

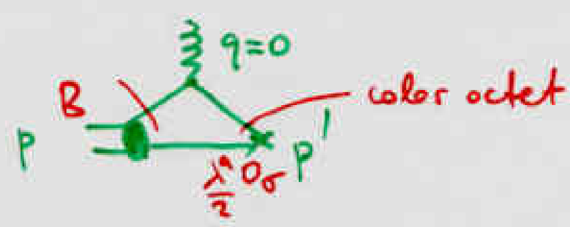
What is known about $B_B(\mu \sim 5\,\mathrm{GeV})$

	B_{B_d}	B_{B_s}	
SR	1.0 ± 0.15		Pich,
	0.95 ± 0.1		Ovchinnikov, Pivovarov, Narison
Lat	$0.92(4) \pm {}^{3}_{0}$	$0.91(2) {}^{+3}_{-0}$	Lellouch, Lin '99
	$0.93(8)$	$0.92(6)$	Bečirević et al '00

All results indeed agree with $\Delta B < 0$

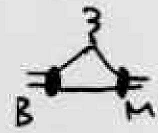
Dispersion approach + constituent quark picture:

- We know numerical parameters from description of SL decays for fermion.



For meson-meson transitions we have

$B \to M$: $f(q^2) = \int ds\, ds'\, \phi_B(s)\, \phi_M(s')\, \Delta(s, s', q^2)$:



Our present quantity is more complicated:

F:

$$F(p^2, p'^2, q^2)\Big|_{\substack{q=0 \\ p^2 = M_B^2 \\ p'^2 = M_B^2}} = \int ds\, \phi_B(s)\, \frac{\lambda^{1/2}(s, m_b^2, m_d^2)}{s}\, D(s, p'^2)$$

$\Rightarrow q^2 = 0,\ p^2, p'^2$

$D(s, p'^2)$ - propagator of a color-octet $b\bar{d}$ state.

We know that
- $D(s, p'^2) \sim \frac{1}{s - p'^2}$ for large $s - p'^2$
- $D(s, p'^2)$ is finite at $s = p'^2$ — no pole

$$D(s, p'^2) = \frac{1}{s - p'^2 + M_0^2}$$

$M_0^2 = O(\Lambda_{QCD} m_Q)$ for $m_Q \to \infty$

$M_0^2 = \omega\, m_b\, m_d$

constit. d-quark mass $\qquad \omega \simeq 1$

$M_0^2 = \omega\, m_d \cdot m_b$, for $\bar{b}d$ system (7)

If we assume for $\bar{d}d$ system the same formula holds, for $D(k^2)$ we find, $\omega \simeq 1$.

$$D(k^2) = 15\ \mathrm{GeV}^2$$

From Lattice gluon propagator $D(k^2=0) \simeq 18\ \mathrm{GeV}^2$, Bonnet et al

quite reasonable, since gluon propagator might be expected to behave similar to $\bar{q}q$ octet state propagator.

If we rely upon this assumption for $\overbrace{D(k^2)}^{\text{form of}}$ and numerical parameters, then we find

$$\boxed{\begin{array}{l} 1+\Delta B_{B_d}(5\ \mathrm{GeV}) \sim 0.94 \pm 0.04 \\ 1+\Delta B_{B_s}(5\ \mathrm{GeV}) \sim 0.95 \pm 0.04 \end{array}}$$

We can then estimate ΔB_K as well

(Notice that $K \to \pi$ ff is well described in our disp. approach)

$$\boxed{\Delta B_K(1\ \mathrm{GeV}) = -0.21 \pm 0.04}$$

Conclusions

- Correction to factorization (= vacuum saturation) can be expressed in terms of the $\langle \alpha_s G G \rangle$. The correction is **NEGATIVE** independent of particular values of the form factors which describe B-meson specific transition into coloured $B\bar{q}$ state.

- The form factors can be calculated in terms of the ϕ_B - meson wave function and propagator of $B\bar{q}$ color-octet state. Assuming that non-perturbative effects in this propagator can be described by a "mass" term, we calculate these corrections:

$$1 + \Delta B_{B_d}(5\,\mathrm{GeV}) = 0.94 \pm 0.04$$

$$1 + \Delta B_{B_s}(5\,\mathrm{GeV}) = 0.95 \pm 0.04$$

$$\Delta B_K(1\,\mathrm{GeV}) = -0.21 \pm 0.04$$