Leptonic CP violation

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Introduction

There are only three ν which couple to the Z^0 and they seem to be massive

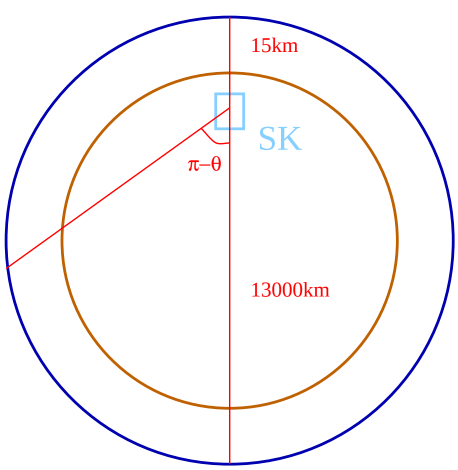
Thanks to their weakness and their lightness they undergo flavour oscillations in their propagation:

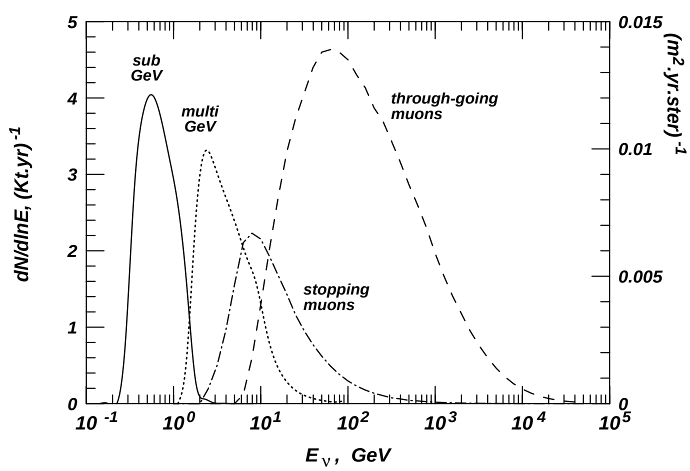
$$P_{\nu_\alpha \nu_\beta} = \sin^2 2\theta \, \sin^2 \frac{\Delta m^2 L}{4E_\nu}$$

- Solar ν deficit $\leftrightarrow$ flavour transition $\nu_e \longrightarrow \nu_{\mu,\tau}$

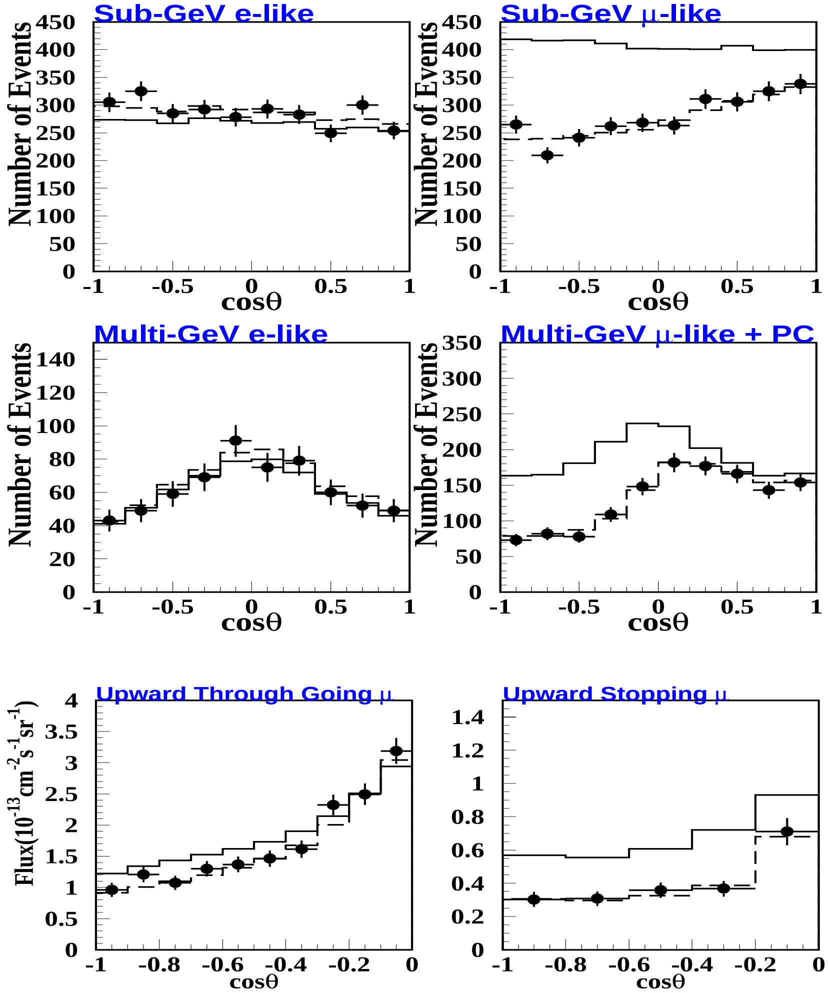
$$\phi^{CC}_{\nu_e} < \phi^{NC}_{\nu_e} \qquad \text{SNO}$$

- Atmospheric anomaly $\leftrightarrow$ up-down and energy asymmetry in the signal $\nu_\mu \longrightarrow \nu_\mu$





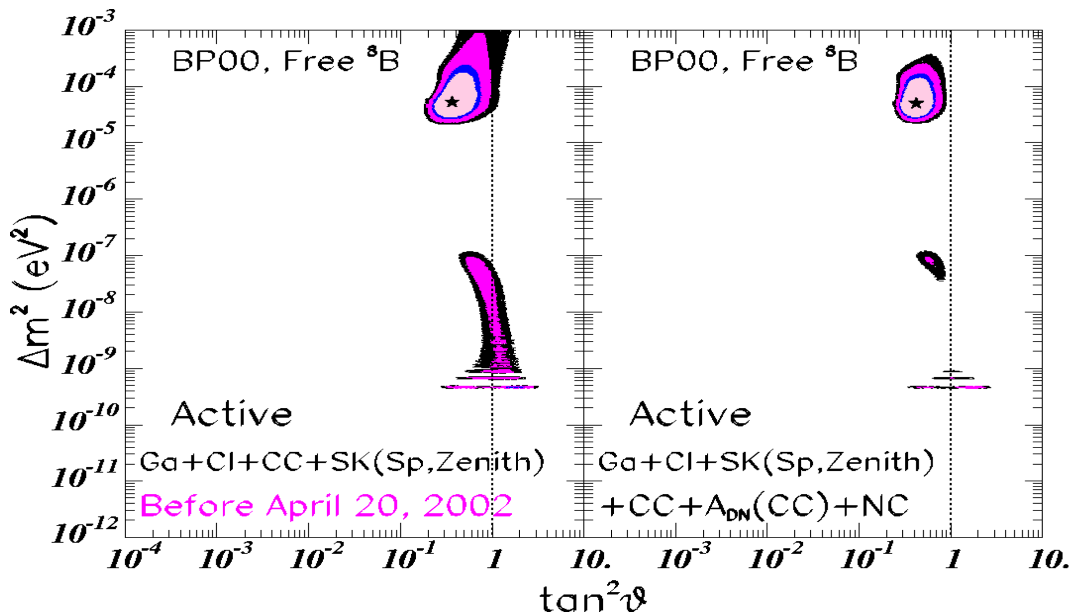
Atmospheric ν anomaly



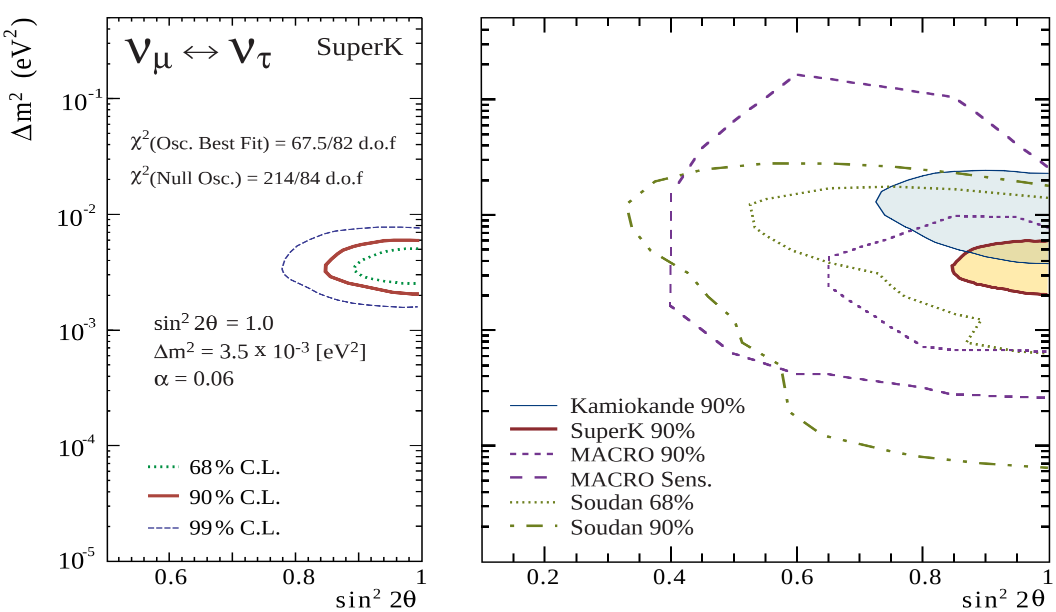
SuperK coll.

The three SM neutrinos fit wonderfully this picture with

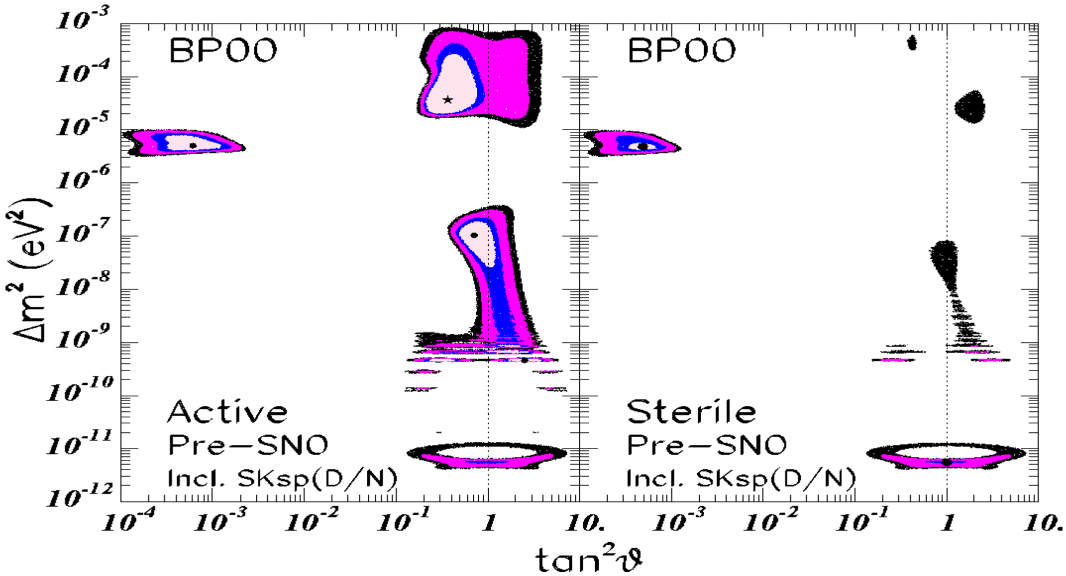
$$|\Delta m^2_{solar}| \ll |\Delta m^2_{atmos}|$$



Bahcall *et al*



Pre-SNO:



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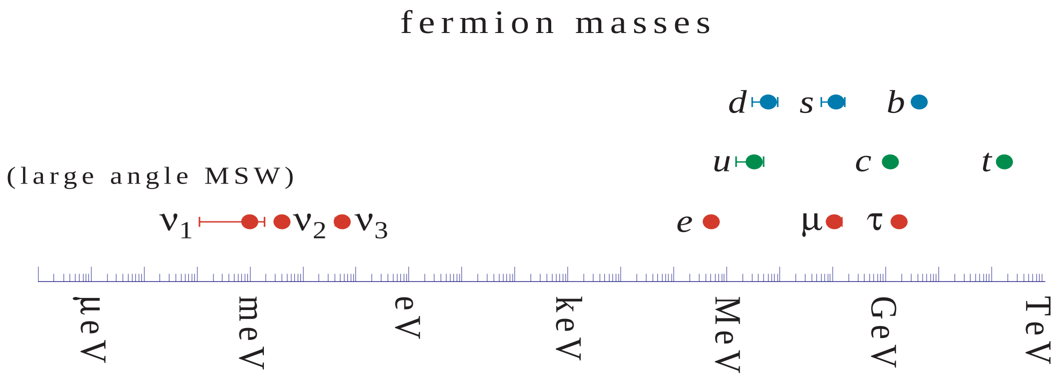
ν masses in SM

If **Dirac** $\leftrightarrow$ (SM + 3 ν_R)

$$\delta \mathcal{L}_{SM} = \bar{L}_L \, \lambda^\nu \, \tilde{\Phi} \, \nu_R + h.c. \quad \rightarrow \quad v \;\, \bar{\nu}_L \, \lambda^\nu \, \nu$$



If neutrino have masses they mix:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{NMS} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

U_{NMS}	Angles	Phases
Dirac	3	1

$U_{NMS}^{\text{Dirac}} \equiv$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} e^{i\delta} & 0 \\ -s_{12} e^{i\delta} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Generically there is leptonic CP-violation in the SM

If **Majorana**

$$\delta\mathcal{L}_{SM} = \frac{\tilde{\lambda}^\nu}{\Lambda}\,\bar{L}_L^C\;\;\tilde{\Phi}^\dagger\,\tilde{\Phi}\,L_L + h.c. \quad\rightarrow\quad \frac{v^2}{\Lambda}\,\bar{\nu}_L^C\,\tilde{\lambda}^\nu\,\nu_L$$

m_ν are naturally small if $\Lambda \gg v$!

- New physics at Λ: SM no longer renormalizable, it is an effective theory valid for $E \ll \Lambda$
- Total lepton number violation
- New sources of leptonic CP violation:

U_{NMS}	Angles	Phases
Majorana	3	3

$$U_{NMS}^{\rm Majorana} \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_1} & 0 \\ 0 & 0 & e^{i\alpha_2} \end{pmatrix} U_{NMS}^{\rm Dirac}(\theta_{12},\theta_{13},\theta_{23},\delta)$$

This is all welcome for baryogenesis !

ν-masses are not just the last fundamental parameters in the SM to add to the Particle Data Book...but a new path towards the physics beyond the SM:

- Data imply there is at least one $m_\nu \geq 0.05$eV.
  If $\lambda_\nu \sim \lambda_u$:

$$v \ < \ M \sim 10^{15}\text{GeV} \ < \ M_{Planck}$$

  close to the typical Grand Unification (GUT) scale !

- Together with the quarks flavour sector, they can give us a clue on the origin of fermion masses in SM:
  $\{\lambda_f\} \rightarrow 22/26$ parameters of the SM

ν-masses might play an important role in astrophysics and cosmology:

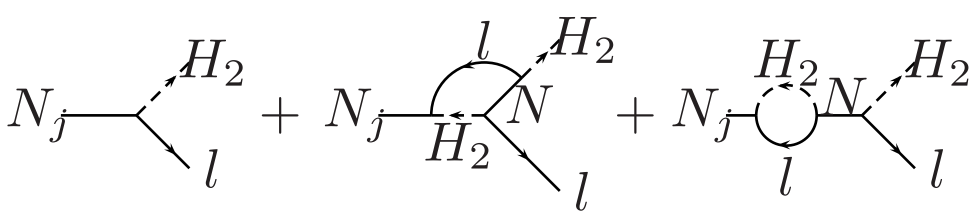
- Leptogenesis: the three conditions of Sakharov for producing a lepton number out of a symmetric initial state are met and a mechanism for baryogenesis naturally emerges even in the simplest sea-saw model

Heavy Majorana neutrinos

$$\delta\mathcal{L} = \lambda_\nu \bar{L}_L\, \tilde{\Phi}\, N_R + \lambda_l\, \bar{L}_L\, \Phi\, N_R + \bar{N}_R^C\, M_R\, N_R$$

The CP-asymmetry in the out of equilibrium decay ($\Gamma_N \ll$ rate of expansion) of the N is the source of $\Delta L = \Delta B$:

$$\Gamma(N \to \Phi\, l) - \Gamma(N \to \Phi\, \bar{l}) \to \Delta L$$



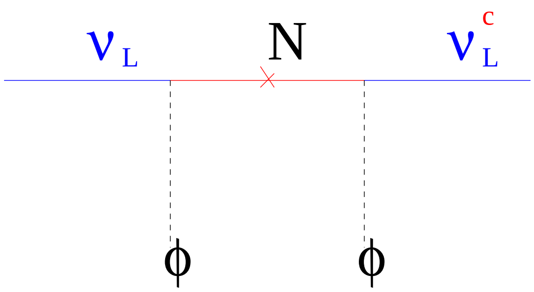
Details are model dependent but it is natural to obtain the observed n_B/n_γ

Fukugita, Yanagida

Buchmüller, Plümacher

Can we test this in the laboratory ?

At energies $E \ll M_R$, the heavy Majoranas can be integrated out leaving a trace of higher dimensional operators:



$$\mathcal{L}_{eff}^{d=5} = \bar{L}_L^C \ \ \tilde{\Phi}^\dagger \ \left(\tilde{\lambda}_\nu^T \frac{1}{M_R} \tilde{\lambda}_\nu\right) \ \tilde{\Phi} \ L_L$$

$$\mathcal{L}_{eff}^{d=6} = \ldots.$$

Param	Masses	Angles	Phases
total	6	6	6
d=5	3	3	3

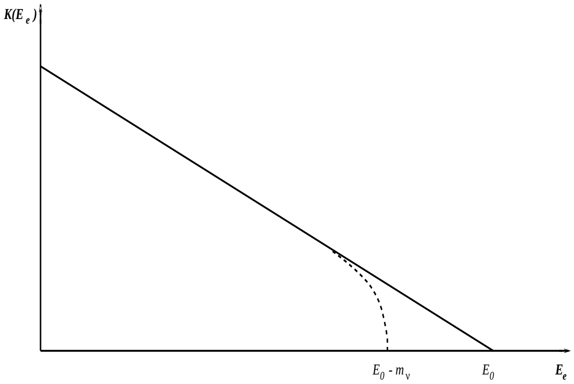
The CP-asymmetry responsible for leptogenesis can in principle depend on more parameters than those at low energy

Branco *et al*

Endoh *et al*

Atmos $\oplus$ Solar oscillations in SM

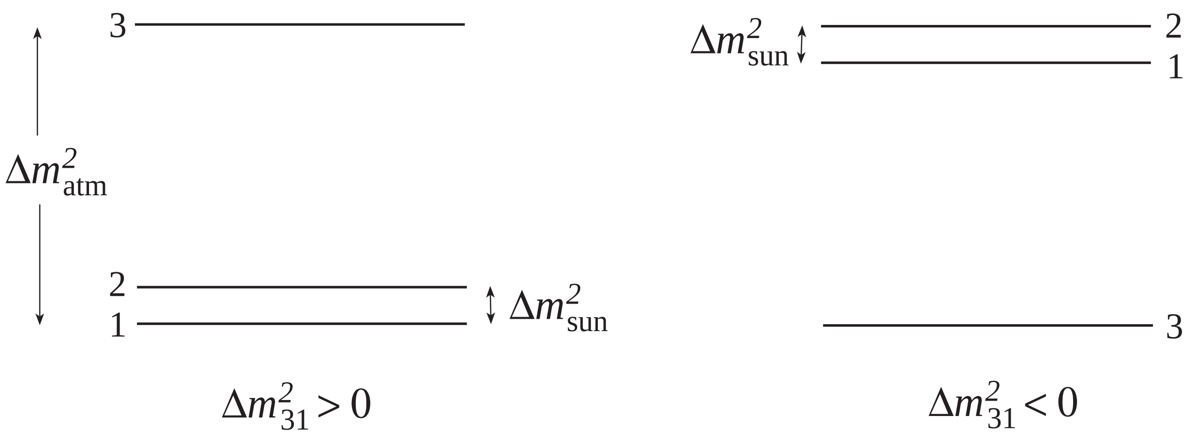
ν mass scale:



$$H^3 \rightarrow {}^3He + e^- + \bar{\nu}_e$$

$m_{\nu_e} < 2.5$eV (Troisk) < 6 eV (Mainz)

ν spectrum:



ν mixing matrix: $\leftarrow$ Chooz
$(P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \simeq 1 \;\; @E_\nu/L \sim |\Delta m^2_{13}| \;)$

$$U_{NMS} \simeq \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_1} & 0 \\ 0 & 0 & e^{i\alpha_2} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & s_{13} \\ -\frac{1}{2}(e^{i\delta} + s_{13}) & \frac{1}{2}(e^{i\delta} - s_{13}) & \frac{1}{\sqrt{2}} \\ \frac{1}{2}(e^{i\delta} - s_{13}) & -\frac{1}{2}(e^{i\delta} + s_{13}) & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$\theta_{13} \leq 10^\circ \quad \delta, \alpha_1, \alpha_2$ unconstrained

Only hope for α_1, α_2 is $0\nu\beta$ decay $\rightarrow$ Petcov's talk

CP violation in neutrino oscillations

Vacuum oscillations ($W^{jk}_{\alpha\beta} \equiv [U_{\alpha j} U^*_{\beta j} U^*_{\alpha k} U_{\beta k}]$)

$$P(\nu_\alpha \;\to\; \nu_\beta) = -4 \sum_{k>j} \mathrm{Re}[\mathrm{W}^{\mathrm{jk}}_{\alpha\beta}] \sin^2\left(\frac{\Delta \mathrm{m}^2_{\mathrm{jk}}\,\mathrm{L}}{4\mathrm{E}_\nu}\right)$$
$$\pm \;\; 2 \sum_{k>j} \mathrm{Im}[\mathrm{W}^{\mathrm{jk}}_{\alpha\beta}] \sin\left(\frac{\Delta \mathrm{m}^2_{\mathrm{jk}}\,\mathrm{L}}{2\mathrm{E}_\nu}\right)$$

- CP and T-odd asymmetries by comparing $\nu_\alpha \to \nu_\beta$ with $\bar{\nu}_\alpha \to \bar{\nu}_\beta$ ($\nu_\beta \to \nu_\alpha$)

$$A^{CP}_{\alpha\beta} \equiv \frac{P(\nu_\alpha \to \nu_\beta) - P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)}{P(\nu_\alpha \to \nu_\beta) + P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)} \quad A^{T}_{\alpha\beta} \equiv \frac{P(\nu_\alpha \to \nu_\beta) - P(\nu_\beta \to \nu_\alpha)}{P(\nu_\alpha \to \nu_\beta) + P(\nu_\beta \to \nu_\alpha)}$$

- By CPT, CP and T-odd terms cancel in survival probabilities $\to$ need appearance measurements: $\alpha \neq \beta$

- CP(T)-odd terms the same for all $\alpha \neq \beta$:

$$P^{\mathrm{CP(T)-odd}}_{\nu_\alpha \nu_\beta} = 2\ J\ \frac{\Delta m^2_{12} L}{4E_\nu}\ \sin^2 \frac{\Delta m^2_{13} L}{4E_\nu}$$

$$J \equiv \sin\delta\ \cos\theta_{13}\ \sin 2\theta_{13}\ \sin 2\theta_{12}\ \sin 2\theta_{23}$$

- Minimize GIM suppression: $E_\nu / L \sim |\Delta m^2_{13}|$

- Effects of δ are more significant in subleading transitions: $\nu_e \to \nu_\mu(\nu_\tau)$:

$$P^{\mathrm{CP-even}}_{\nu_\mu \nu_\tau} = \text{unsuppressed in } \theta_{13} \text{ or } \frac{\Delta m^2_{12} L}{E_\nu}$$

$$A^{\mathrm{CP(T)-odd}}_{\nu_\mu \nu_\tau} \sim \sin 2\,\theta_{13}\ \frac{\Delta m^2_{12} L}{E_\nu}$$

$$P^{\mathrm{CP-even}}_{\nu_e \nu_\mu(\nu_\tau)} = \text{suppressed in } \theta^2_{13} \text{ or } \left(\frac{\Delta m^2_{12} L}{E_\nu}\right)^2$$

$$A^{\mathrm{CP(T)-odd}}_{\nu_e \nu_\mu(\nu_\tau)} \sim \frac{\Delta m^2_{12} L/E_\nu}{\sin 2\theta_{13}} \text{ or } \frac{\sin 2\theta_{13}}{\Delta m^2_{12} L/E_\nu}$$

θ_{13} and δ

$$P_{\nu_e\nu_\mu(\bar\nu_e\bar\nu_\mu)} = s_{23}^2 \sin^2 2\theta_{13} \sin^2\left(\frac{\Delta_{23} L}{2}\right) \equiv P^{atmos}$$

$$+ \; c_{23}^2 \sin^2 2\theta_{12} \sin^2\left(\frac{\Delta_{12} L}{2}\right) \equiv P^{solar}$$

$$+\tilde{J} \cos\left(\pm\delta - \frac{\Delta_{23} L}{2}\right) \frac{\Delta_{12} L}{2} \sin\left(\frac{\Delta_{23} L}{2}\right) \equiv P^{inter}$$

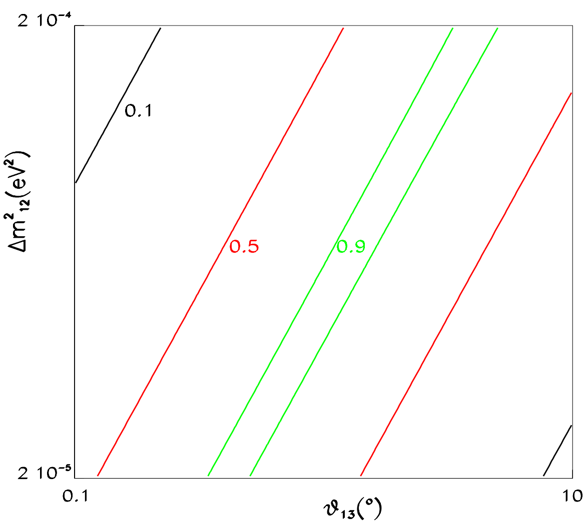
$$(\tilde{J} \equiv c_{13} \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23}, \quad \Delta_{ij} \equiv \frac{\Delta m_{ij}^2}{2E_\nu})$$

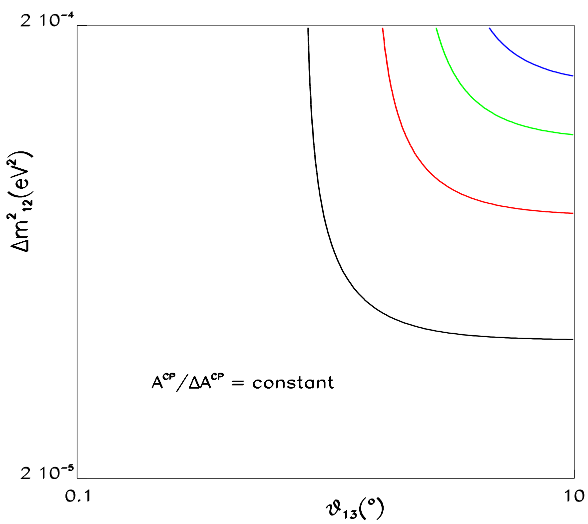
A. Cervera *et al*, hep-ph/0002108

$$P^{atmos} \gg P^{solar} \rightarrow A^{CP,T}_{\nu_e\nu_\mu(\nu_\tau)} \sim \frac{\Delta_{12} L}{\sin 2\theta_{13}}$$

$$P^{solar} \gg P^{atmos} \rightarrow A^{CP,T}_{\nu_e\nu_\mu(\nu_\tau)} \sim \frac{\sin 2\theta_{13}}{\Delta_{12} L}$$

$$P^{solar} \simeq P^{atmos} \rightarrow A^{CP,T}_{\nu_e\nu_\mu(\nu_\tau)} = O(1)$$





The challenge

We need to measure for the first time small oscillation probabilities: need more intense and purer ν sources

- **Superbeams** ν from K, π decay

$$\pi, K \to \quad \nu_\mu \quad O(1\%)\, \nu_e \; \bar{\nu}_\mu;$$
$$\nu_\mu \quad \to \nu_e \to e^-$$
$$\nu_e \to \nu_e \to e^-$$

- **Neutrino factory** ν from muon decay

$$\mu^- \to e^- \quad \nu_\mu \quad \bar{\nu}_e\,;$$
$$\bar{\nu}_e \to \bar{\nu}_\mu \to \mu^+$$
$$\nu_\mu \quad \to \nu_\mu \to \mu^-$$

- β-**beams** from boosted heavy ions decays Zucchelli

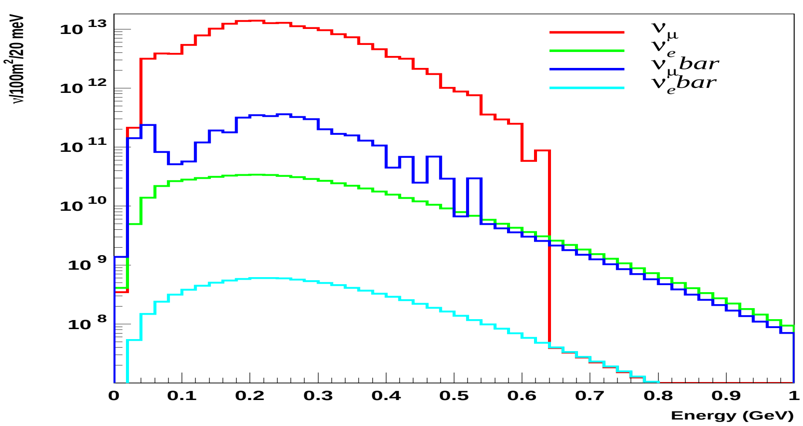
$$^6He^{++} \to ^6_3 Li^{+++}\, e^- \qquad \bar{\nu}_e$$
$$\bar{\nu}_e \to \bar{\nu}_\mu \to \mu^+$$

Optimal E_ν and L ?

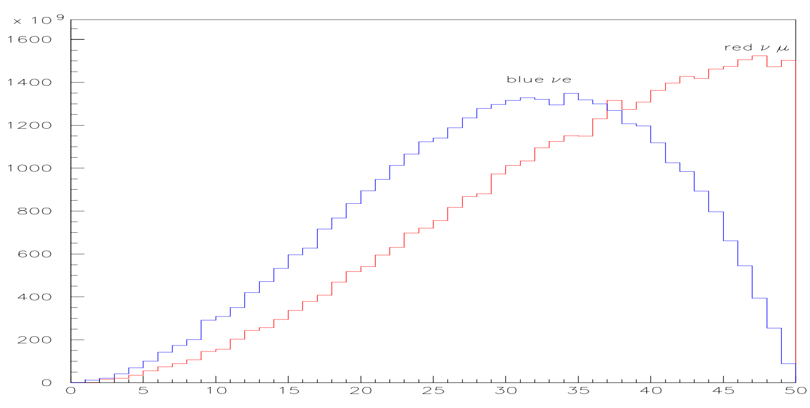
The $\langle E_\nu \rangle$ is optimized to get the maximal signal-to-noise

Superbeams and β-beams:

Superbeam	$\langle E_\nu \rangle$	L$\sim$ $\langle E_\nu \rangle / \Delta m^2_{13}$
JHF	0.7 GeV	295 km (-Kamioka)
SPL	0.25 GeV	130 km (CERN-Frejus)
β-beam	0.35 GeV	130 km (CERN-Frejus)



ν-factory $\langle E_\nu \rangle = O(10)$GeV: $\rightarrow L = O(5000)$ km



Earth matter effects are very important!

Matter Effects

When ν's cross the earth, forward scattering amplitudes are different for the different flavours:

$$M_\nu^2 = U_{NMS} \begin{pmatrix} m_1^2 & & \\ & m_2^2 & \\ & & m_3^2 \end{pmatrix} U_{NMS}^\dagger + \begin{pmatrix} \pm 2E_\nu A & & \\ & 0 & \\ & & 0 \end{pmatrix}$$

with $A \equiv \sqrt{2} G_F n_e$

Approximate formula up to $O(\theta_{13})^3$, $O(\Delta_{12})^3$:

$$P_{\nu_e\nu_\mu(\bar\nu_e\bar\nu_\mu)} = s_{23}^2 \sin^2 2\theta_{13} \left(\frac{\Delta_{13}}{B_\pm}\right)^2 \sin^2\left(\frac{B_\pm L}{2}\right)$$

$$+ c_{23}^2 \sin^2 2\theta_{12} \left(\frac{\Delta_{12}}{A}\right)^2 \sin^2\left(\frac{A L}{2}\right)$$

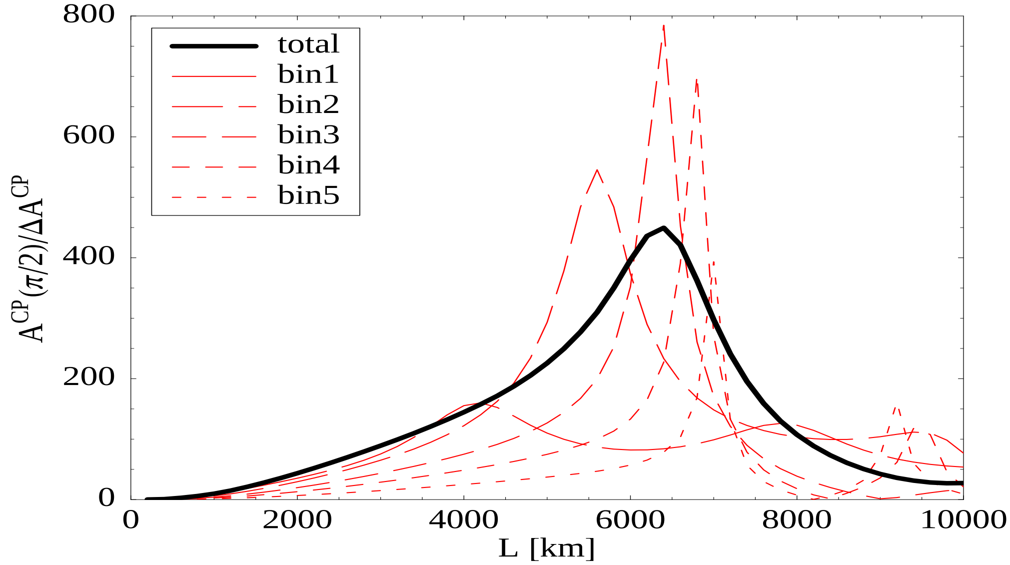
$$+ \tilde{J} \frac{\Delta_{12}}{A} \sin\left(\frac{AL}{2}\right) \frac{\Delta_{13}}{B_\pm} \sin\left(\frac{B_\pm L}{2}\right) \cos\left(\pm\delta - \frac{\Delta_{13} L}{2}\right)$$

$$B_\pm = |A \pm \Delta_{13}| \qquad \Delta_{ij} = \frac{\Delta m_{ij}^2}{2E_\nu}$$

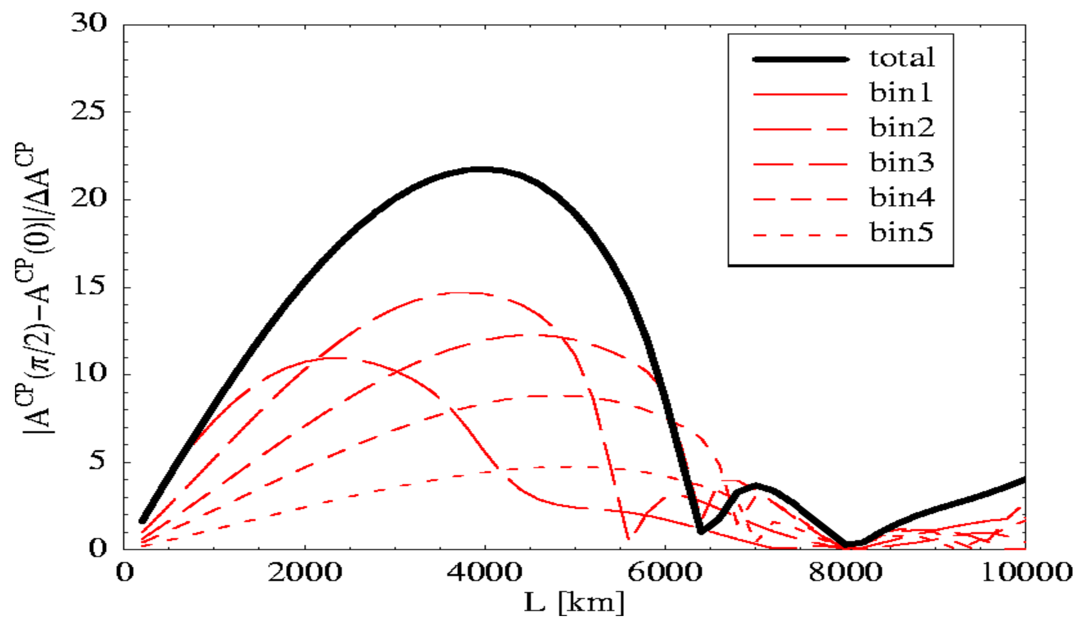
Huge resonance effect in the ν or $\bar\nu$ channel if:

$$2E_\nu A \sim |\Delta m_{13}^2|, \quad E_\nu \sim 10 - 20 GeV$$

Even if $\delta = 0$ there is a large CP-asymmetry induced by matter effects:

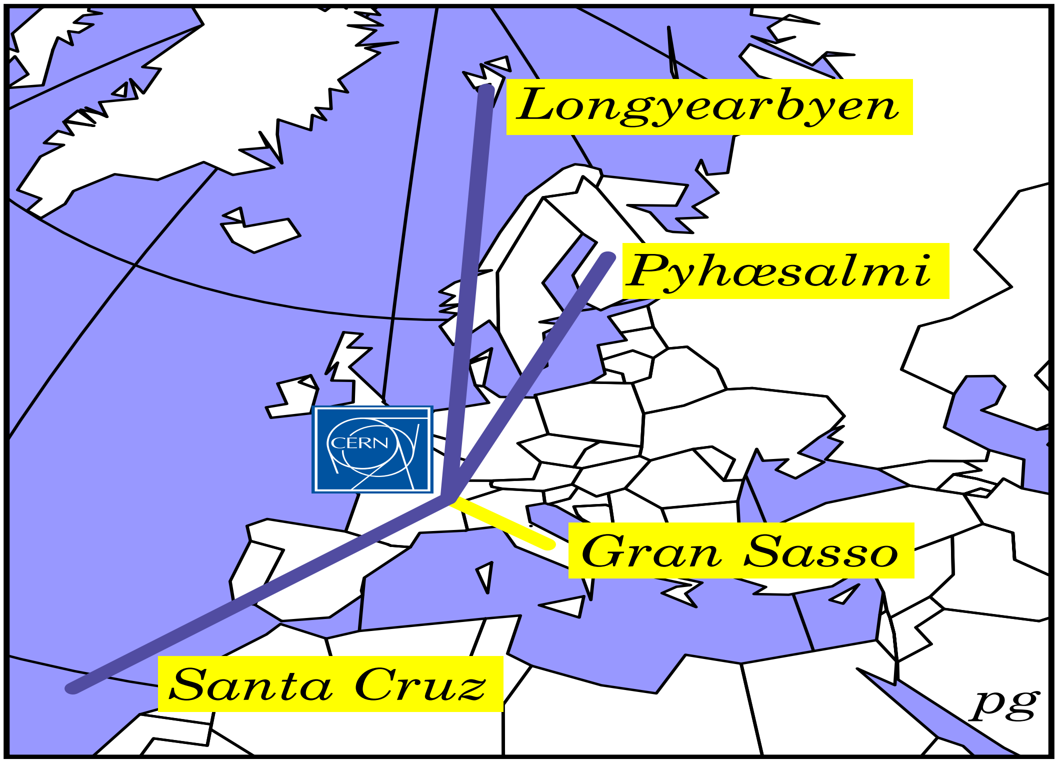


The optimal baseline at the Nufact is $O(3000)$ km $< \frac{\langle E_\nu \rangle}{|\Delta m^2_{13}|}$



A. Cervera *et al*

Sites for underground laboratories



Parameter correlations

At the time of one of these facilities it is expected four parameters will be very precisely known:

$$\text{Solar}: \quad \Delta m^2_{12}, \sin^2 2\theta_{12}$$

$$\text{Atmospheric}: \quad |\Delta m^2_{13}|, \sin^2 2\theta_{23}$$

The measurement of θ_{13} and δ has to be done simultaneously. At fixed E_ν and L there are generically two solutions to the system:

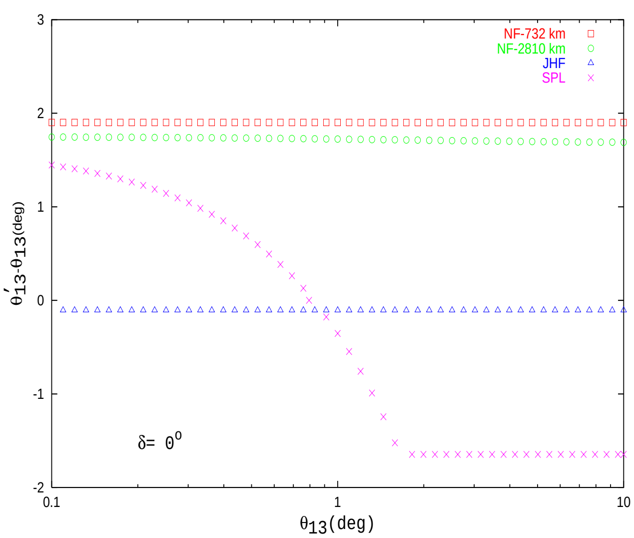
$$\left.\begin{array}{l} P_{\nu_e\nu_\mu}(\theta_{13}', \delta') = P_{\nu_e\nu_\mu}(\theta_{13}, \delta) \\ P_{\bar{\nu}_e\bar{\nu}_\mu}(\theta_{13}', \delta') = P_{\bar{\nu}_e\bar{\nu}_\mu}(\theta_{13}, \delta) \end{array}\right\}$$

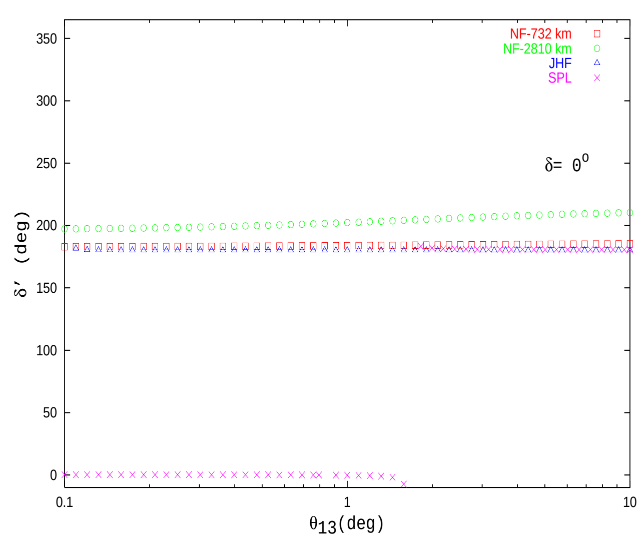
In vacuum:

Solution I: $\theta_{13}' = \theta_{13} \quad \delta' = \delta$

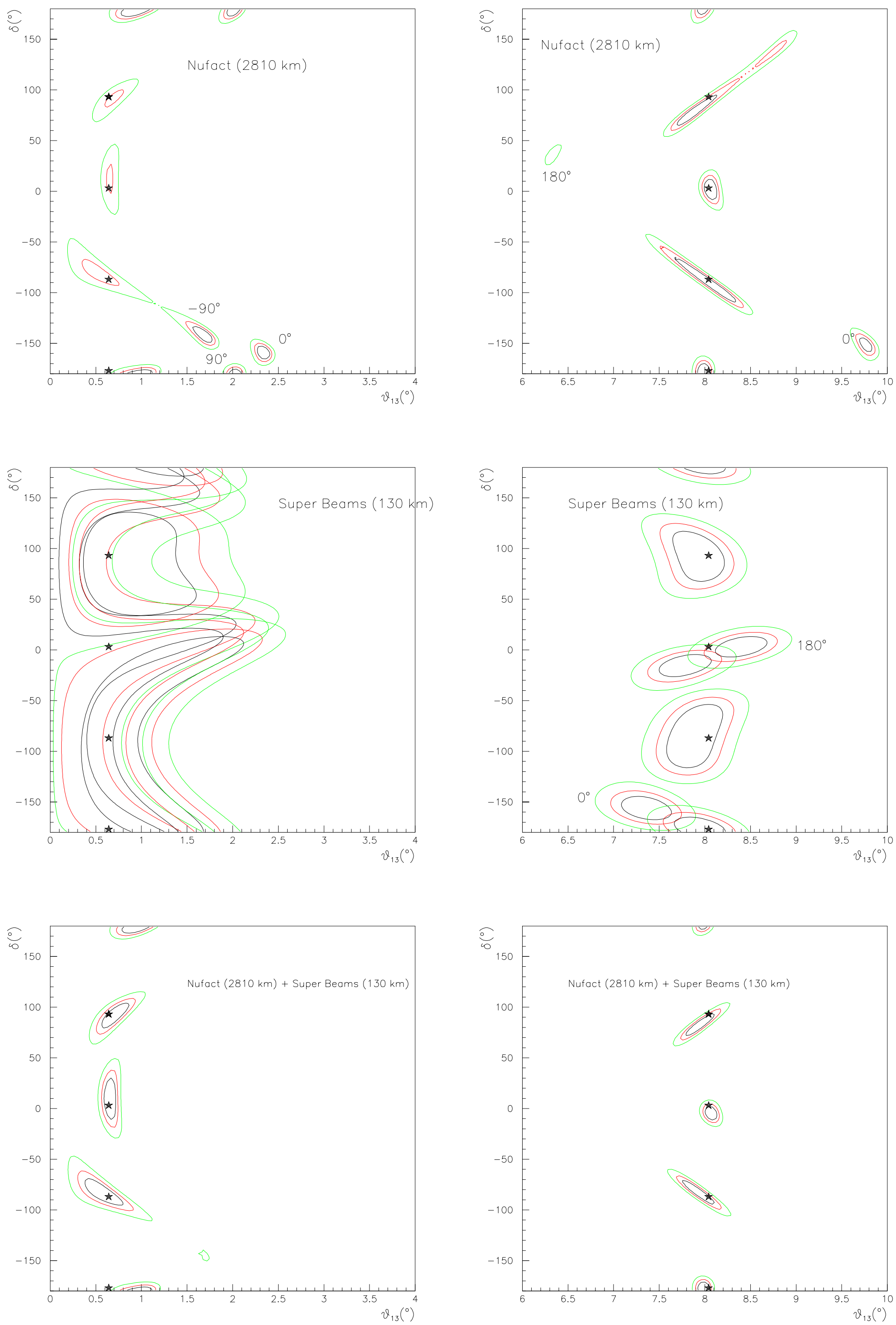
Solution II: $\theta_{13}' = \theta_{13} + \Delta(E_\nu, L) \quad \delta' = \pi - \delta$

Burguet-Castell *et al*





Energy dependence of the signal in one experiment is usually not enough to resolve them → combine experiments!



Parameter correlations

There are two extra discrete ambiguities:

- $\Delta m^2_{13} >$ or < 0
- $\theta_{23} >$ or $< \frac{\pi}{4}$ if $\theta_{23} \neq \pi/4$

Additional fake solutions analogous to I and II are obtained at fixed E_ν and L *in vacuum* when the probabilities for $\Delta m^2_{23} > 0 (\theta_{23} > \pi/4)$ are fitted to those for $\Delta m^2_{23} < 0 (\theta_{23} < \pi/4)$

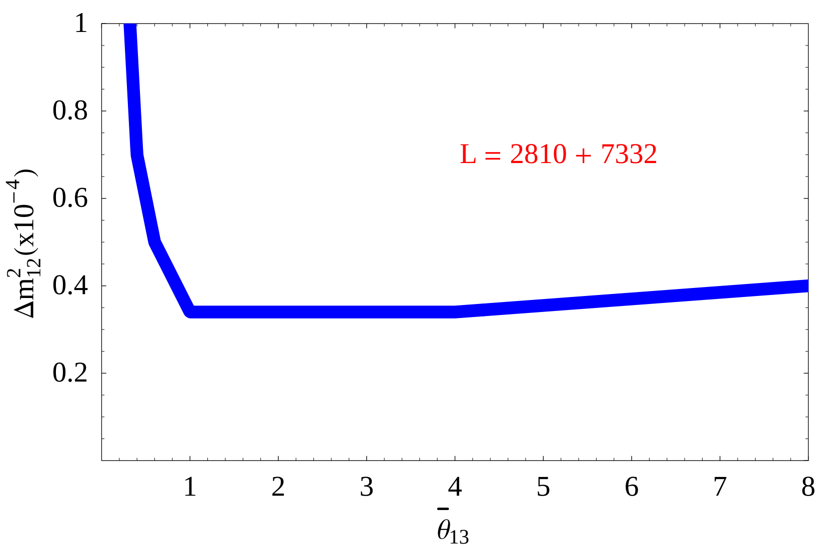
Barger *et al*

The combination of experiments with different $\langle E_\nu \rangle / L$ and with/without matter effects is extremely powerfull to:

- Eliminate the bias in the extraction of θ_{13} and δ
- Determine the $\mathrm{sign}(\Delta m^2_{13})$ and $\mathrm{sign}(\theta_{23} - \pi/4)$

Conclusions

- If ν are massive there is generically CP-violation yet to be discovered in the lepton sector
- Large CP-odd asymmetries are expected in $\nu_e \to \nu_\mu(\nu_\tau)$ oscillations at $\langle E_\nu \rangle / L \sim |\Delta m^2_{\rm atmos}|$
- The measurement of this new source of CP-violation requires more precise ν oscillation experiments: superbeams/beta-beams and ultimately a neutrino factory from μ-decays
- The combination of at least two experiments with different $\langle E_\nu \rangle / L$ is very useful to reduce parameter correlations which might otherwise affect the extraction of θ_{13} and δ
- Given the latest news from solar neutrinos, chances to discover CP-violation in the lepton sector are indeed very good:



- Even though the low energy parameters of the ν mass matrix are probably not enough to predict the baryon asymmetry, the discovery of:
  - Majorana nature of ν's
  - CP-violation in the lepton sector

  would be very strong hints of the viability of this scenario