Experimental Results on T and CPT

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Testing T and CPT symmetries in K^0

- **The remarkable properties of the K^0 system:**
  - Production in S conserving strong interaction
  - Decay through S violating weak interaction
  - The decay states $|K_S\rangle$ and $|K_L\rangle$ form a two particles system:
    - oscillating between $|K^0\rangle$ (S=+1) and $|\overline{K}^0\rangle$ (S=−1)
    - with a frequency $\omega = \Delta m = 3.489\times10^{-12}$ MeV $= .535\times10^{10}\ \hbar s^{-1}$ comparable to $\Delta\Gamma = \Gamma_S - \Gamma_L = 1.119\times10^{10}\ s^{-1}$

      ➡ Oscillation observable before the K_S disappearance.

- **The facility of a strangeness tagged K^0 production**
- **The capability to identify decaying** $\overline{K}^0, K^0$ by $e^{\mp}\pi\nu$

➡ **Variety of high precision tests of discrete symmetries (T and CPT)**

Time evolution of the K^0 system

- The neutral kaon state vectors satisfy the Schrödinger equation:

$$i\frac{d|\psi\rangle}{dt} = \mathcal{H}|\psi\rangle \qquad H = H_S + H_{em} + H_{wk} = H_0 + H_{wk}$$

- The states $|K^0\rangle$, $|\overline{K}^0\rangle$ are eigenstates of H_0 and S

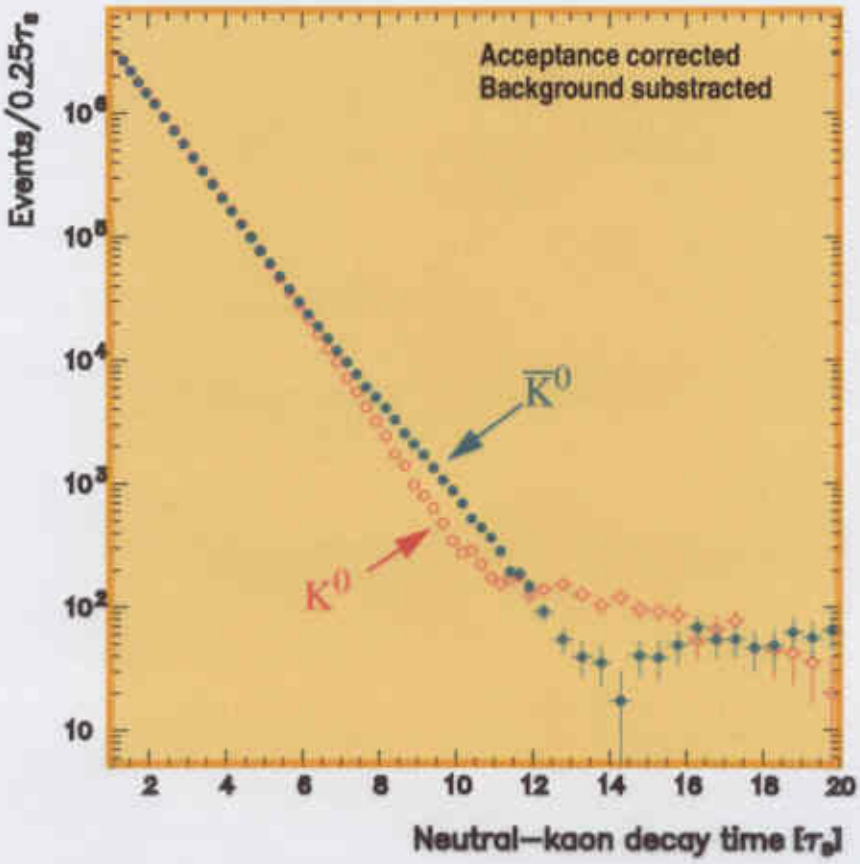
$$\mathcal{H}_0|K^0\rangle = m_0|K^0\rangle \quad \mathcal{H}_0|\overline{K}^0\rangle = m_0|\overline{K}^0\rangle$$

$$S|K^0\rangle = |K^0\rangle \qquad S|\overline{K}^0\rangle = -|\overline{K}^0\rangle$$

- At t=0 $|\psi_0\rangle = \psi_{K^0}(0)|K^0\rangle + \psi_{\overline{K}^0}(0)|\overline{K}^0\rangle$
- Since H_{wk} is not S invariant,
  a neutral kaon will change its strangeness with time $\Rightarrow$ $\Delta S = 2$ transitions

CP violation in $K^0, \overline{K}^0 \rightarrow \pi^+\pi^-$ ...

- **CPLEAR 99:**
  - 70×10^6 **events, selected above** $1\tau_S$



  - **Corrections:**
    - **Differences in the tagging efficiency between** K^0 **and** $\overline{K}^0$
    - **Regeneration effect** (dedicated data taking in 1996)

Time evolution of the K^0 system...

- **Solving the equation ♥ using the** Weisskopf-Wigner **approximation one obtains:**

$\psi = e^{-i\Lambda t}\psi_0$ **which satisfies** $i\frac{d\psi}{dt} = \Lambda\psi$

where $\psi(t) = \begin{pmatrix} \psi_{K^0}(t) \\ \psi_{\overline{K}^0}(t) \end{pmatrix}$ **with** $\Psi(t=0) = \Psi_0$

$\Lambda = M - \frac{i}{2}\Gamma$ is a time independent 2x2 matrix in terms of the Hermitian mass and decay matrices

$M = M^\dagger, \quad \Gamma = \Gamma^\dagger$

$$M_{\alpha\alpha'} = m_0\delta_{\alpha\alpha'} + \langle\alpha|\mathcal{H}_{wk}|\alpha'\rangle + \mathcal{P}\sum_\beta\left(\frac{\langle\alpha|\mathcal{H}_{wk}|\beta\rangle\langle\beta|\mathcal{H}_{wk}|\alpha'\rangle}{m_0 - E_\beta}\right)$$

$$\Gamma_{\alpha\alpha'} = 2\pi\sum_\beta\langle\alpha|\mathcal{H}_{wk}|\beta\rangle\langle\beta|\mathcal{H}_{wk}|\alpha'\rangle\delta(m_0 - E_\beta)$$

$(\alpha, \alpha' = K^0, \overline{K}^0)$

Time evolution of the K^0 system...

- Amplitudes describing the transition probability of an S-eigenstate A into an other one B:

$$\langle B|\, e^{-i\Lambda t}\, |A\rangle$$

are used to define rate asymmetries, measurable experimentally.

- The asymmetry

$$\begin{aligned} A_T &\equiv \frac{|\langle K^0|\, e^{-i\Lambda t}\, |\overline{K}^0\rangle|^2 - |\langle \overline{K}^0|\, e^{-i\Lambda t}\, |K^0\rangle|^2}{|\langle K^0|\, e^{-i\Lambda t}\, |\overline{K}^0\rangle|^2 + |\langle \overline{K}^0|\, e^{-i\Lambda t}\, |K^0\rangle|^2} \\ &= \frac{|\Lambda_{K^0\overline{K}^0}|^2 - |\Lambda_{\overline{K}^0 K^0}|^2}{|\Lambda_{K^0\overline{K}^0}|^2 + |\Lambda_{\overline{K}^0 K^0}|^2} \end{aligned}$$

is the necessary and sufficient condition for T violation.

Time evolution of the K^0 system...

- The asymmetry:

$$A_{\mathrm{CPT}} \equiv \frac{|\langle \overline{\mathrm{K}}^0| \mathrm{e}^{-\mathrm{i}\Lambda t} |\overline{\mathrm{K}}^0\rangle|^2 - |\langle \mathrm{K}^0| \mathrm{e}^{-\mathrm{i}\Lambda t} |\mathrm{K}^0\rangle|^2}{|\langle \overline{\mathrm{K}}^0| \mathrm{e}^{-\mathrm{i}\Lambda t} |\overline{\mathrm{K}}^0\rangle|^2 + |\langle \mathrm{K}^0| \mathrm{e}^{-\mathrm{i}\Lambda t} |\mathrm{K}^0\rangle|^2}$$

is the necessary and sufficient condition for CPT violation.

In an analogue way, we can define the asymmetry:

$$A_{\mathrm{CP}} \equiv \frac{|\langle f| \mathrm{e}^{-\mathrm{i}\Lambda t} |\overline{\mathrm{K}}^0\rangle|^2 - |\langle f| \mathrm{e}^{-\mathrm{i}\Lambda t} |\mathrm{K}^0\rangle|^2}{|\langle f| \mathrm{e}^{-\mathrm{i}\Lambda t} |\overline{\mathrm{K}}^0\rangle|^2 + |\langle f| \mathrm{e}^{-\mathrm{i}\Lambda t} |\mathrm{K}^0\rangle|^2}$$

describing the difference between the transitions of $\overline{\mathrm{K}}^0$ and K^0 into the CP eigenstates $|f\rangle = \frac{|\mathrm{K}^0\rangle \pm |\overline{\mathrm{K}}^0\rangle}{\sqrt{2}}$

Relation between invariance laws and observables

If $\mathcal{H}_{\rm wk}$ has the property	invariance		
$\mathcal{T}^{-1}\,\mathcal{H}_{\rm wk}\;\mathcal{T} = \mathcal{H}_{\rm wk}$	$\mathcal{T}$	$\Lambda_{\rm K^0\overline{K}^0} = \Lambda_{\rm \overline{K}^0K^0}$	$A_{\rm T} = 0$
$(\mathcal{CPT})^{-1}\,\mathcal{H}_{\rm wk}\;(\mathcal{CPT}) = \mathcal{H}_{\rm wk}$	$\mathcal{CPT}$	$\Lambda_{\rm K^0K^0} = \Lambda_{\rm \overline{K}^0\overline{K}^0}$	$A_{\rm CPT} = 0$
$(\mathcal{CP})^{-1}\,\mathcal{H}_{\rm wk}\;(\mathcal{CP}) = \mathcal{H}_{\rm wk}$	$\mathcal{CP}$	$\Lambda_{\rm K^0K^0} = \Lambda_{\rm \overline{K}^0\overline{K}^0}$ and	
		$\Lambda_{\rm K^0\overline{K}^0} = \Lambda_{\rm \overline{K}^0K^0}$	$A_{\rm CP} = 0$

- **The solution of the Schrödinger equation can be expressed as a function of the Λ-eigenvalues:**

$$\psi = e^{-i\Lambda t}\psi_0 = \begin{pmatrix} f_+ + 2\delta f_- & 2(\varepsilon - \sigma) f_- \\ -2(\varepsilon + \sigma) f_- & f_+ - 2\delta f_- \end{pmatrix} \psi_0$$

with:

$$f_\pm(t) = \frac{e^{-i\lambda_S t} \pm e^{-i\lambda_L t}}{2}$$

and

$$\begin{aligned} \delta &\equiv (\Lambda_{\overline{K}^0\overline{K}^0} - \Lambda_{K^0K^0})/(2\Delta\lambda) \\ \varepsilon &\equiv (\Lambda_{\overline{K}^0K^0} - \Lambda_{K^0\overline{K}^0})/(2\Delta\lambda) \\ \sigma &\equiv (\Lambda_{\overline{K}^0K^0} + \Lambda_{K^0\overline{K}^0})/(2\Delta\lambda) \end{aligned}$$

where the eigenvalues

$$\lambda_{L,S} = \frac{1}{2}\left(\Lambda_{K^0K^0} + \Lambda_{\overline{K}^0\overline{K}^0} \pm \sqrt{(\Lambda_{K^0K^0} - \Lambda_{\overline{K}^0\overline{K}^0})^2 + 4\Lambda_{K^0\overline{K}^0}\Lambda_{\overline{K}^0K^0}}\right)$$

$$\lambda_{L,S} = m_{L,S} - \frac{i}{2}\Gamma_{L,S}$$

and

$$\Delta\lambda = \lambda_L - \lambda_S = \Delta m + \frac{i}{2}\Delta\Gamma \quad (\Delta\Gamma = \Gamma_S - \Gamma_L,\ \Delta m = m_L - m_S)$$

The evolution of initial K^0 and $\overline{K}^0$

$$
\begin{aligned}
|K^0(t)\rangle &= [f_+(t) + \frac{\Lambda_{\overline{K}^0\overline{K}^0} - \Lambda_{K^0K^0}}{\Delta\lambda} f_-(t)]\,|K^0\rangle - 2\frac{\Lambda_{K^0\overline{K}^0}}{\Delta\lambda} f_-(t)\,|\overline{K}^0\rangle \\
|\overline{K}^0(t)\rangle &= [f_+(t) - \frac{\Lambda_{\overline{K}^0\overline{K}^0} - \Lambda_{K^0K^0}}{\Delta\lambda} f_-(t)]\,|\overline{K}^0\rangle - 2\frac{\Lambda_{\overline{K}^0K^0}}{\Delta\lambda} f_-(t)\,|K^0\rangle
\end{aligned}
$$

- Using the previous definitions of f, $\varepsilon = (\varepsilon_S + \varepsilon_L)/2$, δ, we obtain in the $\overline{K}^0$, K^0 and K_S, K_L bases:

$$
\begin{aligned}
|K^0(t)\rangle &= [f_+(t) + 2\delta f_-(t)]\,|K^0\rangle + (1 - 2\varepsilon) f_-(t)\,|\overline{K}^0\rangle \\
&= \frac{1}{\sqrt{2}}[(1 - \varepsilon + \delta)e^{-i\lambda_S t}\,|K_S\rangle + (1 - \varepsilon - \delta)e^{-i\lambda_L t}\,|K_L\rangle]
\end{aligned}
$$

$$
\begin{aligned}
|\overline{K}^0(t)\rangle &= [f_+(t) - 2\delta f_-(t)]\,|\overline{K}^0\rangle + (1 + 2\varepsilon) f_-(t)\,|K^0\rangle \\
&= \frac{1}{\sqrt{2}}[(1 + \varepsilon - \delta)e^{-i\lambda_S t}\,|K_S\rangle - (1 + \varepsilon + \delta)e^{-i\lambda_L t}\,|K_L\rangle]
\end{aligned}
$$

Semileptonic decays as strangeness tag

- Four decay amplitudes can be defined:

$$\Delta S = \Delta Q \rightarrow \quad \mathcal{A}_+ = \langle e^+\pi^-\nu | \mathcal{H}_{wk} | K^0 \rangle \qquad \overline{\mathcal{A}}_- = \langle e^-\pi^+\overline{\nu} | \mathcal{H}_{wk} | \overline{K}^0 \rangle$$

$$\Delta S = -\Delta Q \rightarrow \quad \mathcal{A}_- = \langle e^-\pi^+\overline{\nu} | \mathcal{H}_{wk} | K^0 \rangle \qquad \overline{\mathcal{A}}_+ = \langle e^+\pi^-\nu | \mathcal{H}_{wk} | \overline{K}^0 \rangle$$

and decomposed in CPT invariant and CPT violating parts:

$$\Delta S = \Delta Q \rightarrow \quad \mathcal{A}_+ = \overset{\text{CPT}}{a} + \overset{\text{CPT}}{b} \qquad \overline{\mathcal{A}}_- = \overset{\text{CPT}}{a^*} - \overset{\text{CPT}}{b^*}$$

$$\Delta S = -\Delta Q \rightarrow \quad \mathcal{A}_- = c + d \qquad \overline{\mathcal{A}}_+ = c^* - d^*$$

- The following quantities decribe $\Delta S = \Delta Q$ violation in l^+, l^- and ~~CPT~~ in the $\Delta S = \Delta Q$ decay amplitudes

$$x = \frac{\overline{\mathcal{A}}_+}{\mathcal{A}_+} \qquad \overline{x} = \left(\frac{\mathcal{A}_-}{\overline{\mathcal{A}}_-}\right)^* \qquad y = \frac{\overline{\mathcal{A}}_-^* - \mathcal{A}_+}{\overline{\mathcal{A}}_-^* + \mathcal{A}_+}$$

$x_+ = \frac{(x+\overline{x})}{2}$ [$\Delta S = -\Delta Q$, CPT] and $x_- = \frac{(x-\overline{x})}{2}$ [$\Delta S = -\Delta Q$, ~~CPT~~]

Semileptonic decay rates

$$R_+(\tau) \equiv R\left[\mathrm{K}^0{}_{t=0} \to (\mathrm{e}^+\pi^-\nu)_{t=\tau}\right] \qquad \overline{R}_-(\tau) \equiv R\left[\overline{\mathrm{K}}^0_{t=0} \to (\mathrm{e}^-\pi^+\overline{\nu})_{t=\tau}\right]$$
$$R_-(\tau) \equiv R\left[\mathrm{K}^0{}_{t=0} \to (\mathrm{e}^-\pi^+\overline{\nu})_{t=\tau}\right] \qquad \overline{R}_+(\tau) \equiv R\left[\overline{\mathrm{K}}^0_{t=0} \to (\mathrm{e}^+\pi^-\nu)_{t=\tau}\right]$$

- Using $|\overline{\mathrm{K}}^0(t)\rangle$, $|\mathrm{K}^0(t)\rangle$ and phase space integration the rates can be expressed in terms of the previously defined parameters :

$$R(\tau) = \mathrm{f}(\epsilon,\ \delta,\ x,\ \mathfrak{x},\ y)$$

- Their measurement as a function of the decay eigentime τ allows
  - Definition of the rate asymmetries
    - Determination of T and CPT parameters

ex. $$A_T^{exp} = \frac{\overline{R}_+(\Delta\tau) - R_-(\Delta\tau)}{\overline{R}_+(\Delta\tau) + R_-(\Delta\tau)}$$

with reduced systematic errors.

The CPLEAR method

- $K^0, \bar{K}^0$ production through $\bar{p}$ annihilation in H_2 at rest

$$\bar{p}p \longrightarrow \begin{matrix} K^-\pi^+ K^0 \\ K^+\pi^- \bar{K}^0 \end{matrix} \quad \text{golden b.r. } 2\,10^{-3}$$

- S tagging at the production $K^{\mp}\pi^{\pm}$ pair $(\vec{p}_{K^{\mp}}; \vec{p}_{\pi^{\pm}})$
- S tagging at the decay $e^{\pm}$ in $e\pi\nu$ $(\vec{p}_{e^{\pm}}; \vec{p}_{\pi^{\mp}})$
- Trigger/DAQ

multilevel HWP trigger

$N_{tracks}, \Sigma Q_i = 0 \longrightarrow$ full kinem. reconst. $\vec{p}_{\bar{K}^0}$

$\sim 10^6$ annihilations s^{-1}

$3.2\,10^3$ golden ; $\sim 50\%$ decays in CPLEAR.

DAQ rates $\sim 400\,Hz$ (1 Mb/s)

$\longrightarrow N_{\bar{p}} \sim 1.1\,10^{13}$ (20 pg); 12 Tb/57600 cartr.

Event selection $\bar{p}p \to K^{\mp}\pi^{\pm}\overleftrightarrow{K}^{0}$, $\overleftrightarrow{K}^{0} \to e\pi\nu$

- $N_{tracks} = 4$, $\sum Q_i = 0$

  pattern recognition, track fit, vertexing, particle id

  decay $e^{\pm}$ id (NN using $\vec{p}$, dE/dx, N_{pe}, t-o-f)

  $\pi/e < .02$; $\langle \epsilon_e \rangle = .85$

- Constrained fits (MM, IM, topology)

  1C-fit : $\bar{p}p \to K^{\pm}\pi^{\mp}\overleftrightarrow{K}^{0}$ $P > 10\%$

  6C-fit : kinematics, E,P conservation

  $\vec{p}_{\overleftrightarrow{K}^0}$ alignment, good z-intersections $P > 5\%$

  4C-fit : eliminating $\pi^+\pi^-$ background

  kin. constraint $\curvearrowright$ rejection if $P > 10\%$

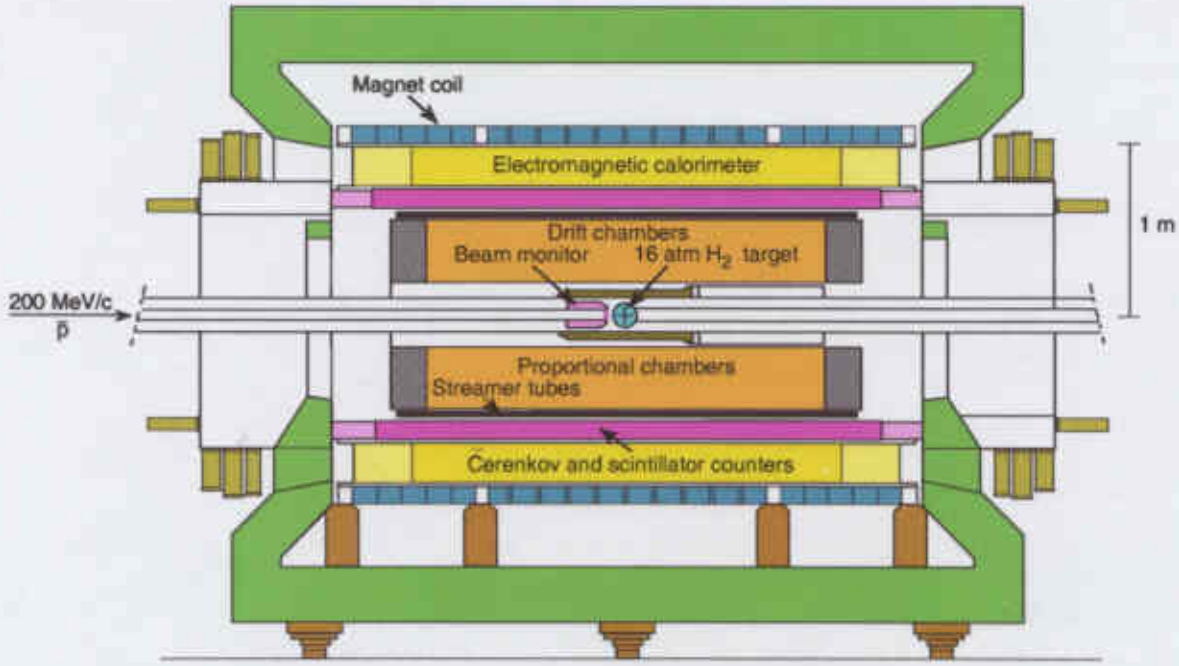
* fitted momenta and vertices (6C-fit)

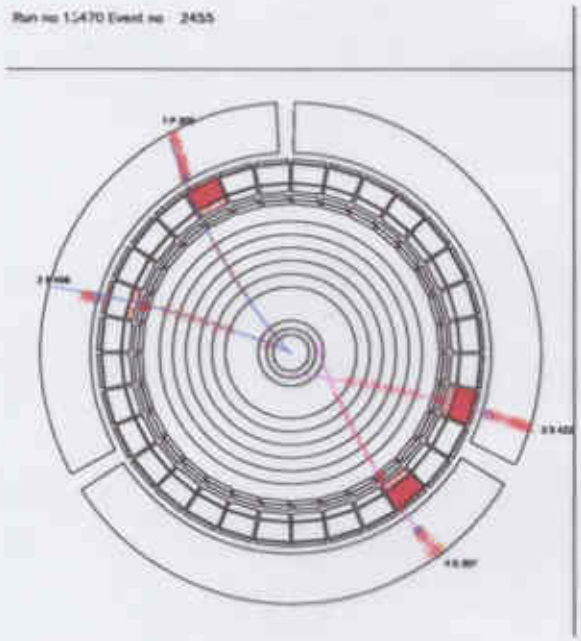
$\Downarrow$

decay time resolution $\Delta\tau$ $.05\,\tau_S$ to $< .3\,\tau_S$

$\longrightarrow$ $1.3\ 10^6$ $e\pi\nu$ events $> 1\,\tau_S$

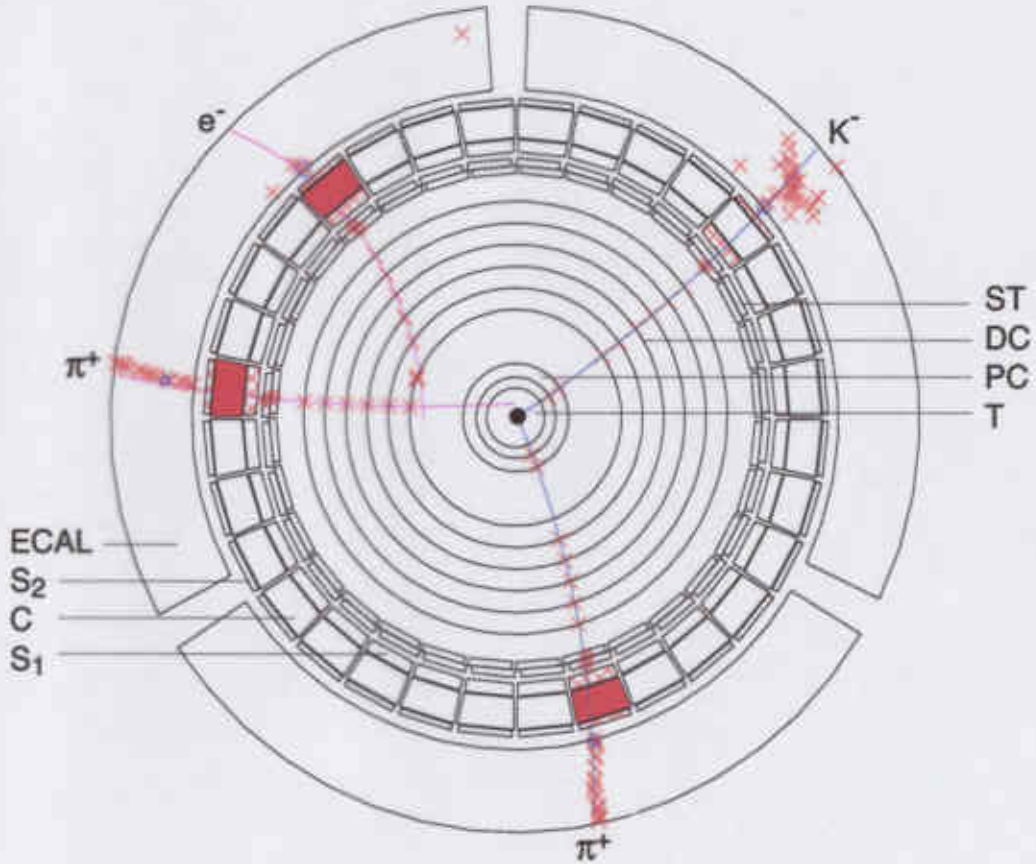
CPLEAR Detector





$$\approx 70M \quad K^\circ(\overline{K}^\circ) \to \pi^+\pi^-$$
$$\approx 1.3M \quad K^\circ(\overline{K}^\circ) \to \pi e \nu$$
$$\approx 0.5M \quad K^\circ(\overline{K}^\circ) \to \pi^+\pi^-\pi^0$$
$$\approx 2M \quad K^\circ(\overline{K}^\circ) \to \pi^0\pi^0$$
$$\approx 17000 \quad K^\circ(\overline{K}^\circ) \to \pi^0\pi^0\pi^0$$

K^o to $\overline{K}^o$ transition with $\overline{K}^o \rightarrow e^- \pi^+ \nu_e$ decay



Normalisation of K^0 and $\bar{K}^0$ rates in asymmetries

** advantage of reduced systematics (due to efficiencies) in the asymmetries!

* compensation of geometrical acceptance differences using both $\vec{B}$ polarities

→ normalisation was done on an event-by-event basis using weights

$$w_{\pm} = \xi \cdot \eta \cdot \omega_r$$

$\xi = \epsilon(\vec{p}_{K^+}, \vec{p}_{\pi^-}) / \epsilon(\vec{p}_{K^-}, \vec{p}_{\pi^+})$ at the production vertex

determined from $\pi^+\pi^-$ $< 4\tau_S$ stats, S/B, small ~~CP~~

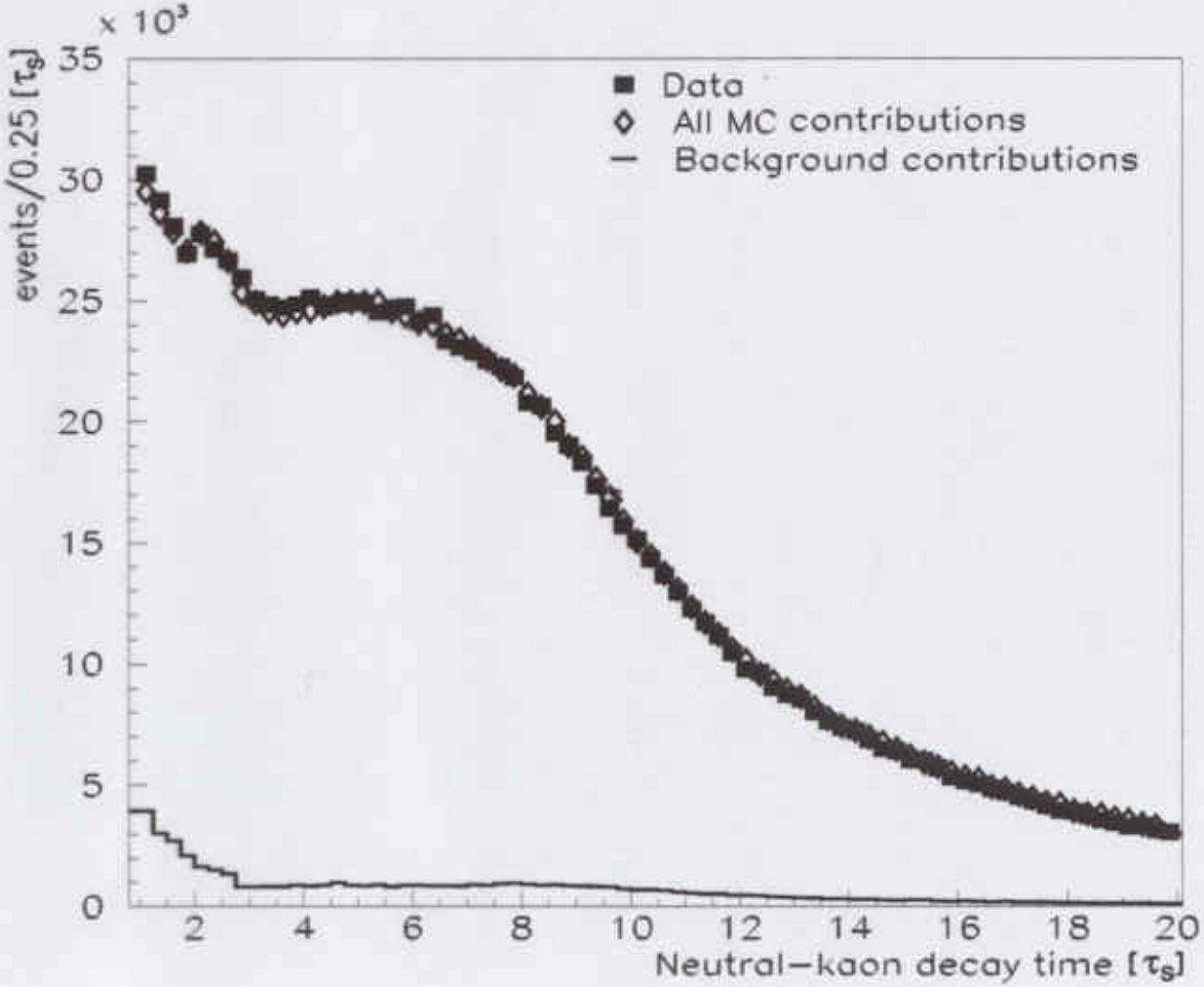
$\eta = \epsilon(\vec{p}_{e^-}, \vec{p}_{\pi^+}) / \epsilon(\vec{p}_{e^+}, p_{\pi^-})$ at the decay vertex

determined from $(\gamma \to e^+e^-)$ data + MC

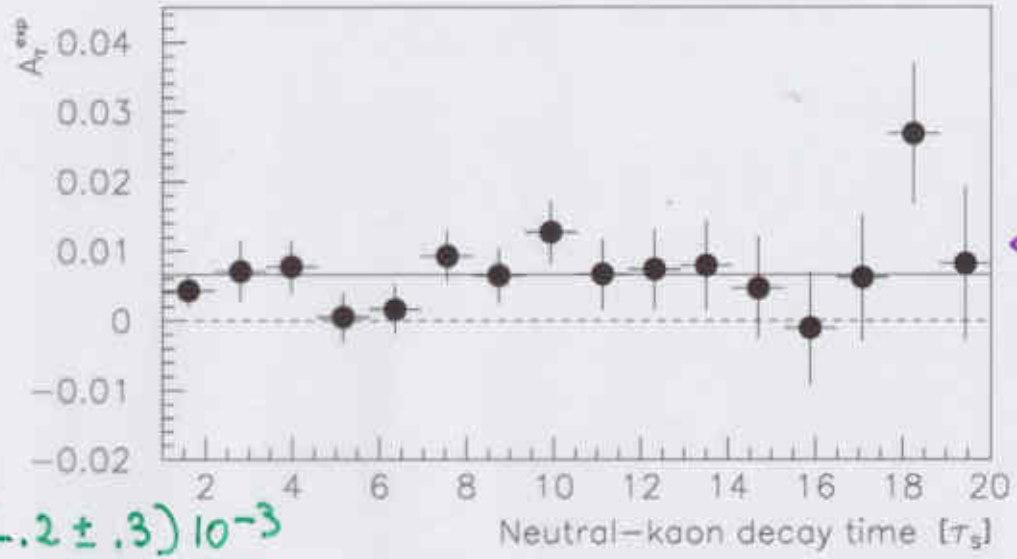
$\omega_r \equiv$ the ratio of decay probabilities of an initial K^0 ($\bar{K}^0$) propagating in vacuum and through the detector.

determined from MC and later from data.

Decay-time distribution for real and simulated $e\pi\nu$ data



A_T asymmetry Results



$\langle A_T^{exp} \rangle = (6.6 \pm 1.3 \pm 1.0)\, 10^{-3}$

the ~~CPT~~ terms

$\mathrm{Re}(y) + \mathrm{Re}(x_-) = (-.2 \pm .3)\, 10^{-3}$

$$A_T(\tau) = 4(\mathrm{Re}(\varepsilon) - \mathrm{Re}(y) - \mathrm{Re}(x_-)) + 2\frac{\mathrm{Re}(x_-)(e^{-\frac{1}{2}\Delta\Gamma\tau} - \cos(\Delta m\tau)) + \mathrm{Im}(x_+)\sin(\Delta m\tau)}{\cosh(\frac{1}{2}\Delta\Gamma\tau) - \cos(\Delta m\tau)}$$

for $t \gg \tau_S$ $\quad A_T^{exp} \to 4\,\mathrm{Re}(\epsilon) - 4\,\mathrm{Re}(y) - 4\,\mathrm{Re}\,x_-$

$$\begin{aligned} 4\mathrm{Re}(\varepsilon) &= (6.2 \pm 1.4_{\mathrm{stat}} \pm 1.0_{\mathrm{syst}}) \times 10^{-3} \\ \mathrm{Im}(x_+) &= (1.2 \pm 1.9_{\mathrm{stat}} \pm 0.9_{\mathrm{syst}}) \times 10^{-3} \\ \chi^2/\mathrm{ndf} &= 0.84\,, \quad \mathrm{ndf} = 606 \end{aligned}$$

Summary of systematic errors on A_T^{exp}

Source	Precision	$\langle A_T^{exp} \rangle$ $[10^{-3}]$	$\mathrm{Im}(x_+)$ $[10^{-3}]$
Background level *	$\pm 10\%$	± 0.03	± 0.2
Background asymmetry	± 0.01	± 0.02	± 0.5
Normalization (ξ)	$\pm 4.3 \times 10^{-4}$	± 0.2	± 0.1
Normalization (η)	$\pm 2.0 \times 10^{-3}$	± 1.0	± 0.4
Decay-time resolution *	$\pm 10\%$	negligible	± 0.6
Regeneration	Ref. [30]	± 0.1	± 0.1
Total syst.		± 1.0	± 0.9

Concerns or doubts on the interpretation of A_T^{exp}

see L. Alvarez-Gaumé et al
Physics Letters B458, 347 (1999)

(or the effect of using $l_{\pi\nu}$ in the observation !)

→ Since the Hamiltonian H of the K system

$H \neq H^+$ non Hermitean

$\left\{ \begin{array}{ll} \text{masses} & m \\ \text{widths} & \Gamma \\ \text{eigenstates} & \psi \end{array} \right\}$ have to be found by V_L, V_R matrices such that $V_L H V_R$ diagonal.

i.e. looking at the correct orthogonality conditions for incoming and outgoing states in a "dual complex space" L- and R-.

Direct test of time reversibility, T

→ A system at a final configuration retraces its way back to some initial configuration by reversing the $\vec{v}_i$

$$T\left[\langle \psi^{out}(t_f) | \phi^{in}(t_i)\rangle\right] = \langle \phi^{out}(t_f) | \psi^{in}(t_i)\rangle$$

initial and final quantum mechanical states are interchanged with identical positions and $-\vec{v}_i$.

→ Time reversal invariance means

$$|\langle \bar{K}^{0\,out}(t_f) | K^{0\,in}(t_i)\rangle|^2 = |\langle K^{0\,out}(t_f) | \bar{K}^{0\,in}(t_i)\rangle|^2 .$$

* Departure from T-invariance is given by the Kabir asymmetry,

$$A_T^{th} = \frac{P_{\bar{K}K}(\Delta t) - P_{K\bar{K}}(\Delta t)}{P_{\bar{K}K}(\Delta t) + P_{K\bar{K}}(\Delta t)}$$

$$= \frac{|\langle \bar{K}^{0\,out}(t_f) | K^{0\,in}(t_i)\rangle|^2 - |\langle K^{0\,out}(t_f) | \bar{K}^{0\,in}(t_i)\rangle|^2}{+}$$

** A_T^{th} is proven to be t-independent.

to go $\longrightarrow 4\,\mathrm{Re}(\epsilon)$ without any assumption CPT or unitarity.

The CPLEAR measurement, A_T^{exp}

The CPLEAR asymmetry

$$\rightarrow A_T^{exp} = \frac{\bar{R}_+(\Delta t) - R_-(\Delta t)}{\bar{R}_+(\Delta t) + R_-(\Delta t)}$$

where $\bar{R}_+(\Delta t) = |\langle e^+\pi^-\nu(t_f)|\overset{\Delta S = -\Delta Q}{\bar{K}^{0\,in}(t_i)}\rangle$

$$+ \langle e^+\pi^-\nu(t_f)|\underset{\Delta S = \Delta Q}{K^{0\,in}(t_f)}\rangle \times \overbrace{\langle K_0^{out}(t_f)|\bar{K}^{0\,in}(t_i)\rangle|^2}^{\bar{K}^0 \to K^0 \text{ oscillation}}$$

$$R_-(\Delta t) = |\langle e^-\pi^+\nu(t_f)|K_0^{in}(t_i)\rangle$$

$$+ \langle e^-\pi^+\nu(t_f)|\bar{K}_0^{in}(t_i)\rangle \times \langle \bar{K}^{0\,out}(t_f)|K^{0\,in}(t_i)\rangle|^2$$

$$* \quad A_T^{exp} \longrightarrow \simeq 4\,\mathrm{Re}(\epsilon) - 4\,\mathrm{Re}(y) - 4\,\mathrm{Re}(x_-)$$ proper normalization of the experiment

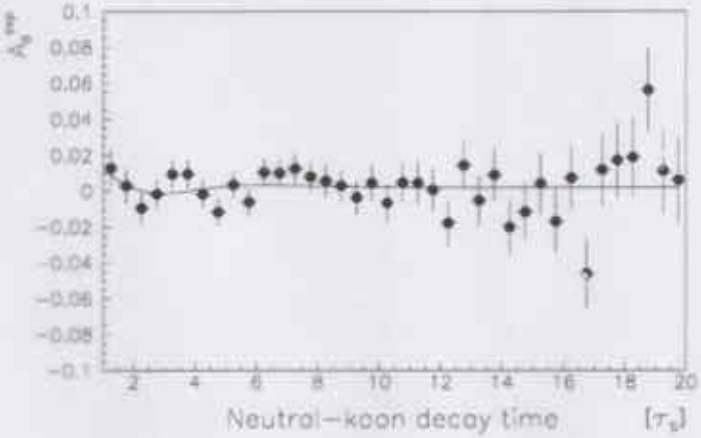
a) $\Delta S = -\Delta Q$ absent ($\sim 10^{-7}$ in SM)

b) A_T^{exp} (ϵ and y) is time independent!

c) since not Σ decays $\curvearrowright$ no unitarity or CPT condition enter the analysis

$\curvearrowright$ A_T^{CPLEAR} is a direct measurement of $\cancel{T}$.

A_δ asymmetry Results



$$A_\delta(\tau) = 4\mathrm{Re}(\delta) + 4\frac{\mathrm{Re}(\delta)\sinh(\Delta\Gamma\tau/2) + \mathrm{Im}(\delta)\sin(\Delta m\tau)}{\cosh(\Delta\Gamma\tau/2) + \cos(\Delta m\tau)} - 4\frac{\mathrm{Re}(x_-)\cos(\Delta m\tau)\sinh(\Delta\Gamma\tau/2) - \mathrm{Im}(x_+)\sin(\Delta m\tau)\cosh(\Delta\Gamma\tau/2)}{[\cosh(\Delta\Gamma\tau/2)]^2 - [\cos(\Delta m\tau)]^2}$$

for $t \gg \tau_S$: $A_\delta^{exp}(t) \to 8\,\mathrm{Re}(\delta)$

$$\begin{aligned}
\mathrm{Re}(\delta) &= (\ \ 3.0 \pm 3.3_{\mathrm{stat}} \pm 0.6_{\mathrm{syst}}) \times 10^{-4} \\
\mathrm{Im}(\delta) &= (-1.5 \pm 2.3_{\mathrm{stat}} \pm 0.3_{\mathrm{syst}}) \times 10^{-2} \\
\mathrm{Re}(x_-) &= (\ \ 0.2 \pm 1.3_{\mathrm{stat}} \pm 0.3_{\mathrm{syst}}) \times 10^{-2} \\
\mathrm{Im}(x_+) &= (\ \ 1.2 \pm 2.2_{\mathrm{stat}} \pm 0.3_{\mathrm{syst}}) \times 10^{-2} \\
\chi^2/\mathrm{ndf} &= 1.14, \quad \mathrm{ndf} = 604
\end{aligned}$$

$\mathrm{Re}(x_-) = \mathrm{Im}(x_+) = 0$

$$\begin{aligned}
\mathrm{Re}(\delta) &= (\ \ 2.9 \pm 2.6_{\mathrm{stat}} \pm 0.6_{\mathrm{syst}}) \times 10^{-4} \\
\mathrm{Im}(\delta) &= (-0.9 \pm 2.9_{\mathrm{stat}} \pm 1.0_{\mathrm{syst}}) \times 10^{-3}
\end{aligned}$$

Study of T and CPT symmetries with Bell-Steinberger relation.

The relation

$$\mathrm{Re}(\epsilon) - i\mathrm{Im}(\delta) = \frac{1}{2(i\Delta m + \frac{1}{2}\gamma)} \times \Big(\sum\big(|\mathrm{A_S}|^2\eta_{\pi\pi}\big) + \sum\big(|\mathrm{A_L}|^2\eta_{\pi\pi\pi}{}^{*}\big)$$
$$+2\big[-i(\mathrm{Im}(\mathrm{x}_+) + \mathrm{Im}(\delta)) + \mathrm{Re}(\epsilon) + \mathrm{Re}(\mathrm{y})\big]|f_{\pi\ell\nu}|^2\Big)$$

$$|\mathrm{A_S}|^2 = \mathrm{BR}(\mathrm{K_S} \to \pi\pi)\Gamma_\mathrm{S}$$
$$|\mathrm{A_L}|^2 = \mathrm{BR}(\mathrm{K_L} \to \pi\pi\pi)\Gamma_\mathrm{L}$$
$$|f_{\pi\ell\nu}|^2 = \mathrm{BR}(\mathrm{K_L} \to \pi\ell\nu)\Gamma_\mathrm{L}$$

Experimental input

→ $\lvert\eta_{+-}\rvert$	$(2.283 \pm 0.025) \times 10^{-3}$	CPLEAR & PDG
→ φ_{+-}	$43.6^\circ \pm 0.6^\circ$	CPLEAR fit
→ Δm	$(530.19 \pm 1.54) \times 10^7 \hbar s^{-1}$	CPLEAR fit without CPT
$\Delta\phi$	$-0.3^\circ \pm 0.8^\circ$	PDG(E731/NA31)
$r = \frac{\lvert\eta_{00}\rvert}{\lvert\eta_{+-}\rvert}$	0.9930 ± 0.0020	PDG(E731/NA31)
→ $\mathrm{Re}(\eta_{+-0})$	$(-2 \pm 8) \times 10^{-3}$	CPLEAR
→ $\mathrm{Im}(\eta_{+-0})$	$(-2 \pm 9) \times 10^{-3}$	CPLEAR
$\mathrm{Re}(\eta_{000})$	0.08 ± 0.11	CPLEAR & Barmin
$\mathrm{Im}(\eta_{000})$	0.07 ± 0.16	CPLEAR & Barmin
$\mathrm{BR}^{\mathrm{S}}{}_{\pi^\circ\pi^\circ}$	$(31.39 \pm 0.28)\%$	PDG
$\mathrm{BR}^{\mathrm{L}}{}_{\pi^+\pi^-\pi^\circ}$	$(12.56 \pm 0.20)\%$	PDG
$\mathrm{BR}^{\mathrm{L}}{}_{\pi^\circ\pi^\circ\pi^\circ}$	$(21.12 \pm 0.27)\%$	PDG
$\mathrm{BR}^{\mathrm{L}}{}_{\pi\ell\nu}$	$(65.95 \pm 0.37)\%$	PDG
τ_S	$(0.8934 \pm 0.0008) \times 10^{-10} s$	PDG
τ_L	$(5.17 \pm 0.04) \times 10^{-8} s$	PDG

Global fit

☞ CPLEAR semileptonic asymmetries: fitted simultaneously

$$\frac{\overline{R}_+(\tau) - R_-(\tau)}{\overline{R}_+(\tau) + R_-(\tau)}, \quad \frac{\overline{R}_-(\tau) - R_+(\tau)}{\overline{R}_-(\tau) + R_+(\tau)}$$

☞ Constraints:

- Bell-Steinberger Relation
- Charge asymmetry:
  $\delta_l = 2\mathrm{Re}(\epsilon) - 2\mathrm{Re}(\delta) - 2\mathrm{Re}(\mathrm{y}) - 2\mathrm{Re}(\mathrm{x}_-) = (3.27 \pm 0.12) \times 10^{-3}$

Results

$$\mathrm{Re}(\epsilon) = (164.9 \pm 2.5) \times 10^{-5}$$
$$\mathrm{Im}(\delta) = (\ \ 2.4 \pm 5.0) \times 10^{-5}$$

$$\mathrm{Re}(\delta) = (\ \ 2.4 \pm 2.8) \times 10^{-4}$$
$$\mathrm{Im}(\mathrm{x}_+) = (-2.0 \pm 2.7) \times 10^{-3}$$
$$\mathrm{Re}(\mathrm{x}_-) = (-0.5 \pm 3.0) \times 10^{-3}$$
$$\mathrm{Re}(\mathrm{y}) = (\ \ 0.3 \pm 3.1) \times 10^{-3}$$

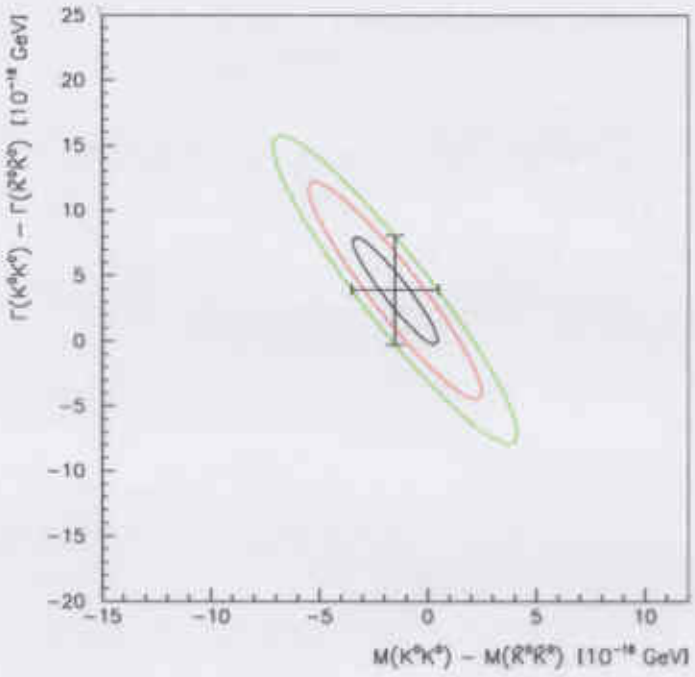
☞ *Phys. Lett. B456(1999) 297.*

K° and $\overline{K}^\circ$ mass and width equality

The projections of δ on φ_{SW} axe.

$$\delta_{\parallel} = \frac{1}{4}\frac{\Gamma_{K^\circ K^\circ} - \Gamma_{\overline{K}^\circ \overline{K}^\circ}}{\sqrt{\Delta m^2 + (\frac{\Delta\gamma}{2})^2}} = \mathrm{Re}(\delta)\cos\varphi_{SW} + \mathrm{Im}(\delta)\sin\varphi_{SW}$$

$$\delta_{\perp} = \frac{1}{2}\frac{M_{K^\circ K^\circ} - M_{\overline{K}^\circ \overline{K}^\circ}}{\sqrt{\Delta m^2 + (\frac{\Delta\gamma}{2})^2}} = -\,\mathrm{Re}(\delta)\sin\varphi_{SW} + \mathrm{Im}(\delta)\cos\varphi_{SW}$$



$$\Gamma_{K^\circ K^\circ} - \Gamma_{\overline{K}^\circ \overline{K}^\circ} = (\ \ 3.9 \pm 4.2) \times 10^{-18}\ \mathrm{GeV}$$
$$M_{K^\circ K^\circ} - M_{\overline{K}^\circ \overline{K}^\circ} = (-1.5 \pm 2.0) \times 10^{-18}\ \mathrm{GeV}$$

Assume CPT-invariance in decay and ignore contributions of K_L decay channels:

$$\frac{M_{K^\circ K^\circ} - M_{\overline{K}^\circ \overline{K}^\circ}}{m_K} = (-0.2 \pm 5.4) \times 10^{-19}$$

The measurement of δ_ℓ by KTeV

The charge asymmetry in the semileptonic K_L decays

$$\delta_\ell = \frac{BR(e^+\pi^-) - BR(e^-\pi^+)}{BR(e^+\pi^-) + BR(e^-\pi^+)}$$

has been measured :

$$\delta_\ell = 3{,}322 \pm \underbrace{.058 \pm .047}_{\pm\ .074\ 10^{-3}\ \text{combined}}\ 10^{-3}$$

$3.27 \pm .12\ 10^{-3}$ PDG '00

$3.409 \pm .171\ 10^{-3}$ CERN-Heidelberg '74

* Expressing δ_ℓ in terms of the ~~CP~~ and ~~CPT~~ parameters

$$\delta_\ell = 2\,\mathrm{Re}\,\epsilon_L - 2\,\mathrm{Re}\,y - 2\,\mathrm{Re}\,x_-$$

$$= 2\,\mathrm{Re}(\epsilon - \delta) - 2\,\mathrm{Re}\,y - 2\,\mathrm{Re}\,x_-$$

$\rightarrow$ $\mathrm{Re}(y + x_- + \delta) = \mathrm{Re}\,\epsilon - \delta_\ell/2$

$\rightarrow$ $\mathrm{Re}(y + x_- + \delta) = -3 \pm 35\ 10^{-6}$

or $|\mathrm{Re}(y + x_- + \delta)| < 61\ 10^{-6}$ 90% CL.

from CPLEAR : $\mathrm{Re}(y + x_- + \delta) = 4 \pm 42\ 10^{-5}$!

Summary of measurements

Parameter	Symmetry	Measurement	Experiment
$\lvert\eta_{+-}\rvert$	CP	$(2.264\pm0.035)\times10^{-3}$	CPLEAR
φ_{+-}	CP	$(43.19\pm0.60)°$	CPLEAR
$\lvert\eta_{00}\rvert$	CP	$(2.33\pm0.18)\times10^{-3}$	Christenson 79
φ_{00}	CP	$(44.5\pm2.5)°$	NA31
$\varphi_{00}-\varphi_{+-}$	CPT	$(-0.30\pm0.88)°$	E731, E773
Δm		$(529.5\pm2.0)\times10^{7}\hbar s^{-1}$	CPLEAR
φ_{SW}		$(43.50\pm0.08)°$	Compilation
$\varphi_{+-}-\varphi_{SW}$	CPT	$(0.1\pm0.6)°$	Compilation
$\mathrm{Re}(\eta_{+-0})$	CP	$(-2\pm8)\times10^{-3}$	CPLEAR
$\mathrm{Im}(\eta_{+-0})$	CP	$(-2\pm9)\times10^{-3}$	CPLEAR
$\mathrm{Re}(\eta_{000})$	CP	0.18 ± 0.15	CPLEAR
$\mathrm{Im}(\eta_{000})$	CP	0.15 ± 0.20	CPLEAR
A_T	T	$(6.6\pm1.6)\times10^{-3}$	CPLEAR
$\mathrm{Re}(\delta)$	CPT	$(3.0\pm3.3)\times10^{-4}$	CPLEAR
$\mathrm{Im}(\delta)$	CPT	$(2.4\pm5.0)\times10^{-5}$	CPLEAR
$\mathrm{Re}(\varepsilon)$	T	$(164.9\pm2.5)\times10^{-5}$	CPLEAR
$\mathrm{Re}(y)$	CPT	$(0.3\pm3.1)\times10^{-3}$	CPLEAR
$m_{K^0}-m_{\bar{K}^0}$	CPT	$(-1.5\pm2.0)\times10^{-18}$ GeV	CPLEAR
$\Gamma_{K^0}-\Gamma_{\bar{K}^0}$	CPT	$(3.9\pm4.2)\times10^{-18}$ GeV	CPLEAR

SUMMARY and CONCLUSIONS

Some remarquable CPLEAR measurements

- direct measurement of ~~T~~ and CPT invariance in the K^0 mixing,
- determination of ~~CPT~~ parameters $\mathrm{Re}\,\delta$ and $\mathrm{Im}\,\delta$ using unitarity,
- determination of the MASS and DECAY WIDTH difference between K^0 and $\overline{K}^0$.

illustrating that:

* intense Particle-Antiparticle sources
* dedicated detectors for initial and final flavour tagging : unique for direct access to ~~T~~, CPT

VERY IMPATIENT TO HAVE A LUMINOUS DAΦNE!

- Indirect tests of discrete symmetries (CPT)

$\varphi_{00} - \varphi_{+-}$; $\varphi_{+-} - \varphi_{SW}$; δ_{ℓ} .

will improve thanks to the high precisions of Kaon beam experiments.

→ In the B sector ...

beyond establishing ~~CP~~ and determining CKM

PRECISION FLAVOUR TAGGING

is necessary for direct T and CPT studies.