PaH

Pure Phase Mass Matrices from 6 Dimensions

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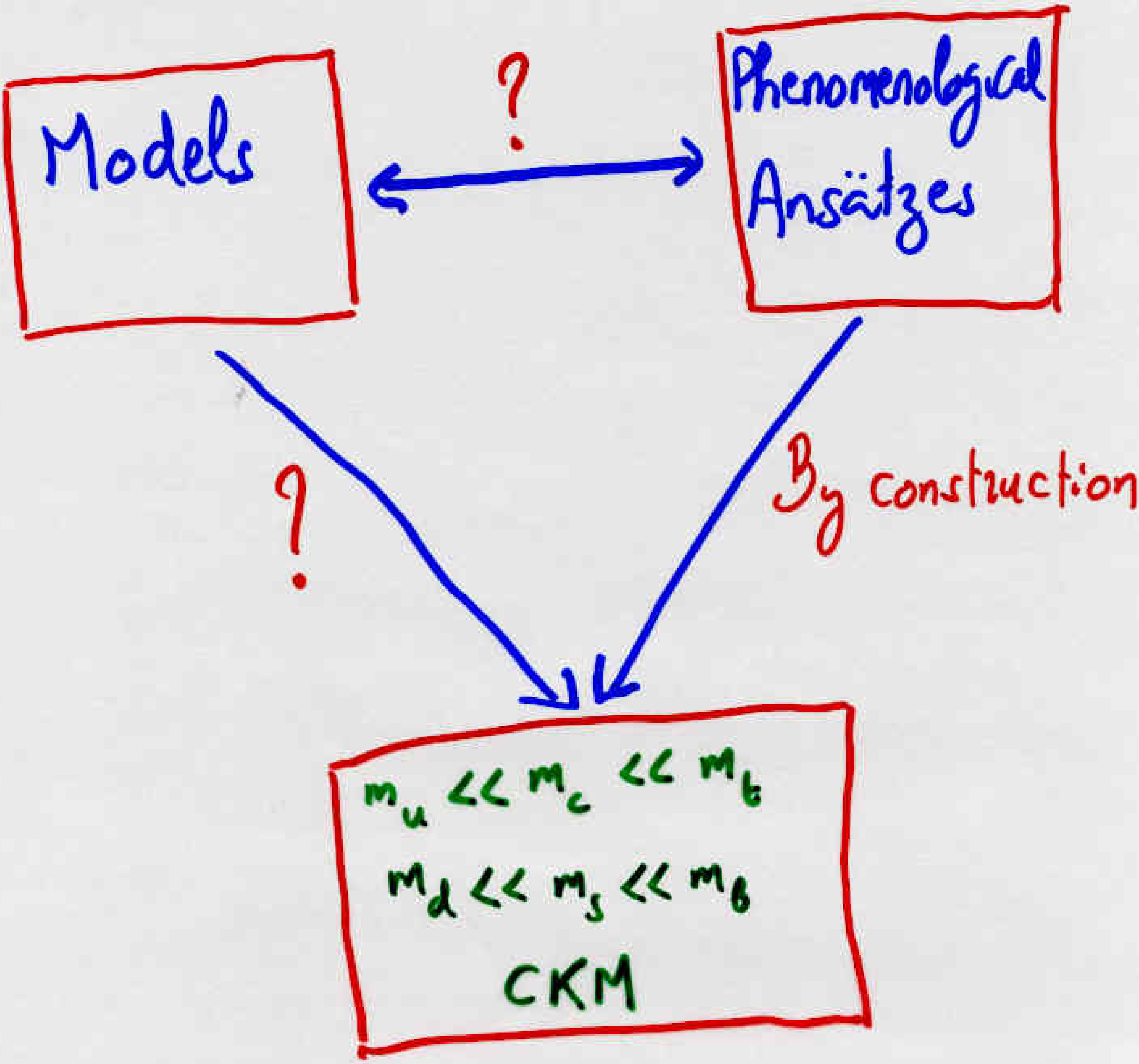
- Hierarchy of quark masses : Pure Phase Mass Matrices (PPMM).
- Construction of PPMM from 6 Dimensions : Why and How.
- Implications.

"Mass hierarchy Triangle"



Quark Mass Hierarchy

- Why: $m_u \ll m_c \ll m_t$;
  $m_d \ll m_s \ll m_b$?

- Origin of mixing angles and CP phase in CKM matrix ?

- Many models: A majority of the models uses a top-down approach.

- Phenomenological Ansatzes: Devised to fit masses, mixing angles, CP phase. Most are rather ad-hoc. No clear physics behind.

- Can one build models to justify these ansatzes? In general, it's very complicated!

Complicated ansätz $\Rightarrow$ Complicated model

- One particularly attractive ansätz:

The so-called Pure Phase Mass Matrix (Branco, Rebelo, Silva-Marcos).

It is attractive for its "simplicity".

$$m_{U,D} = g_{U,D} \left(\frac{v}{\sqrt{2}}\right)_{(U,D)} \left\{ e^{i\theta_{ij}} \right\} \qquad i,j: 1,2,3$$

$g_{U,D}$: Universal Yukawa Coupling

$\left\{ e^{i\theta_{ij}} \right\}$: Matrix elements with Unit modulii

- Equally successful is an almost-Pure Phase mass matrix: $e^{i\theta_{ij}} \rightarrow (1-\rho_{ij})e^{i\theta_{ij}}$

$$|\rho| \ll 1$$

- Model(s) for PPMM?

- Need to explain:
  - $g_{U,D}$
  - $\left(\frac{v}{\sqrt{2}}\right)_{U,D}$ ← This could come from one or two Higgses.
  - $e^{i\theta_{ij}}$ (or $(1-\rho_{ij})e^{i\theta_{ij}}$)
- One attempt (Fishbane & Hung):
  Very complicated Higgs structure!
  No clear lesson learned.
- How can one $\boxed{\text{rephrase}}$ the problem in such a way as to better $\boxed{\text{focus}}$ on the origins of $g_{U,D}$; $\left(\frac{v}{\sqrt{2}}\right)_{U,D}$; $e^{i\theta_{ij}}$?
- Think out of the box or rather
  Think out of four dimensions.

Pure Phase Mass Matrix from 6 dimensions

* P.Q.H. & M.Seco ; hep-ph/0111013

Requirement: Field theory in 4 + n (extra spatial dimensions) ⇒ Field theory in 4 dimensions with chiral fermions (for the SM)

- Why 6 dimensions ?
  - One (extra spatial) dimension to get the Universal Yukawa coupling $g_{U,D}$
  - Another (extra spatial) dimension to get the phases $e^{i\theta_{ij}}$ or $(1-\rho_{ij})e^{i\theta_{ij}}$

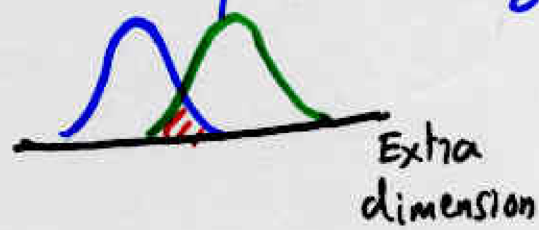
- How ? Bottom-up approach:

Add extra spatial dimensions one by one
Compactify them
See what kind of effective theory in 4 dim one obtains.

• Key concepts ;

• Localization of chiral zero modes along the extra dimension(s).

• Effective coupling strengths in 4-dim comes from wave function overlap(s) along the extra dimension(s).



(Arkani-Hamed & Schmaltz)

Three essential steps :

1) Get the chiral zero modes.

2) Localize them

3) Compute the wave function overlaps.

Chiral Zero Modes

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I) 4 + 1 dim. : (x^μ, y) (Flat)

- Compactify y on S_1/Z_2 with $y \in [0, L_5]$
- The free Lagrangian has the symmetry:

← Dirac Fermion

$$\Psi(x^\mu, y) \to \Psi(x^\mu, y) = \pm \gamma_5 \Psi(x^\mu, L_5 - y) \qquad Z_2 \qquad (1)$$

- Periodic B.C. : $\Psi(x^\mu, y) = \Psi(x^\mu, L_5 + y) = \Psi(x^\mu, 2L_5 + y)$ (2)
- With $P_L\Psi = \Psi_L$; $P_R\Psi = \Psi_R$

zero mode

(1)+(2) give: $\boxed{\Psi_R^{(0)}(x,y) = \Psi_R^{(0)}(x) \quad ; \quad \Psi_L^{(0)}(x,y) = 0}$

for $\Psi \to \gamma_5 \Psi$

or

$\boxed{\Psi_R^{(0)}(x,y) = 0 \quad ; \quad \Psi_L^{(0)}(x,y) = \Psi_L^{(0)}(x)}$

for $\Psi \to -\gamma_5 \Psi$

Fixed pts: 0, L_5

$\Rightarrow$ Appearance of a chiral zero mode!

II) 4 + 2 dim : (x^μ, y, z) (Flat)

- $y \in [0, L_5]$ on S_1/Z_2 ; $z \in [0, L_6]$ on $S_1/Z_2 \times Z_2'$
- Dirac spinor in 6-dim :

$$\underset{\substack{\uparrow \\ \text{8-comp.}}}{\Psi} = \begin{pmatrix} \Psi_+ \\ \Psi_- \end{pmatrix}$$

Gamma matrices:

$\Gamma_0, \Gamma_1, \dots, \Gamma_3$

$$\Gamma_y = \begin{pmatrix} 0 & -i\gamma_5 \\ i\gamma_5 & 0 \end{pmatrix} \; ; \; \Gamma_z = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

- Invariances of the free Lagrangian :

$$\Psi(x, y, z) \xrightarrow[Z_2]{} \Gamma_z \Psi(x, y, L_6 - z)$$

$$\Psi(x, y, z') \xrightarrow[Z_2']{} \Gamma_z \Psi(x, y, L_6 - z') \qquad z' = L + z$$

$$\Psi(x, y, z) \xrightarrow[Z_2(y)]{} \pm\tilde{\gamma}_5 \Psi(x, L_5 - y, z)$$

$$\tilde{\gamma}_5 = i \Gamma_y \Gamma_7 \qquad \Gamma_7 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

• Rewrite:

$$\Psi_\pm = \frac{1}{\sqrt{2}}(\chi \pm i\eta)$$

$\Rightarrow$ Z_2-symmetries + B.C. give one remaining chiral zero mode in 4 dim.:

$$\chi_L^{(0)}(x,y,z) \text{ or } \chi_R^{(0)}(x,y,z) \qquad (\mp\gamma_5)$$

which can be written as

$$\boxed{\chi_{L,R}^{(0)}(x,y,z) = \Psi_{L,R}(x)\,\xi_{L,R}^{(5)}(y)\,\xi_{L,R}^{(6)}(z)}$$

• Without interactions, $\xi_{L,R}$'s are constant.

Proceed to Step # 2.

Localization of fermion zero modes

I) In 4 + 1 dim :

- Introduce a background scalar field ← i.e. it has no zero modes

  $\phi \xrightarrow[Z_2]{} -\phi$

- Yukawa coupling: $f\bar{\Psi}\Psi\phi$ ← Z_2- invariant coupling (1)

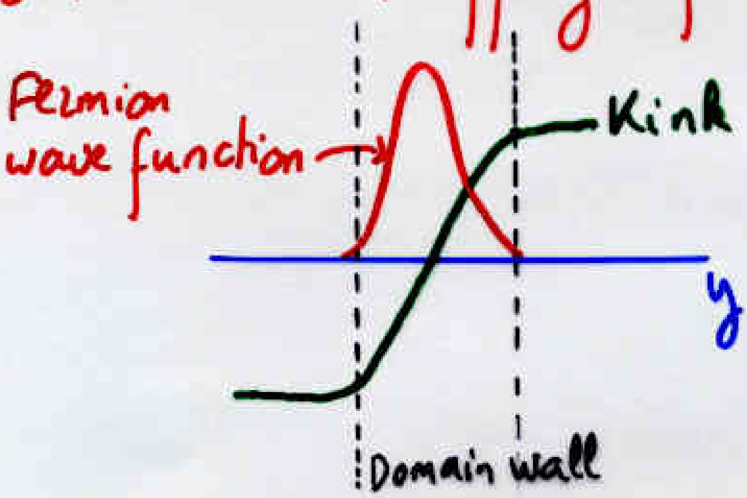
- Kink solution: $\langle\phi\rangle = h(y)$

- This traps the fermion through (1):

$$\xi(y) = C\, e^{-\int_0^y f h(y')dy'} \sim C' e^{-\mu^2 y^2}$$

$(f h(y) \sim 2\mu^2 y)$

Domain wall trapping of chiral fermions



II) In 4+2 dim:

- Background scalar field

$$\phi'(x,y,z) \xrightarrow[\mathbb{Z}_2(y)]{} -\phi'(x, L_5 - y, z)$$

Invariant under the $\mathbb{Z}_2'$s along z.

- Yukawa coupling:

$$f'\bar{\Psi}\Gamma_7\phi'\Psi = f(\bar{\chi}\phi'\chi - \bar{\eta}\phi'\eta)$$

$\Rightarrow$ usual coupling in 5 dimensions

$\Rightarrow$ usual localization along y i.e. $\boxed{\xi^{(5)}(y) \propto e^{-\mu^2 y^2}}$

- How can one get $\xi^{(6)}(z) \propto e^{i(\cdots)}$?

- Simplest way: Define $\boxed{\tilde{\Psi}(x,y,z) \equiv \Psi(x,y,L_6+z)}$

Introduce $\phi \to \phi \quad \mathbb{Z}_2$

$\phi \to -\phi \quad \mathbb{Z}_2'$

- Coupling: $\boxed{f(\bar{\Psi}\tilde{\Psi} - \bar{\tilde{\Psi}}\Psi)\phi}$ $\quad \leftarrow \langle\phi\rangle = h(z)$

- $-\partial_z \xi^{(6)}(z) + i f h(z) \xi^{(6)}(L_6+z) = 0$

Assume $\xi^{(6)}(z) = \xi^{(6)}(L_6 + z)$; $h(z) = v \tanh(\mu z)$ ← Kink

$$\Rightarrow \boxed{\xi^{(6)}(z) = \frac{1}{\sqrt{L_6}} e^{i f v \ln(\cosh(\mu z))/\mu}}$$

see remarks

Proceed to Step # 3 .

- Fermion zero mode :

$$\Psi^{(0)}(x,y,z) = \Psi(x)\, \xi^{(5)}(y)\, \xi^{(6)}(z)$$

- 4- dim effective Lagrangian involving 2 fermion fields :

$$\propto \bar{\Psi}_i(x)\, O\, \Psi_j(x) \underbrace{\overbrace{\int dy\, \xi_i^{(5)}(y)\, \xi_j^{(5)}(y)}^{\Rightarrow\, y_{v,0}} \overbrace{\int dz\, \xi_i^{*(6)}(z)\, \xi_j^{(6)}(z)}^{\Rightarrow\, \text{phase}}}_{\text{overlaps}}$$

overlaps ⇒ Effective interaction strength.

Remarks

- $\tilde{\psi}(x,y,z) = \psi(x,y,L_6+z) \xrightarrow[L_6\to\infty]{} 0$

$\Rightarrow f(\bar{\psi}\tilde{\psi} - \bar{\tilde{\psi}}\psi)\phi \to 0$

$\Rightarrow \xi^{(6)}(z) \propto e^{i g(z)}$ in this scenario

comes from a compactified sixth dimension.

- A hermitian coupling of the form

$i f \bar{\psi}(x,y,z)\psi(x,y,z)\phi$ (for appropriate ϕ)

$\Rightarrow \xi^{(6)}(z) \not\Rightarrow e^{i(\cdots)}$

but rather $\xi^{(6)}(z) \propto e^{-(\cdots)}$

(Almost) Pure Phase Mass Matrices

$\#$

- Assign a symmetry $S_{3L} \times S_{3R}$ ← (simplest choice)
- Write a SM Yukawa coupling

$$g_{Y,U} \sum_i \bar{Q}^i_L \Phi \sum_j U^j_R + \text{h.c.} \Rightarrow g_{Y,U} \sum_{i=1}^{3} \bar{q}^i_L h \sum_{j=1}^{3} u^j_R$$

$$\times \int dy\, \xi^i_L(y)\, \xi^j_R(y) \times \int dz\, \xi^i_L(z)\, \xi^j_R(z)$$

- Localize kinks at various places along the extra dimensions $(y^i_{L,R}\ ;\ z^i_{L,R})$ ← Geography of extra dimensions
- In order to see explicitly the phases in the mass matrix, assume thickness of domain walls $\sim L_6$ (along z)

The mass matrix takes the form

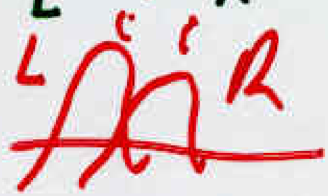
$$m_\nu = g^{eff}_{y,\nu} \frac{v}{\sqrt{2}} \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

Results

I) If $\xi_L^{i}(y) = \overset{\#}{\xi}_L(y)$ and $\xi_R^{j}(y) = \xi_R(y)$

⇒ Universal Yukawa Coupling

$$g^{eff}_{y,\nu} = g_{y,\nu} \int_0^{L_5} dy \, \xi_L(y)\, \xi_R(y)$$



II) If $z_{Li} = z_{Ri} = z_i$

$|\rho_{ij}| \ll 1$

$$a_{ii} = 1$$

$$a_{ij} = (1-\rho_{ij})\, e^{i\theta_{ij}} = \int_0^{L_5} dz \, \xi^{i}(z)\, \xi^{j}(z)$$

$$= a^{*}_{ji}$$

Almost Pure Phase

⇒ M_ν is hermitian ⇒ Real $\det M_\nu$.

$\Rightarrow \arg(\det M_U) = 0$

Any connection with Strong CP? $(\bar\theta = \theta + \arg\det M)$

III) If $z_{Li} \neq z_{Ri}$,

M_U is non-hermitian.

Similar considerations apply to the Down sector.

Questions from New Perspectives

- What fixes the locations of various kinks?
- When $z_{Li} = z_{Ri} = z \Rightarrow a_{ij} = 1$ all elements are 1.

$\Rightarrow$ Democratic mass matrix $M = g_y \frac{v}{\sqrt{2}} \{1\}$
(does not work!)

What splits the families $\ni$ $\mathcal{M}$ becomes an almost pure phase mass matrix?

- $\mathcal{M}$ is non-hermitian when there is left-right splitting. How?
- Connections to the strong CP problem?
- Can one produce "low-scale" Baryogenesis à la SU(5)?
- What about the lepton sector, in particular neutrinos?

Last but not Least

Would the phase factors $e^{i\theta_{ij}}$ be indirect signals of a compactified extra spatial dimension?