Unitarity Triangles and the Search for New Physics

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B-factories will provide a wealth of experimental data on B-Physics and CP violation

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Provide a stringent test of one of the experimentally least constrained aspects of the SM, namely the Flavour sector and the Kobayashi-Maskawa mechanism of CP violation

So far, all (laboratory) experimental data on flavour physics and CP violation are in agreement with the Standard Model and its KM mechanism of CP violation.

Motivation for considering Physics Beyond the SM, in particular New Sources of CP violation.

i) By now, it has been established that the strength of CP violation in the SM is <u>not sufficient</u> to generate the observed Baryon Asymmetry of the Universe (BAU), thus suggesting the need for new sources of CP violation.

(ii) Almost all extensions of the SM, including supersymmetric extensions, have new sources of CP violation

(iii) CP violation is closely connected to the most intriging sector of the SM, namely, Yukawa couplings, Higgs sector

Consider the charged weak current in its **most general form:**

$$\begin{bmatrix}\bar{u} & \bar{c} & \bar{t} & T & T' & \cdots\end{bmatrix}_L \begin{bmatrix} V_{ud} & V_{us} & V_{ub} & \cdots \\ V_{cd} & V_{cs} & V_{cb} & \\ V_{td} & V_{ts} & V_{tb} & \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \gamma_\mu \begin{bmatrix} d \\ s \\ b \\ B \\ B' \\ \vdots \end{bmatrix}$$

V_{CKM}

The full quark mixing may not be a unitary matrix or even a **square matrix.**

Let us consider the 3×3 V_{CKM} matrix as a set of nine complex numbers.

9 moduli , 9 phases

By rephasing quark fields one can eliminate 5 phases of V_{CKM} :

$$9 - 5 = 4 \longrightarrow \text{\# of independent phases}$$

Number of independent parameters:

$$\underbrace{9}_{\text{moduli}} + \underbrace{4}_{\text{phases}} = 13$$

In the SM, as a result of unitarity:

$$13 - \underbrace{9}_{\substack{\text{unitarity}\\ \text{constraints}}} = 4 \text{ independent parameters.}$$

In the SM, the CKM may be constructed using as **input parameters**

3 angles and 1 phase – standard parametrization

4 moduli : $|V_{us}|, |V_{cb}|, |V_{td}|, |V_{ub}|$ Laurora, GCB

4 independent phases $\beta, \gamma, \chi, \chi'$

Aleksan, London, Kayser

In the future maybe

2 phases 2 moduli: $|V_{us}|, |V_{cb}|, \beta, \gamma$

Choice of rephasing invariant phases

$$\gamma \equiv \arg\left(-V_{ud} V_{cb} V_{ub}^{*} V_{cd}^{*}\right) = \arg\left(-\frac{V_{ud} V_{ub}^{*}}{V_{cd} V_{cb}^{*}}\right)$$

$$\beta \equiv \arg\left(-V_{cd} V_{tb} V_{cb}^{*} V_{td}^{*}\right) = \arg\left(-\frac{V_{cd} V_{cb}^{*}}{V_{td} V_{tb}^{*}}\right)$$

$$\chi \equiv \arg\left(-V_{cb} V_{ts} V_{cs}^{*} V_{tb}^{*}\right) = \arg\left(-\frac{V_{cb} V_{cs}^{*}}{V_{tb} V_{ts}^{*}}\right)$$

$$\chi' \equiv \arg\left(-V_{us} V_{cd} V_{ud}^{*} V_{cs}^{*}\right) = \arg\left(-\frac{V_{us} V_{ud}^{*}}{V_{cs} V_{cd}^{*}}\right)$$

Apart from normalization of rows and columns, the Standard Model predicts a series of exact relations involving moduli and $\gamma, \beta, \chi, \chi'$

Choice of Phase convention:

(No loss of generality even if there is New Physics)

$$\arg V_{CKM} = \begin{bmatrix} 0 & \chi' & -\gamma \\ \pi & 0 & 0 \\ -\beta & \pi+\chi & 0 \end{bmatrix}$$

Note that this phase convention is close but does not coincide with that of the "standard parametrization"

$$V = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix}$$

Exact relations among measurable quantities, derived from 3×3 unitarity:

(uc) $$\boxed{\sin\chi' = \frac{|V_{ub}|}{|V_{us}|}\frac{|V_{cb}|}{|V_{cs}|}\sin\gamma} \quad (7)$$

(ut) $$\boxed{|V_{ud}||V_{td}|\sin\beta - |V_{us}||V_{ts}|\sin(\chi'-\chi) - |V_{ub}||V_{tb}|\sin\gamma = 0} \quad (8)$$

(ct) $$\boxed{\sin\chi = \frac{|V_{td}|}{|V_{ts}|}\frac{|V_{cd}|}{|V_{cs}|}\sin\beta} \quad (9)$$

(db) $$\boxed{\frac{|V_{ub}|}{|V_{td}|} = \frac{\sin\beta}{\sin\gamma}\frac{|V_{tb}|}{|V_{ud}|}} \quad (10)$$

(ds) $$\boxed{\sin\chi' = \frac{|V_{td}||V_{ts}|}{|V_{ud}||V_{us}|}\sin(\beta+\chi)} \quad (11)$$

(sb) $$\boxed{\frac{\sin\chi}{\sin(\gamma+\chi')} = \frac{|V_{us}|}{|V_{ts}|}\frac{|V_{ub}|}{|V_{tb}|}} \quad (12)$$

By applying the **law of sines** to the (db) and (sb) triangles, one obtains:

(db) $$\boxed{|V_{ub}| = \frac{|V_{cd}||V_{cb}|}{|V_{ud}|} \frac{\sin\beta}{\sin(\beta+\gamma)}} \qquad (13)$$

(db) $$\boxed{|V_{td}| = \frac{|V_{cd}||V_{cb}|}{|V_{tb}|} \frac{\sin\gamma}{\sin(\gamma+\beta)}} \qquad (14)$$

(sb) $$\boxed{\sin\chi = \frac{|V_{us}|}{|V_{cs}|} \frac{|V_{ub}|}{|V_{cb}|} \sin(-\chi+\chi'+\gamma)} \qquad (15)$$

Important question:

How will these equations be affected by New Physics?

In order to answer this question one has to make some assumptions about the nature of New Physics

Assumptions about New Physics

- Assume that the tree level weak-decays are **dominated** by the Standard Model **W exchange diagrams.**

$\Downarrow$

Extraction of $|V_{us}|$, $|V_{cb}|$, $|V_{ub}|$ from experiment continues to be **valid** even in the presene of **New Physics**

- Allow for significant **New Physics** contributions to $B_d - \bar{B}_d$ mixing, $B_s - \bar{B}_s$ mixing

$\downarrow$

The SM only contributes at loop level

Parametrization of New Physics

$$M_{12}^{B_d-\bar{B}_d} = \left(M_{12}^{B_d-\bar{B}_d}\right)^{SM} r_d^{2}\, e^{-2i\phi_d}$$

$$M_{12}^{B_s-\bar{B}_s} = \left(M_{12}^{B_s-\bar{B}_s}\right) r_s^{2}\, e^{-2i\phi_s}$$

r_d and/or $r_s \neq 1 \Rightarrow$ New Physics

ϕ_d and/or $\phi_s \neq 0 \Rightarrow$ New Physics

SM	β	γ	χ	χ'	$\alpha=\pi-\beta-\gamma$
NP	$\bar{\beta}=\beta-\phi_d$	γ	$\bar{\chi}=\chi+\phi_s$	χ'	$\bar{\alpha}=\alpha+\phi_d$

$$\gamma = \pi - \frac{1}{2}\left[\arcsin a_{J/\psi K_S} + \arcsin a_{\pi^+\pi^-}\right]$$

Paramitrization independent tests of the Standard Model

Since we aim at **precision tests** it is important to use **exact relations** predicted by the SM.

Example :

$$\boxed{|V_{ub}| = \frac{|V_{cd}||V_{cb}|}{|V_{ud}|}\,\frac{\sin\beta}{\sin(\beta+\gamma)}} \qquad (13)$$

From Eqs (8), (10) :

$$\sin(\chi-\chi') = r\sin\beta\left|\frac{V_{us}}{|V_{ud}|}\right|\left[1-\frac{|V_{cb}|^2}{|V_{us}|^2}\right] \qquad (16)$$

Combining Eqs (12), (13) :

$$\sin\chi = \frac{|V_{us}|}{|V_{ts}|}\,\frac{|V_{cd}|}{|V_{tb}|}\,\frac{|V_{cb}|}{|V_{ud}|}\,\frac{\sin\beta\,\sin(\gamma+\chi')}{\sin(\gamma+\beta)}$$

well approximated by : Alexan, Kayser, London Silva, Wolfenstein

$$\sin\chi \cong \frac{|V_{us}|^2}{|V_{ud}|^2}\,\frac{\sin\beta\,\sin\gamma}{\sin(\gamma+\beta)}$$

How to extract (or put bounds) on

ϕ_d , ϕ_s :

$$|V_{ub}| = \frac{|V_{cd}||V_{cb}|}{|V_{ud}|} \frac{\sin\beta}{\sin(\gamma+\beta)}$$

only affected by ϕ_d, allows for the extraction of ϕ_d :

$$\tan\phi_d = \frac{R_u \sin(\gamma+\bar{\beta}) - \sin(\bar{\beta})}{\cos(\bar{\beta}) - R_u \cos(\gamma+\bar{\beta})}$$

$$R_u \equiv \frac{|V_{ud}||V_{ub}|}{|V_{cd}||V_{cb}|}$$

$$\bar{\beta} = \frac{1}{2} \arcsin a_{J/\psi K_S}$$

Similarly, from:

$$\boxed{\sin \chi = \frac{|V_{us}||V_{ub}|}{|V_{cs}||V_{cb}|} \sin(-\chi + \chi' + \gamma)} \quad (15)$$

and neglecting χ', one can extract ϕ_s:

$$\boxed{\tan \phi_s = \frac{\sin \bar{\chi} - C \sin(\gamma - \bar{\chi})}{C \cos(\gamma - \bar{\chi}) + \cos \bar{\chi}}}$$

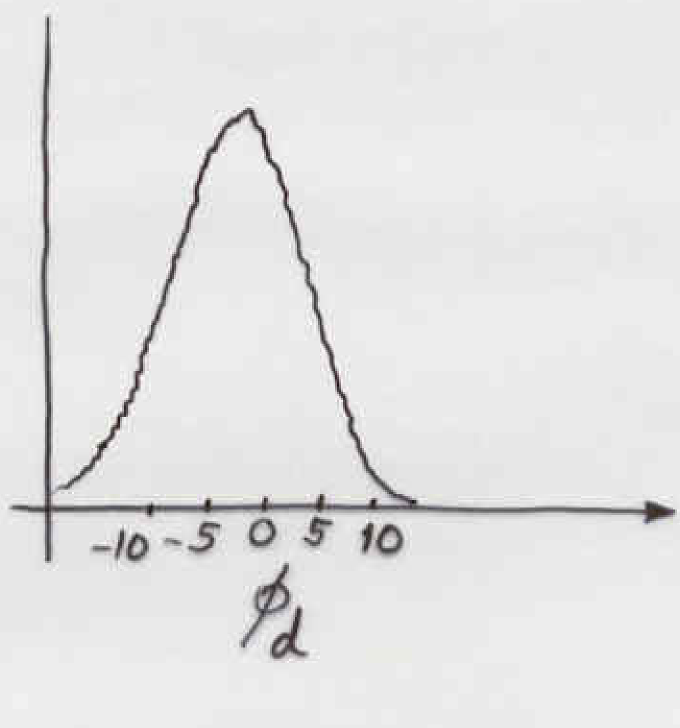
$$C \equiv \frac{|V_{us}||V_{ub}|}{|V_{cs}||V_{cb}|}$$

Some examples

Example 1

Input data:

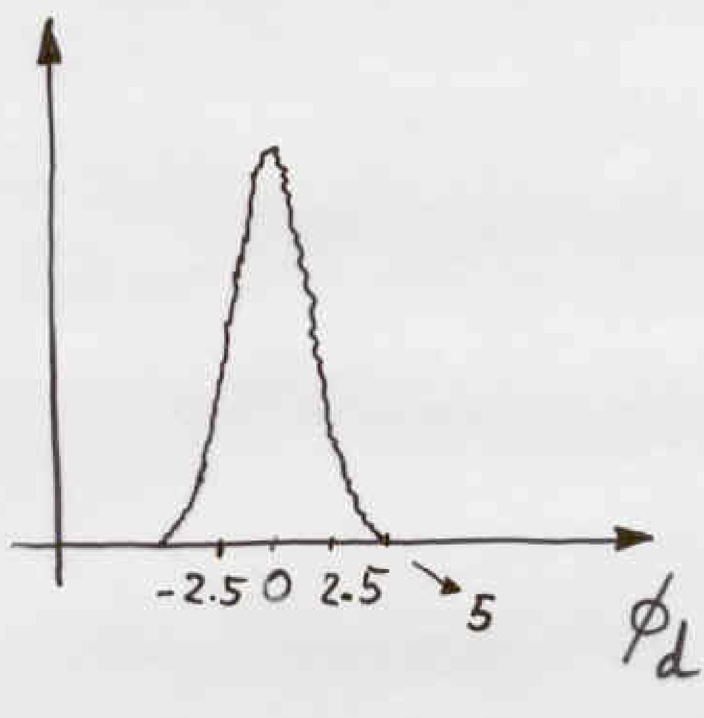
$$\left.\begin{aligned}
|V_{us}| &= 0.221 \pm 0.002 \\
|V_{cb}| &= 0.0417 \pm 0.0010 \\
|V_{ub}| &= (4.05 \pm 0.42) \times 10^{-3} \\
\bar{\beta} &= (26.9 \pm 5.0)^\circ \\
\gamma &= (55.4 \pm 11.9)^\circ
\end{aligned}\right\} \Rightarrow \phi_d = (-2.6 \pm 6)^\circ$$



Example 2 (more precise data)

Input :

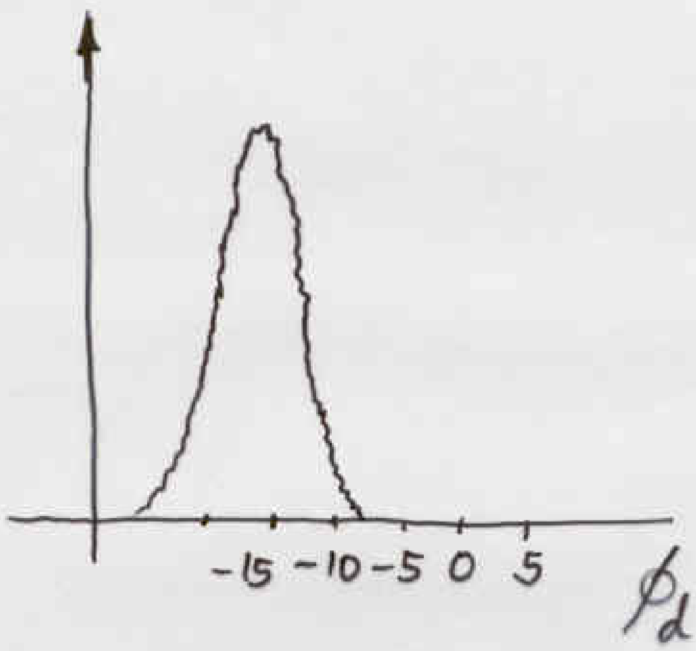
$$\left.\begin{aligned} |V_{us}| &= 0.221 \pm 0.002 \\ |V_{cb}| &= 0.0417 \pm 0.0010 \\ |V_{ub}| &= (4.05 \pm 0.21) \times 10^{-3} \\ \bar{\beta} &= (25.1 \pm 0.25)^\circ \\ \gamma &= (56.6 \pm 5.6)^\circ \end{aligned}\right\} \longrightarrow \phi_d = (-0.1 \pm 1.7)^\circ$$



Example 3 : A more optimistic scenario:

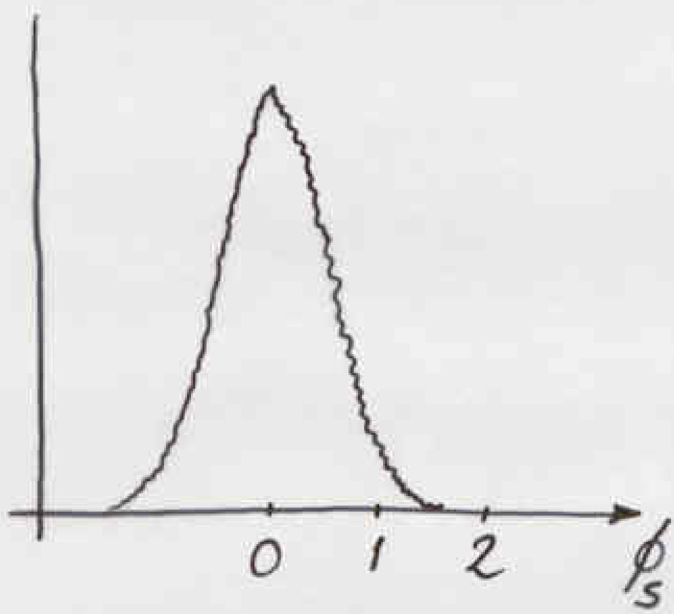
Discovery of New Physics

$$
\left.\begin{array}{l}
|V_{us}| = 0.221 \pm 0.002 \\
|V_{cb}| = 0.0417 \pm 0.0010 \\
|V_{ub}| = (4.05 \pm 0.21) \times 10^{-3} \\
\bar{\beta} = (30 \pm 0.3)^\circ \\
\gamma = (20 \pm 5)^\circ
\end{array}\right\} \longrightarrow \phi_d = (-16.3 \pm 3.2)^\circ
$$



Extraction of ϕ_s :

$$\left.\begin{aligned}
|V_{us}| &= 0.221 \pm 0.002 \\
|V_{cb}| &= 0.0417 \pm 0.0010 \\
|V_{ub}| &= (4.05 \pm 0.42) \times 10^{-3} \\
\gamma &= (56.6 \pm 5.6)^\circ \\
\bar{\chi} &= (1.06 \pm 0.50)^\circ
\end{aligned}\right\} \rightarrow \phi_s = (.03 \pm 0.51)^\circ$$



$\Downarrow$

Assuming γ and $\bar{\chi}$ to be known with a precision of $\sim 20\%$ and 50%, it will be possible to discover NP contribution to $B_s - \bar{B}_s$ mixing for $\phi_s \gtrsim 1.5^\circ$

Extraction of r_d/r_s

Once $x_s \equiv \Delta M_{B_s} \tau_{B_s}$ is measured, one can determine r_d/r_s from

$$\frac{x_d}{x_s} = \left|\frac{V_{td}}{V_{ts}}\right|^2 \left(\frac{r_d}{r_s}\right)^2 \left(\frac{M_{B_d}\,\tau_{B_d}}{M_{B_s}\,\tau_{B_s}}\right) \frac{1}{\xi^2}$$

From Eq. (14) $$|V_{td}| = \frac{|V_{cd}||V_{cb}|}{|V_{tb}|} \frac{\sin\gamma}{\sin(\gamma+\beta)}$$

One derives:

$$\frac{|V_{td}|}{|V_{ts}|} = \frac{|V_{us}|}{1+|V_{us}|\left[\frac{|V_{ub}|}{|V_{cb}|}\cos\gamma - \frac{1}{2}|V_{us}|\right]} \frac{\sin\gamma}{\sin(\gamma+\beta)} + O(\lambda^5)$$

$$f(\phi_d) \times \frac{r_d}{r_s} = \left(\xi^2 \frac{x_d\, M_{B_s}\,\tau_{B_s}}{x_s\, M_{B_d}\,\tau_{B_d}}\right)^{1/2} \left(\frac{\sin(\gamma+\bar\beta)}{|V_{us}|\sin\gamma}\right) \times \left[1+|V_{us}|\left[\frac{|V_{ub}|}{|V_{cb}|}\cos\gamma - \frac{1}{2}|V_{us}|\right]\right]$$

$$f(\phi_d) = \left[\cos\phi_d\left(1+\frac{\tan\phi_d}{\tan(\gamma+\bar\beta)}\right)\right]^{-1}$$

Summary and Conclusions

- Without theoretical input, the **quark mixing matrix V_{CKM}** contains $13 = 9 + 4$

  $9 \rightarrow$ moduli

  $4 \rightarrow$ invariant phases

  measurable quantities

- The **Standard** Model predicts a series of exact relations among above quantities which allow for **parametrization independent** tests of the SM

$\downarrow$

complements the Standard Analysis in the ρ, η plane

(see A. Buras talk)