Fermion Masses and Mixing in Thick Branes

XIVth RENCONTRES DE BLOIS
MATTER-ANTIMATTER ASYMMETRY

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"Split fermions in extra dimensions and CP violation"
by G.C. Branco, A. de Gouvêa, MNR
Phys. Lett. 506B (2001) 115

"Neutrino masses and lepton flavour violation in thick brane scenarios" by G. Barenboim, G.C. Branco, A. de Gouvêa and MNR
Phys Rev D 64 (2001) 073005

EXTRA DIMENSIONS

Old Idea Kaluza, Klein, Nördstrom
(compactification) $R \lesssim 10^{-17}$ cm

Strings require extra dimensions
revival of interest in the eighties

Some papers on SM fields in extra dim already in the 80's

Great revolution occurred in the late nineties prompted by

- Arkani-Hamed, Dimopoulos and Dvali (ADD)
  "Large" extra dimensions (eg milimetric size)
  provided only gravity populates them
- Randall and Sundrum (RS)
  Infinite extra dimensions with non
  factorizable geometries

A lot of Work has been done on
Large Extra Dimensions and

- gauge hierarchy problem
- symmetry and supersymmetry breaking
- cosmology
- fermion masses and mixing

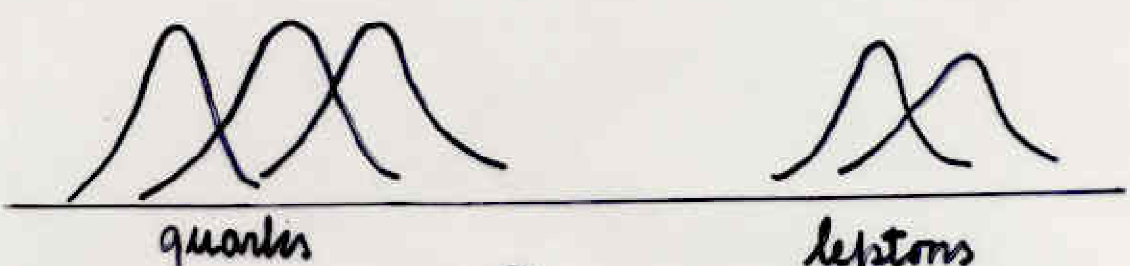
ADD and RS

- We live on a 3-brane, ie four dimensional spacetime (domain wall) located in the extra dimensions; gravity propagates in the whole space
- from the viewpoint of our 3-brane the fundamental mass scale is the Tev scale
- at the Tev scale quantum gravity gets strong

SPECIAL FRAMEWORK (AS)

Arkani-Hamed, Schmaltz

- SM fields are constrained to live on a wall in n extra dimensions; gravity and perhaps other SM singlet fields are free to propagate
- Our wall is slightly thick in one (or more) of the extra dimension
- Wall substructure: Higgs and SM gauge fields are free to propagate SM fermions are fixed at different points in the wall, with wave functions given by narrow gaussians.
- Direct couplings between fermions suppressed by the exponentially small overlapp of wave functions



IMPORTANT ISSUES ADDRESSED SUCESSFULLY

- Proton stability
- suppression of FCNC
- FERMION MASS HIERARCHY

AS scenario leads to startling consequences which may be observed at next-generation collider experiments

Proton stability requires suppression of operator of the form $\frac{1}{\Lambda^2} QQQL$

in normal GUT's $\Lambda \simeq M_{GUT} \simeq 10^{16}$ GeV

(proton lifetime $> 3 \times 10^{33}$ yr)

In these extra dimension scenarios $\Lambda \sim 1$ TeV in the AS scenario, with different fermions located at different points in the extra dimensions, efficient suppression due to Q's and L's living in different "Worlds"
(distances between quarks and leptons $\sim 10 \mu^{-1}$)

Lepton flavour violating muon decays

$\mu \to e\gamma$, $\mu \to eee$

and tau decays

$\tau \to \mu\gamma$, $\tau \to e\gamma$, $\tau \to \mu\mu\mu$, $\tau \to eee$

($\tau \to \mu ee$, $e\mu\mu$)

must also be suppressed

Different Yukawa couplings result from different overlaps of field wave functions inside the four dimensional domain wall in higher dimensional space

EFFECTIVE FOUR-DIMENSIONAL YUKAWA COUPLING:

$$\lambda_{ij} = e^{-\frac{1}{2}\mu^2(\ell_i-\ell_j)^2}\cdot \kappa_{ij}\ ; \quad \mu^{-1} \text{ gaussian width}$$

FLAVOUR PUZZLE addressed through

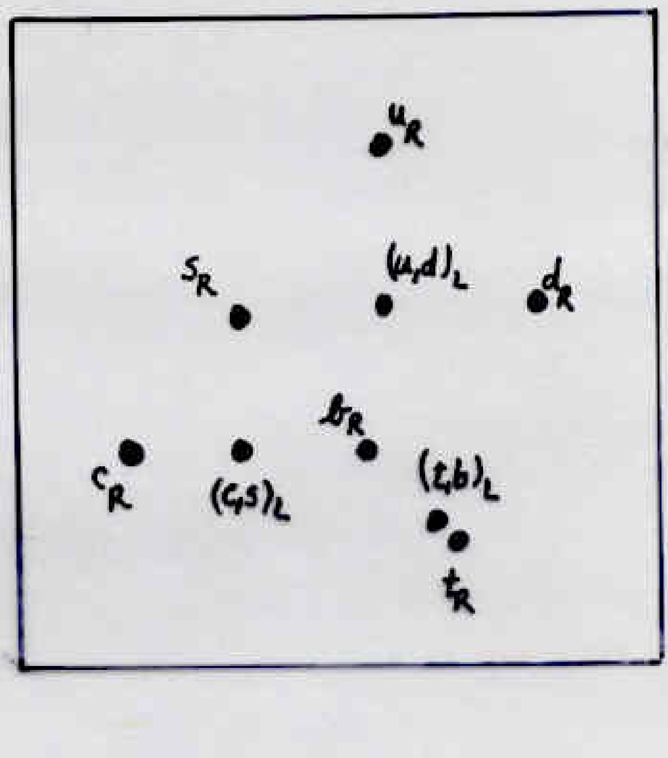
GEOMETRY RATHER THAN SYMMETRY ($\kappa_{ij}\sim 1$)

- FORMALISM FAVOURING MASS MATRICES WITH SEVERAL ZERO ENTRIES
- NO RATIONAL FOR EQUALITIES AMONG NON ZERO ENTRIES

SOME POSSIBLE MASS TEXTURES: (Mirabelli and Schmaltz)

$$\begin{pmatrix} m^u_{11} & 0 & 0 \\ 0 & m^u_{22} & 0 \\ 0 & 0 & m^u_{33} \end{pmatrix}$$

$$\begin{pmatrix} m^d_{11} & m^d_{12} & m^d_{13} \\ 0 & m^d_{22} & m^d_{23} \\ 0 & 0 & m^d_{33} \end{pmatrix}$$



TWO EXTRA DIMENSIONS are needed

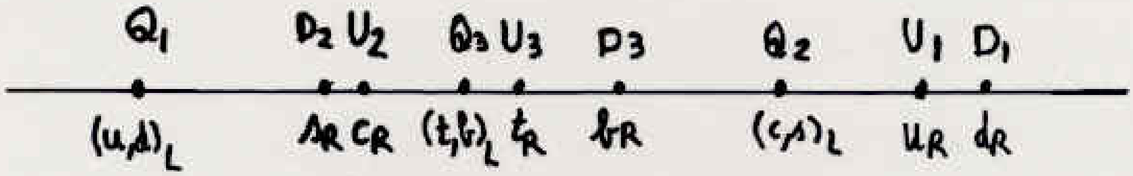
Triangular basis (T)

Nearest-Neighbour-Interaction, NNI, basis

$$M_d = \begin{pmatrix} 0 & a & 0 \\ a' & 0 & b \\ 0 & b' & c \end{pmatrix}, \quad M_u = \begin{pmatrix} 0 & d & 0 \\ d' & 0 & e \\ 0 & e' & f \end{pmatrix}$$

no Hermiticity assumed

Possible to realize with only ONE EXTRA DIMENSION



In the SM the NNI and T forms simply correspond to a choice of weak basis (WB)

$$M_u \to M_u' = W_L^{+} M_u W_R^{u}$$
$$M_d \to M_d' = W_L^{+} M_d W_R^{d}$$

Mirabelli and Schmaltz found

$$M_d = \begin{pmatrix} 0 & 16.974 & 0 \\ 14.510 & 0 & 123.42 \\ 0 & 1373.2 & 2370.2 \end{pmatrix} \text{MeV}$$

$$M_u = \begin{pmatrix} 1.7630 & 0 & 0 \\ 0 & 576.06 & 2.7882 \times 10^{-3} \\ 5902.8 & 0 & 165900 \end{pmatrix} \text{MeV} \simeq \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}$$

geometry similar to picture above, $U_1 \leftrightarrow U_2$

almost unique possible configuration in one extra dimension in order to reproduce masses and $|V_{ij}|$

QUESTION? CAN THERE BE ENOUGH CP VIOLATION WITH ONLY ONE EXTRA DIMENSION

Assume all entries in M_d and M_d complex with arbitrary phases

$$H_u \equiv M_u M_u^\dagger \simeq \begin{pmatrix} m_u^2 & 0 & 0 \\ 0 & m_c^2 & 0 \\ 0 & 0 & m_t^2 \end{pmatrix} ; H_d \equiv M_d M_d^\dagger = \begin{pmatrix} H_{11}^d & 0 & H_{13}^d \\ 0 & H_{22}^d & H_{23}^d \\ H_{13}^{d*} & H_{23}^{d*} & H_{33}^d \end{pmatrix}$$

WB where H_u is diagonal, all $|H_d|_{ij}$ have physical meaning and can be expressed as a function of quark masses and mixing angles

e.g.
$$H_{12}^d = m_d^2 V_{ud} V_{cd}^* + m_s^2 V_{us} V_{cs}^* + m_b^2 V_{ub} V_{cb}^*$$

V_{ij} are V_{CKM} elements

$H_{12}^d = 0$ together with unitarity:

$$\frac{m_s^2 - m_d^2}{m_b^2 - m_d^2} = \frac{|V_{ub}||V_{cb}|}{|V_{us}||V_{cs}|}$$

and

$$\arg\left(V_{ub} V_{cs} V_{us}^* V_{cb}^*\right) = \pi \quad \text{(no CP violation)}$$

Yet $m_{3_1}^u \neq 0$

but still $J \equiv \mathrm{Im}\left(V_{ub} V_{cs} V_{us}^* V_{cb}^*\right) \lesssim 5 \times 10^{-9}$

to be compared to $J \simeq 10^{-5}$ required from ε_K

WB invariant controlling CP violation

$$\mathrm{Tr}[H_u, H_d]^3 = 0$$

Bernabeu, Branco, Gronau

IF M_u diag

$$\mathrm{Tr}[H_u, H_d]^3 = 6i\,(m_u^2 - m_c^2)(m_t^2 - m_c^2)(m_t^2 - m_u^2) \times \mathrm{Im}\left(H_{12}^d H_{23}^d H_{13}^{d*}\right)$$

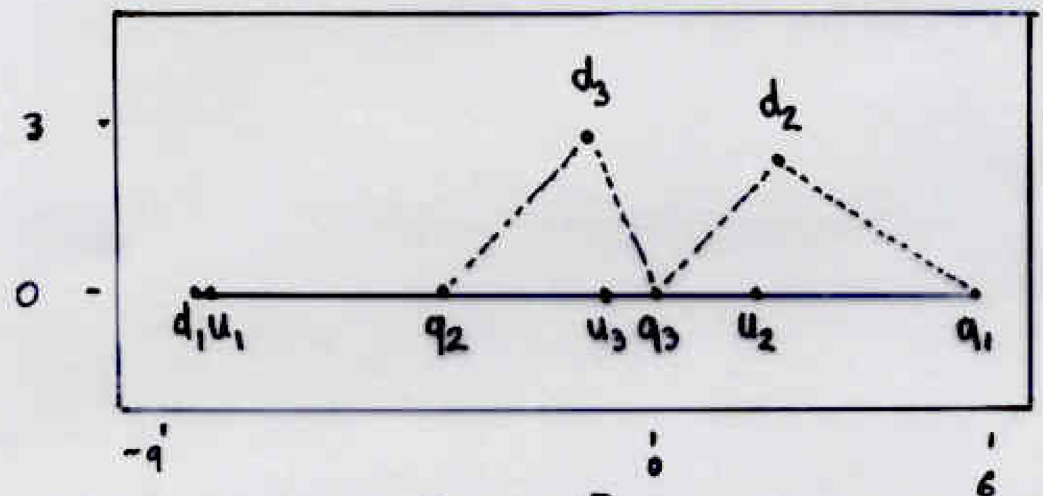
in NNI form for M_d $\qquad H_{12}^d = 0$

In M.S. $m_{3,1}^u$ is "too small"

MORE MIXING REQUIRED FROM M_u!

HENCE BIGGER OFF DIAGONAL ENTRIES NEEDED IN M_u

IMPLYING THE NEED FOR U_i TO GET CLOSER TO Q_i → TWO EXTRA DIMENSIONS NEEDED!



NNI basis :

$$H_{u,d} = \begin{pmatrix} H_{11}^{(u,d)} & 0 & H_{13}^{(u,d)} \\ 0 & H_{22}^{(u,d)} & H_{23}^{(u,d)} \\ H_{13}^{(u,d)*} & H_{23}^{(u,d)*} & H_{33}^{(u,d)} \end{pmatrix}$$

possible to eliminate all complex phases from H_u

$H_u \to K^{\dagger} H_u K \quad , \quad H_d \to K^{\dagger} H_d K$

phases in H_d still arbitrary

Diagonalization

$O_u^{\dagger} H_u O_u = \mathrm{diag}(m_u^2, m_c^2, m_t^2)$

$O_d^{\dagger} K'^{\dagger} H_d K' O_d = \mathrm{diag}(m_d^2, m_s^2, m_b^2)$

$K' = \mathrm{diag}(1, e^{i\phi}, e^{i\sigma})$ with loss of generality

$V_{CKM} = O_u^{\dagger} K' O_d$ $\quad\quad$ ϕ, σ free parameters

Expanding V_{CKM}

eg $V_{us} = O_{11}^{u} O_{12}^{d} + O_{21}^{u} O_{22}^{d} e^{i\phi} + O_{31}^{u} O_{32}^{d} e^{i\sigma}$

....

Series of guided trials using these equations together with quark mass spectrum

Results

- impossibility of generating sufficient CP violation with correct masses and mixing in one dimension extra
- no solution to masses and mixing significantly different from MS

Striking connection between $|V_{us}|$ and strength of ~~CP~~

Strategy in the search for solutions

Diagonalization of mass matrices

$$O_u^{\dagger} H_u O_u = \mathrm{diag}(m_u^2, m_c^2, m_t^2)$$

$$O_d^{\dagger} K'^{\dagger} H_d K' O_d = \mathrm{diag}(m_d^2, m_s^2, m_b^2)$$

$$K' = \mathrm{diag}(1, e^{i\phi}, e^{i\sigma})$$

ϕ, σ free parameters

$$V_{CKM} = O_u^{\dagger} K' O_d$$

In one extra dimension V_{CKM} dominated by O_d

Existence of non negligible contribution from O_u required

Interesting set of locations found:

$$q_i = \frac{1}{\mu}\begin{pmatrix} 5.941;0 \\ -4.008;0 \\ 0;0 \end{pmatrix};\quad u_i = \frac{1}{\mu}\begin{pmatrix} -8.347;0 \\ 1.815;0 \\ -0.941;0 \end{pmatrix};\quad d_i = \frac{1}{\mu}\begin{pmatrix} -8.421;0 \\ 2.219;2.332 \\ -1.253;2.767 \end{pmatrix}$$

assuming $K^{if} v = 1.5\, m_t$

$$M_d = \begin{pmatrix} 0 & 16.112 & 0 \\ 14.690 & 0 & 121.77 \\ 0 & 1400 & 2467.8 \end{pmatrix} \mathrm{MeV}$$

$$M_u = \begin{pmatrix} 0 & 50.0 & 0 \\ 20.3 & 0 & 2258 \\ 0 & 48000 & 160000 \end{pmatrix} \mathrm{MeV}$$

$m_u = 1.5$ MeV
$m_d = 3.2$ MeV
$m_s = 63.3$ MeV
$m_c = 651$ MeV
$m_b = 2839$ MeV
$m_t = 167059$ MeV

corresponding to the quark masses evaluated at a common scale m_t. These are in agreement with the experimental values of the quark masses (computed in the $\overline{MS}$ renormalization scheme, with up and down and strange quark masses evaluated at a scale of 2 GeV and the other evaluated at a scale equal to $\overline{MS}$ mass), as computed by the Particle Data Group.

large value of ϕ crucial for CP

choice $\phi = 85°$, $\sigma = 0°$

$$|V_{CKM}| = \begin{pmatrix} 0.9753 & 0.2208 & 0.0034 \\ 0.2205 & 0.9746 & 0.0384 \\ 0.0108 & 0.0370 & 0.9993 \end{pmatrix}$$

$$J \equiv |\mathrm{Im}(V_{ub} V_{cs} V_{us}^{*} V_{cb}^{*})| \simeq 2.2 \times 10^{-5}$$

$$\beta \equiv \arg\left(-\frac{V_{cd} V_{cb}}{V_{td} V_{tb}^{*}}\right)$$

$\sin(2\beta) = 0.47$ $\quad \beta \approx 14°$

Same idea applied to leptonic masses and mixing

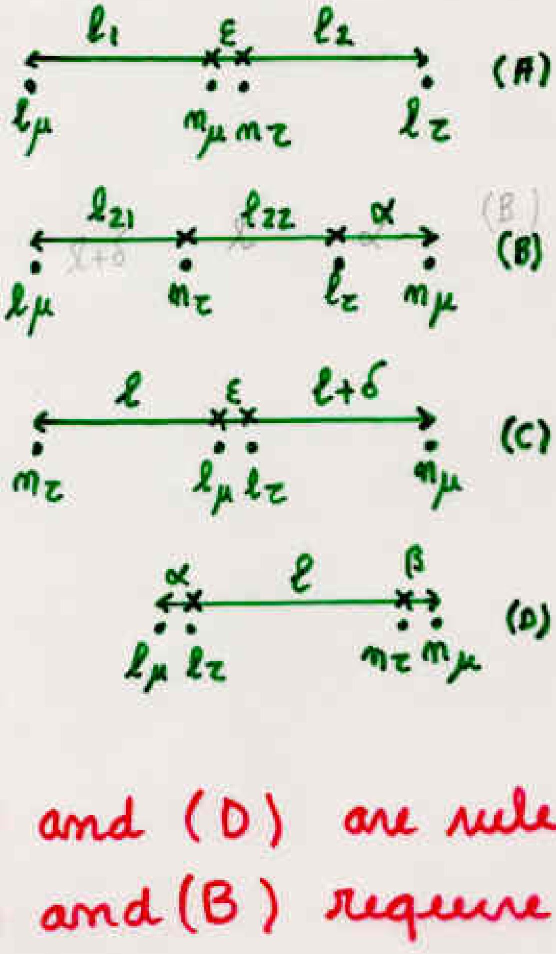
- existence of three righthanded neutrinos assumed
- only Dirac masses

n_i (ν_R) ; ℓ_i (doublets) ; e_i (ℓ_R) . $i = e, \mu, \tau$

Smallness of neutrino masses related to separation between n's and ℓ's

Large mixing requires some elements of M_ν of the same order (for M_e diagonal)

First step : two family analysis (μ, τ) large mixing



(A) Fine tuning between ℓ_1, ℓ_2 required. Charged lepton mass matrix almost diagonal

(B) Fine tuning between α, ℓ_{22}. Charged lepton mass matrix almost diagonal

(C) $\mu\varepsilon \gtrsim 1$ required ($\tau \to \mu\gamma$) yet not possible together with large mixing

(D) $\mu\alpha \gtrsim 1$ required ($\tau \to \mu\mu$) no good solutions

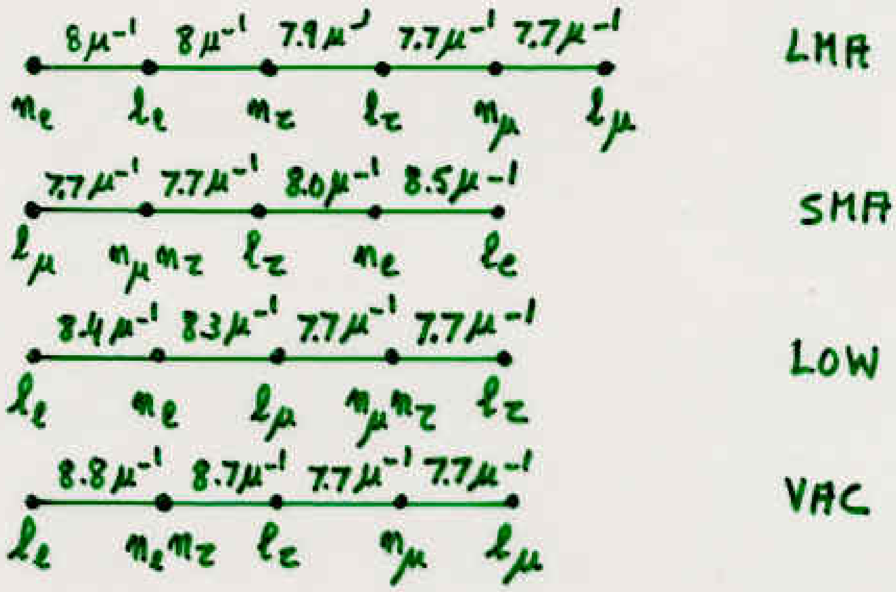
(C) and (D) are ruled out

(A) and (B) require fine-tuning

Three family analysis

- Realistic solutions exist LMA, SMA, LOW, VAC
- Fine tuning as for two generations required
- Only configurations yielding hierarchical neutrino masses were found.
- The charged lepton mass matrix is virtually diagonal for almost all solutions

Four examples which yield neutrino masses and mixing angles satisfying all neutrino data



In all these examples the brane is too thick
Motivation to introduce more than one extra dimension

Neutrino Masses without right-handed neutrinos

- See saw mechanism not an option for generating very small neutrino masses (no high energy scale)
- Mechanism to obtain small Majorana neutrino masses
  (suppression factor due to lepton number conserved global symmetry broken at some far away brane)
  Arkani-Hamed et al
  Arkani-Hamed and Dimopoulos

Difficult to obtain phenomenologically acceptable parameters

Large enough atmospheric mixing angles forces closeness to current experimental bounds on flavour violating μ and τ decays

Very hierarchical neutrino mass-squared differences were not obtained : LOW and VAC not accommodated

SMA and LMA solutions still viable

Conclusions

Flavour puzzle addressed through geometry rather than symmetry

Mass matrices with zero textures are favoured

Non equivalence of WB ie different choices of WB correspond to different geometries leading to different physics

With only one extra dimension it is possible to obtain correct masses and mixing in quark sector

Yet sufficient CP violation requires two extra dimensions

Leptonic sector:

- fine tuning required for large neutrino mixing
- in one dimension the brane is too thick
- with Dirac masses all solar neutrino solutions can be accommodated
- with Majorana masses the model is very constrained, only SMA and LMA solutions are still viable