Geometric Origin of CP Violation in an Extra-Dimensional Brane World

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Problem:

Understanding the fermion mass hierarchy and mixings. Flavor symmetry

Problematic problem:

Understanding the hierarchy between the Planck scale (rest mass of a flea) and particle physics scales.

string theory $\rightarrow$ extra dim.

This hierarchy controlled by features of the extra dim.

Large Extra Dimensions (ADD)
(flat space)
Hierarchy generated by the large volume of the extra dim.
(large w.r.t. $\frac{1}{246\text{GeV}} \rightarrow$ reshuffles the hierarchy problem, does not solve it)

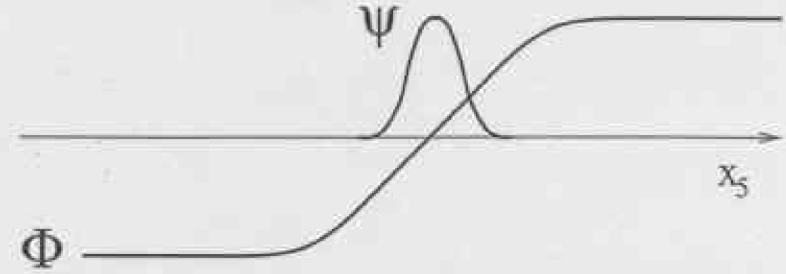
Warped extra dimensions (RS)
hierarchy generated by an exponential damping.
Extra dim. well-motivated by string theory

Extra dim. not so well motivated to address the fermion mass hierarchy.

Outline

- Summarizing the salient points of what has been done to address the quark mass hierarchy within the extra. dim approach.

- 5-d, 6-d, flat and warped. 6 Dimensions can account for CP violation Why are The Yukawa couplings complex?
(all other couplings in the SM are manifestly real...)



Flat, 5-d

Hierarchies without symmetries from Extra dim.

(N. Arkani-Hamed, M. Schmaltz, hep-ph/9903417)

Chiral Fermions in 5 dim.

$$S = \int \mathrm{d}^4x \mathrm{d}x_5 \overline{\psi} \left[i\gamma^\mu \partial_\mu + i\gamma^5 \partial_5 + \Phi(x_5) \right] \psi$$

$$\psi(x^\mu, x_5) = \sum_n \langle x_5 | L_n \rangle P_L \psi_n(x^\mu) + \sum_n \langle x_5 | R_n \rangle P_R \psi_n(x^\mu)$$

$$\overline{\psi}(x^\mu, x_5) = \sum_n \overline{\psi}_n(x^\mu) P_R \langle L_n | x_5 \rangle$$

$$+ \sum_n \overline{\psi}_n(x^\mu) P_L \langle R_n | x_5 \rangle$$

Zero Modes

$$\langle x_5 | L_0 \rangle \approx \exp\left[-\int_0^{x_5} \Phi(s)\,\mathrm{d}s\right]$$

$$\langle x_5 | R_0 \rangle \approx \exp\left[+\int_0^{x_5} \Phi(s)\,\mathrm{d}s\right]$$

Adding more fermions $\cdots$

$$S = \int \mathrm{d}^5 x \sum_{i,j} \overline{\psi}_i \left[i\gamma^5 \partial_5 + \lambda \Phi(x_5) - m \right]_{ij} \psi_j$$

SM constructed from 5-d Dirac spinors $\sim$

$$(Q, U^c, D^c, L, E^c)$$

transforming like the 4-d left-handed SM Weyl fermions $\sim$

$$(q, u^c, d^c, l, e^c)$$

SM Yukawa couplings

$$S = \int \mathrm{d}^5 x \overline{L} \left[i\gamma^5 \partial_5 + \Phi(x_5) \right] L +$$

$$\overline{E}^c \left[i\gamma^5 \partial_5 + \Phi(x_5) - m \right] E^c$$

$$+ \kappa H L^T C_5 E^c$$

$$C = \gamma^0 \gamma^2 \gamma^5$$

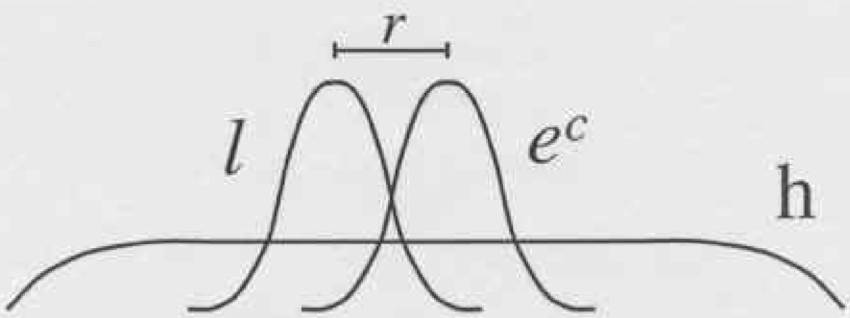
Assume Higgs is *delocalized* $\cdots$

Expand L and E^c and replace H by its x_5-independent zero mode $\rightarrow$

$$S_{Yuk} = \int \mathrm{d}^4 x \kappa h\left(x^\mu\right) l\left(x^\mu\right) e^c\left(x^\mu\right) \int \mathrm{d}x_5 \phi_l\left(x_5\right) \phi_{e^c}\left(x_5\right)$$

S.H.O. approximation $\rightarrow$

$$\Phi\left(x_5\right) = 2\mu^2 x_5 \rightarrow$$



Fermions are then gaussian in x_5

$$\int \mathrm{d}x_5 \phi_l\left(x_5\right) \phi_{e^c}\left(x_5\right) =$$

$$\frac{\sqrt{2}\mu}{\sqrt{\pi}} \int \mathrm{d}x_5 e^{-\mu^2 x_5^2} e^{-\mu^2 (x_5 - r)^2}$$

$$= \exp^{-\mu^2 \frac{r^2}{2}}$$

where r is related to m

An attractive idea.
Unattractive features:

Additional physics must be assumed in order to localize the fermions (kink solution from an additional scalar field that is not the Higgs)
Does not include gravity.

5-d gauge field theory is not renormalizable $\rightarrow$ how to UV extend it $\cdots$ Deconstruction? (Arkani-Hamed, Georgi et al)

Can get this exponential damping automatically from a non-factorizable geometry, no need to assume new physics.

Fermions masses, Mixings
and Proton Decay in a Randall-Sundrum Model
(S. Huber and Q. Shafi, hep-ph/0010195)

General Set-Up:

$$ds^2 = e^{-2\sigma(y)}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2$$

$$S = \int d^4x \int dy\sqrt{G}\{E^A_\alpha \left[\frac{i}{2}\bar{\Psi}\gamma^\alpha\left(\overrightarrow{\partial_A} - \overleftarrow{\partial_A}\right)\Psi + \frac{\omega_{bcA}}{8}\bar{\Psi}\{\gamma^\alpha, \sigma^{bc}\}\Psi\right] - m(y)\bar{\Psi}\Psi\}$$

Z_2 symmetry: $(y \to -y)$.

$$\psi(-y) = \gamma_5\psi(y)$$

$$\psi_L(-y) = -\psi(y)$$

$$\psi_R(-y) = \psi_R(y)$$

Masses for bulk quarks

$$\int d^4x \int dy \sqrt{-g} \lambda_{ij}^{(5)} H \bar{\Psi}_{i+} \Psi_{j-} \equiv \int d^4x \, m_{ij} \bar{\Psi}_{iR}^{(0)} \Psi_{jL}^{(0)} + \cdots$$

The 4d Dirac masses are given by

$$m_{ij} = \int_{-\pi R}^{\pi R} \frac{dy}{2\pi R} \lambda_{ij}^{(5)} e^{-4\sigma} H(y) f_{0i+}(y) f_{0j-}(y).$$

Higgs profile:

$$H(y) = H_0 e^{ak(|y|-\pi R)}.$$

Numerical example

$$\begin{array}{lll} c_{Q1} = 0.72 & c_{D1} = 0.57 & c_{U1} = 0.63 \\ c_{Q2} = 0.60 & c_{D2} = 0.57 & c_{U2} = 0.30 \\ c_{Q3} = 0.35 & c_{D3} = 0.60 & c_{U3} = 0.10, \end{array}$$

$\rightarrow$

Many-Brane, Six-Dimensional Extension of the Randall-Sundrum Solution

$$S = S_{gravity} + \sum_{i=1}^{N} S_i + \sum_{j=1}^{M} S_j$$

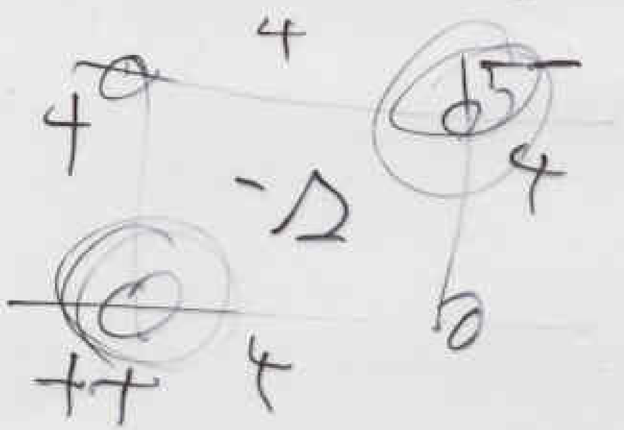
$$S_{grav} = \int d^4x \int_0^{2\pi} d\phi \int_0^{2\pi} d\rho \sqrt{-g} \left(\frac{1}{\kappa_6^2} R - \right.$$

$$\sum_{i,j} \Lambda_{ij} \left[\Theta\left(\phi - \phi_i\right) - \Theta\left(\phi - \phi_{i+1}\right) \right] \times$$

$$\left. \left[\Theta\left(\rho - \rho_j\right) - \Theta\left(\rho - \rho_{j+1}\right) \right] \right)$$

$$S_i = - \int d^4x \int_0^{2\pi} d\rho \sqrt{g^{(\phi=\phi_i)}} T_{\phi_i}$$

$$S_j = - \int d^4x \int_0^{2\pi} d\phi \sqrt{g^{(\rho=\rho_j)}} T_{\rho_j},$$



$$\frac{\lambda_D^{(5)}}{g^{(5)}} = \begin{pmatrix} 0.50 & -2.00 & -2.00 \\ 1.48 & -0.90 & 2.00 \\ 0.52 & -0.50 & 0.70 \end{pmatrix}$$

$$\frac{\lambda_U^{(5)}}{g^{(5)}} = \begin{pmatrix} 0.80 & -1.90 & -2.00 \\ 1.23 & 1.20 & -1.04 \\ 1.85 & 1.66 & -0.80 \end{pmatrix}$$

These parameters predict

$$\begin{aligned} m_u &= 2.9 \text{ MeV}, & m_c &= 1.3 \text{ GeV}, \\ m_d &= 3.8 \text{ MeV}, & m_b &= 4.4 \text{ GeV}, \\ m_s &= 78 \text{ MeV}, & m_t &= 165 \text{ GeV}, \end{aligned}$$

$$|V_{\text{CKM}}| = \begin{pmatrix} 0.9744 & 0.2248 & 0.0045 \\ 0.2248 & 0.9736 & 0.0392 \\ 0.0045 & 0.0392 & 0.9992 \end{pmatrix}.$$

Maybe a phase will give even closer agreement.
$\rightarrow$ go to 6D.

6D Dirac Fermions

$$S = \int d^4x \int d\phi d\rho \sqrt{G} \left\{ E^A_\alpha \left[\frac{i}{2} \overline{\psi} \gamma^\alpha \left(\overrightarrow{\partial_A} - \overleftarrow{\partial_A} \right) \psi \right] \right.$$

$$\left. - m\left(\phi, \rho\right) \overline{\psi}\psi \right\}$$

$$ds^2 = A^2\left(\phi, \rho\right) \eta_{\mu\nu} dx^\mu dx^\nu - B^2\left(\phi, \rho\right) d\phi^2 - C^2\left(\phi, \rho\right)^2$$

$$A = \frac{1}{e^\sigma + e^\gamma - 1}$$

$$B = e^\sigma A$$

$$C = e^\gamma A$$

$$\sqrt{g} = \frac{e^\sigma e^\gamma}{\left(e^\sigma + e^\gamma - 1\right)^6}$$

Dirac algebra in 6-D $\rightarrow$ Complex fermion profiles.

Can write the action as a sum over Kaluza-Klein modes:

$$
\begin{aligned}
S = \sum_m \sum_n \int \mathrm{d}^4x \Big\{ & i\overline{\psi_{n,m+}}(x)\,\gamma^\mu \partial_\mu \psi_{n,m+}(x) \\
& + i\overline{\psi_{n,m-}}(x)\,\gamma^\mu \partial_\mu \psi_{n,m-}(x) \\
& - m_{n,m+}\overline{\psi_{n,m+}}(x)\,\psi_{n,m+}(x) \\
& - m_{n,m-}\overline{\psi_{n,m-}}(x)\,\psi_{n,m-}(x) \Big\}
\end{aligned}
$$

KK decomposition:

$$\Psi_{(R,L)+}(x,\phi,\rho) =$$

$$\sum_m \sum_n \psi^{R,L}_{n,m+}(x) \left(\frac{e^\sigma e^\gamma}{(e^\sigma + e^\gamma - 1)^6} \right)^{-\frac{1}{2}} f^{R,L}_{n,m+}(\phi,\rho)$$

and

$$\Psi_{(R,L)-}(x,\phi,\rho) =$$

$$\sum_m \sum_n \psi^{R,L}_{n,m-}(x) \left(\frac{e^\sigma e^\gamma}{(e^\sigma + e^\gamma - 1)^6} \right)^{-\frac{1}{2}} f^{R,L}_{n,m-}(\phi,\rho)$$

Because $f_{R+} = f^*_{R-}$, we are free to write

$$\begin{aligned} f_{R+} &= U + iV \\ f_{R-} &= U - iV \end{aligned}$$

where U and V are real, with similar expressions for the left-handed modes. This decomposition leads to the following e.o.m. for the zero modes $\rightarrow$

These two Z_2's correspond to parity transformations in the extra dim. We must find matrices S_ϕ, S_ρ s.t.

$$S_\phi^{-1}\Gamma^M S_\phi = \Lambda^M_N \Gamma^N$$

where Λ corresponds to the Lorentz transformation

$$\Lambda = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

and

$$L = i\overline{\psi}\Gamma^N \partial_N \psi$$

is invariant.

and

$$S_\rho^{-1} \Gamma^M S_\rho = \Lambda^M_N \Gamma^N$$

where Λ corresponds to the L.T.

$$\Lambda = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

and

$$L = i\overline{\psi}\Gamma^N \partial_N \psi$$

is invariant. For any S_ϕ, S_ρ satisfying these conditions, one finds that they anticommute. Those given in hep-ph/0111013 commute, and one may check that the lagrangian is not invariant... $\left\{S_\phi, S_\rho\right\} = 0$ necessarily, $\rightarrow$

As is well-known, fermions have the odd property of going into minus themselves when rotated by 2π (pun intended). Applying successively the discrete transformations $\phi \to -\phi$ followed by $\rho \to -\rho$ is equivalent to a rotation of π radians in the $\phi - \rho$ plane. Applying these discrete transformations in the opposite order is equivalent to a rotation of $-\pi$ radians in the same plane. The difference in angles of these two rotations is 2π. Applying both transformations, first in the order $S_\rho S_\phi$ and then in the order $S_\phi S_\rho$, is equivalent to a rotation of 2π in the $\phi - \rho$ plane, and must result in an overall minus sign for the fermion field. $\rightarrow$

Not only are the e.o.m. coupled, but the b.c. are relating different components of the zero modes.

Analytically, an intractable system of equations. Analytic solutions are possible, however, for a Scalar field (Higgs) $\rightarrow$

Scalar Field Zero Mode and Quark Mass Matrices

$$S = \frac{1}{2}\int \mathrm{d}^4x \int \mathrm{d}\phi \mathrm{d}\rho \sqrt{G}\left(G^{AB}\partial_A\Phi\partial_B\Phi - m_\Phi^2\Phi^2\right)$$

Decompose the 6D scalar into KK modes:

$$\Phi\left(x,\phi,\rho\right) = \sum_{n,m}\phi_{n,m}\left(x\right) f_{n,m}\left(\phi,\rho\right)$$

$$S = \frac{1}{2}\sum_{n,m}\int \mathrm{d}^4x \left\{\eta^{\mu\nu}\partial_\mu\phi_{n,m}\left(x\right)\partial_\nu\phi_{n,m}\left(x\right)\right.$$

$$\left. - m_{n,m}^2\phi_{n,m}^2\left(x\right)\right\}$$

Canonical 4d K.E. terms $\rightarrow$

$$\int\int \mathrm{d}\phi \mathrm{d}\rho \frac{e^\sigma e^\gamma}{\left(e^\sigma + e^\gamma - 1\right)^4} f_{m,i}^*\left(\phi,\rho\right) f_{n,k}\left(\phi,\rho\right) = \delta_{mn}, \delta_{ik}$$

Zero Mode e.o.m. :

$$\partial_\phi \left(\frac{e^\gamma}{e^\sigma \left(e^\sigma + e^\gamma - 1\right)^4} \partial_\phi f_0 \right)$$

$$+\partial_\rho \left(\frac{e^\sigma}{e^\gamma \left(e^\sigma + e^\gamma - 1\right)^4} \partial_\rho f_0 \right)$$

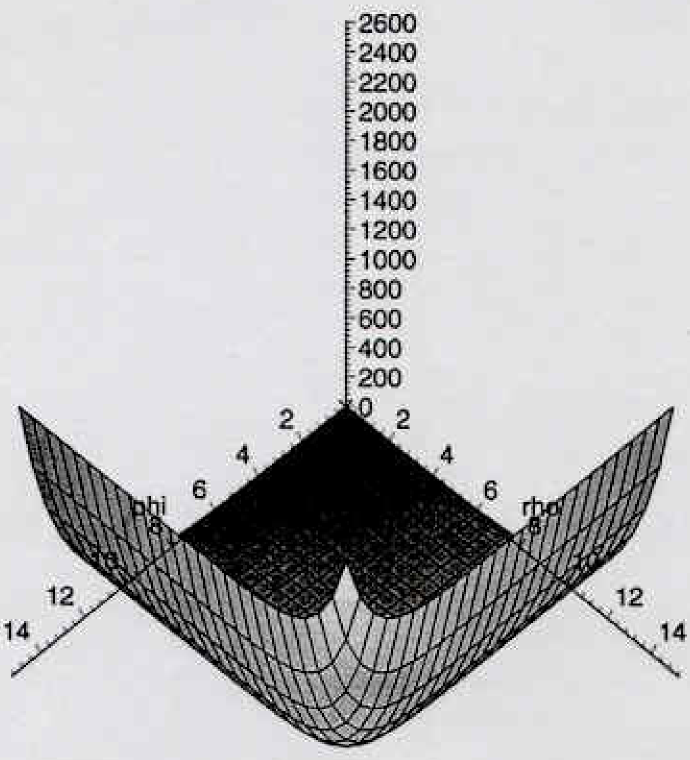
$$-\frac{m^2 e^\sigma e^\gamma}{\left(e^\sigma + e^\gamma - 1\right)^6} f_0 = 0$$

For the special case when

$$2k^2 m^2 = 25k^4$$

we can find an analytic solution:

$$f(\phi, \rho) = e^{\left(\frac{\ln\left(e^{k\phi} + e^{k\rho} - 1\right)}{2k^2} \right)}$$



As in the 5D case, we construct a six dimensional lorentz invariant Yukawa interaction from a term such as

$$\sqrt{-G}\lambda_{ij}^{(6)} H\overline{\psi_i}\psi_j$$

Conclusions / further directions We have shown in this work that CP violation may be understood as a natural consequence of the Dirac algebra in six dimensions. Another motivation for going to six dimensions concerns the mystery of the generation index. As shown by Kogan et al. (Nucl. Phys., B615:191-218, 2001) , multibrane world scenarios imply the existence of light KK modes that are suggestive of family replication. Going to six dimensions may alleviate some of the difficulties encountered in trying to implement this program. This possibility is currently under investigation.