1

A Seesaw-invariant Texture of Lepton Mass Matrices for Leptogenesis & Neutrino Oscillations

Zhi-zhong Xing (IHEP, Beijing)

Motivation: to simultaneously interpret the observed

$\boxed{\text{baryon asymmetry}}$ $Y_B \equiv \frac{n_B - n_{\bar{B}}}{s} \approx (1-10)\times 10^{-11}$

(Olive, Steigman, Walker 2000)

&

$\boxed{\text{neutrino oscillations}}$ Solar: $\Delta m^2_{sun} = (3.3-17)\times 10^{-5}\,eV^2$

$\tan^2\theta_{sun} = 0.30-0.58$ (Smirnov)

Atmospheric: $\Delta m^2_{atm} = (1.6-3.9)\times 10^{-3}\,eV^2$

(Neutrino 02, 90% C.L.) $\sin^2 2\theta_{atm} > 0.92$ (Shiozawa)

- in a minimal extension of the SM (adding 3 right-handed neutrinos).
- Two necessary ingredients: Seesaw : to relate M_ν to M_R.

  Leptogenesis: to translate ε_1 into Y_B.

1. The Ansatz

- A minimal extensition of the SM:

$$-\mathcal{L}_{\rm mass} = \overline{(e\ \mu\ \tau)_L}\, M_l \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_R + \overline{(\nu_e\ \nu_\mu\ \nu_\tau)_L}\, M_D \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}_R + \frac{1}{2}\overline{(\nu_e^c\ \nu_\mu^c\ \nu_\tau^c)_L}\, \underset{\text{higher}}{M_R} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}_R + h.c.$$

$v \doteq 175\,\mathrm{GeV}$

- Seesaw mechanism: $\boxed{M_\nu \approx -M_D \frac{1}{M_R} M_D^T}$ (Gell-Mann, Ramond, Slansky, Yanagida 1979)

Question: Textures of M_D and M_R: from qualitative to quantitative.

- A hint from the texture of quark mass matrices: (Fritzsch, Xing 1995, 1999)

$$M_u = \begin{pmatrix} \circ & \blacktriangle & \circ \\ \blacktriangle & \bullet & \blacktriangledown \\ \circ & \blacktriangledown & \blacksquare \end{pmatrix}, \quad M_d = \begin{pmatrix} \circ & \blacktriangle & \circ \\ \blacktriangle & \bullet & \blacktriangledown \\ \circ & \blacktriangledown & \blacksquare \end{pmatrix} \rightarrow \left|\frac{V_{ub}}{V_{cb}}\right| \approx \sqrt{\frac{m_u}{m_c}}, \quad \left|\frac{V_{td}}{V_{ts}}\right| \approx \sqrt{\frac{m_d}{m_s}}, \text{etc.}$$

- $\boxed{\text{Conjecture}}$: M_D, M_R and M_l might have the same texture as M_u and M_d.
Then, M_ν must have the same texture! — seesaw-invariant texture.
(Fritzsch, Xing 2000)

- Consider

$$\begin{cases} m_e : m_\mu : m_\tau \sim \lambda^6 : \lambda^2 : 1 \\ m_u : m_c : m_t \sim \lambda^8 : \lambda^4 : 1 \\ m_d : m_s : m_b \sim \lambda^4 : \lambda^2 : 1 \end{cases}$$

where $\lambda \approx 0.22$ (Wolfenstein 1983)

③ — We propose:

$$M_D = m_0 \begin{pmatrix} 0 & \hat{\lambda}^3 & 0 \\ \hat{\lambda}^3 & x\hat{\lambda}^2 & \hat{\lambda}^2 \\ 0 & \hat{\lambda}^2 & e^{i\delta} \end{pmatrix} , \quad M_l = m_\tau \begin{pmatrix} 0 & \lambda^4 & 0 \\ \lambda^4 & y\lambda^2 & \lambda^3 \\ 0 & \lambda^3 & 1 \end{pmatrix}$$

where $\hat{\lambda} \equiv \lambda e^{i\omega}$ and $x, y \sim O(1)$, and $m_0 \approx v$ holds. — Non-SO(10)!

[Buchmüller & Wyler 2001: SO(10) relation $M_D = M_u$ and $M_l = M_d$ leads to

$$\sqrt{\frac{m_u}{m_c}} \approx \frac{(1+\tan^2\theta_{atm})^3}{|\tan^2\theta_{sun} - \cot^2\theta_{sun}|} \cdot \frac{\Delta m^2_{sun}}{\Delta m^2_{atm}} \longrightarrow \underbrace{0.04 \sim 0.08}_{\text{conflict}} \approx \underline{>0.1} \ ! \ ]$$

— Then the hierarchy of M_R can well be constrained by the requirement of a sufficiently large mixing angle in the $\underline{\nu_\mu - \nu_\tau}$ sector of M_ν.

$$M_R = M_0 \begin{pmatrix} 0 & \lambda^5 & 0 \\ \lambda^5 & z\lambda^4 & \lambda^4 \\ 0 & \lambda^4 & 1 \end{pmatrix} , \quad M_\nu = \frac{v^2}{M_0} \begin{pmatrix} 0 & \hat{\lambda} & 0 \\ \hat{\lambda} & z' & 1 \\ 0 & 1 & e^{i2\varphi} \end{pmatrix}$$

where $z \sim O(1)$, $z' \equiv 2x - ze^{i\omega}$ with $|z'| \sim O(1)$, and $2\varphi \equiv 2\delta - 5\omega$.

— To get a large mixing angle in the $\underline{\nu_e - \nu_\mu}$ sector of M_ν, the condition $|z' e^{i2\varphi} - 1| \equiv \delta \sim O(\lambda)$ must be satisfied (Vissani 1998).

— $\boxed{\text{Comments}}$: ① the ansatz only has 5 unknown parameters $(x, z, \omega, \delta, M_0)$;
② No fine-tuning is needed to satisfy relevant conditions;
③ A particularly interesting parameter space is

$$x = \frac{1}{\sqrt{2}} , \quad z = 1 + \sqrt{2}\lambda , \quad \delta = -\omega = \frac{\pi}{4} \longrightarrow \delta = \sqrt{2}\lambda , \quad \arg(z' e^{i2\varphi} - 1) = \frac{\pi}{2} \ !$$

2. Neutrino Mixing

- The flavor mixing matrix: $V^{\dagger} M_{\nu} V^{*} = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}$,

contribution from M_1 is negligible due to its strong hierarchy.

where
$$V \simeq \begin{pmatrix} c_x e^{i(\alpha-\gamma+\pi/2)} & s_x e^{i(\alpha-\gamma)} & s_z e^{i\alpha} \\ -s_x c_y e^{i(\beta+\gamma+\pi/2)} & c_x c_y e^{i(\beta+\gamma)} & s_y e^{i\beta} \\ s_x s_y e^{i(\varsigma+\gamma+\pi/2)} & -c_x s_y e^{i(\varsigma+\gamma)} & c_y e^{i\varsigma} \end{pmatrix}$$

with $\alpha \equiv \omega + \varsigma$, $\beta \equiv 5\omega - \varsigma$ and $2\gamma \equiv \arg(z' e^{i2\varphi} - 1)$.

- Explicit results:
($\delta = \sqrt{2}\lambda$)

① mixing angles
$$\begin{cases} \theta_x \simeq \frac{1}{2}\arctan\left(2\sqrt{2}\frac{\lambda}{\delta}\right) \simeq 31.7^\circ \\ \theta_y \simeq \frac{1}{2}\arctan\left(\frac{2}{\delta}\right) \simeq 40.6^\circ \\ \theta_z \simeq \frac{1}{2}\arctan\left(\frac{\lambda}{\sqrt{2}}\right) \simeq 4.4^\circ \end{cases}$$

$$\sin^2 2\theta_{sun} \simeq \frac{8\lambda^2}{8\lambda^2+\delta^2} \simeq 0.8$$
$$\sin^2 2\theta_{atm} \simeq \frac{4}{4+\delta^2} \simeq 0.98$$
$$\sin^2 2\theta_{CHOOZ} \simeq \frac{\lambda^2}{2} \simeq 0.024$$

② neutrino masses
($m_3 \simeq \sqrt{\Delta m^2_{atm}} \simeq (4.0 \sim 6.2)\times 10^{-2}\,\mathrm{eV}$)
$$\begin{cases} m_1 \simeq \left(\frac{\lambda}{2\sqrt{2}}\tan\theta_x\right) m_3 \simeq (1.9 \sim 3.0)\times 10^{-3}\,\mathrm{eV} \\ m_2 \simeq \left(\frac{\lambda}{2\sqrt{2}}\cot\theta_x\right) m_3 \simeq (5.0 \sim 7.8)\times 10^{-3}\,\mathrm{eV} \\ m_3 \simeq 2\frac{v^2}{M_0} \end{cases}$$
$$\longrightarrow \quad M_0 \simeq 2\frac{v^2}{m_3} \simeq (4.9 \sim 7.6)\times 10^{14}\,\mathrm{GeV}$$

③ $R \equiv \frac{\Delta m^2_{sun}}{\Delta m^2_{atm}} \simeq \frac{\delta}{16}\sqrt{8\lambda^2+\delta^2} \simeq 1.4\times 10^{-2}$ for solar vs atmospheric neutrinos

④ neutrinoless double-β decay: $\langle m \rangle_{ee} \equiv \sum_{k=1}^{3}(m_k V_{ek}^2) \simeq \frac{\lambda^2 m_3}{\sqrt{8\lambda^2+\delta^2}} \simeq (2.8-4.3)\times 10^{-3}\,\mathrm{eV}$

⑤ CP Violation in neutrino oscillations: $J \equiv |\mathrm{Im}(V_{e2} V_{\mu 3} V_{e3}^{*} V_{\mu 2}^{*})| \simeq \frac{\lambda^2 \sin 2\gamma}{4\sqrt{8\lambda^2+\delta^2}} \simeq 2\%$

3. Baryon Asymmetry

- Diagonalization: $U^{\dagger} M_R U^{*} = \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{pmatrix}$ using $U \simeq \begin{pmatrix} i & \frac{\lambda}{z} & 0 \\ -i\frac{\lambda}{z} & 1 & \lambda^4 \\ i\frac{\lambda^5}{z} & -\lambda^4 & 1 \end{pmatrix}$,

we obtain: $M_1 \simeq \frac{\lambda^6}{z} M_0$, $M_2 \simeq z\lambda^4 M_0$, $M_3 \simeq M_0$. Hierarchy!

(typically) $\simeq 5.2\times10^{10}$ GeV, $\simeq 1.8\times10^{12}$ GeV, $\simeq 6.0\times10^{14}$ GeV.

- Lepton asymmetry: the interference between tree-level and one-loop amplitude of the decay of the lightest heavy Majorana neutrino with mass M_1.

$$\varepsilon_1 \simeq -\frac{3}{16\pi v^2}\cdot\frac{M_1}{[U^T M_D^{\dagger} M_D U^*]_{11}} \sum_{j=2}^{3} \frac{\mathrm{Im}\left([U^T M_D^{\dagger} M_D U^*]_{1j}\right)^2}{M_j}$$

(Fukugita, Yanagida 1986) Leptogenesis

$$\simeq -\frac{\lambda^6}{4\pi}\left(1 - \frac{23\sqrt{2}\lambda}{12} + \frac{67\lambda^2}{18}\right) \simeq -5.2\times10^{-6}$$

- Wash-out (Kolb, Turner 1990): $\kappa \simeq \frac{0.3}{K}\cdot\frac{1}{(\ln K)^{0.6}} \simeq 1.7\times10^{-3}$, where

$$K \equiv \frac{[U^T M_D^{\dagger} M_D U^*]_{11}}{8\pi v^2}\cdot\frac{M_{Pl}}{1.66\sqrt{g_*}\,M_1} \simeq \frac{3-\sqrt{2}\lambda+6\lambda^2}{16\pi}\cdot\frac{M_{Pl}}{1.66\sqrt{g_*}\,M_0} \simeq 73.$$

- Baryon asymmetry: (Kuzmin, Rubakov, Shaposhnikov 1985) $Y_B = \frac{c\kappa\varepsilon_1}{g_*} \simeq 4.7\times10^{-11}$ where $\begin{cases} c = -8/15 \\ g_* \simeq 100 \end{cases}$

Conclusion: ★ A "complete" Ansatz for neutrino mixing and "thermal Leptogenesis" — a non-SO(10) modification of the Buchmüller-Wyler model. [to appear in PRD]