MAJORANA NEUTRINOS, NEUTRINO MASS SPECTRUM, CP-VIOLATION IN THE LEPTON SECTOR AND $(\beta\beta)_{0\nu}$-DECAY

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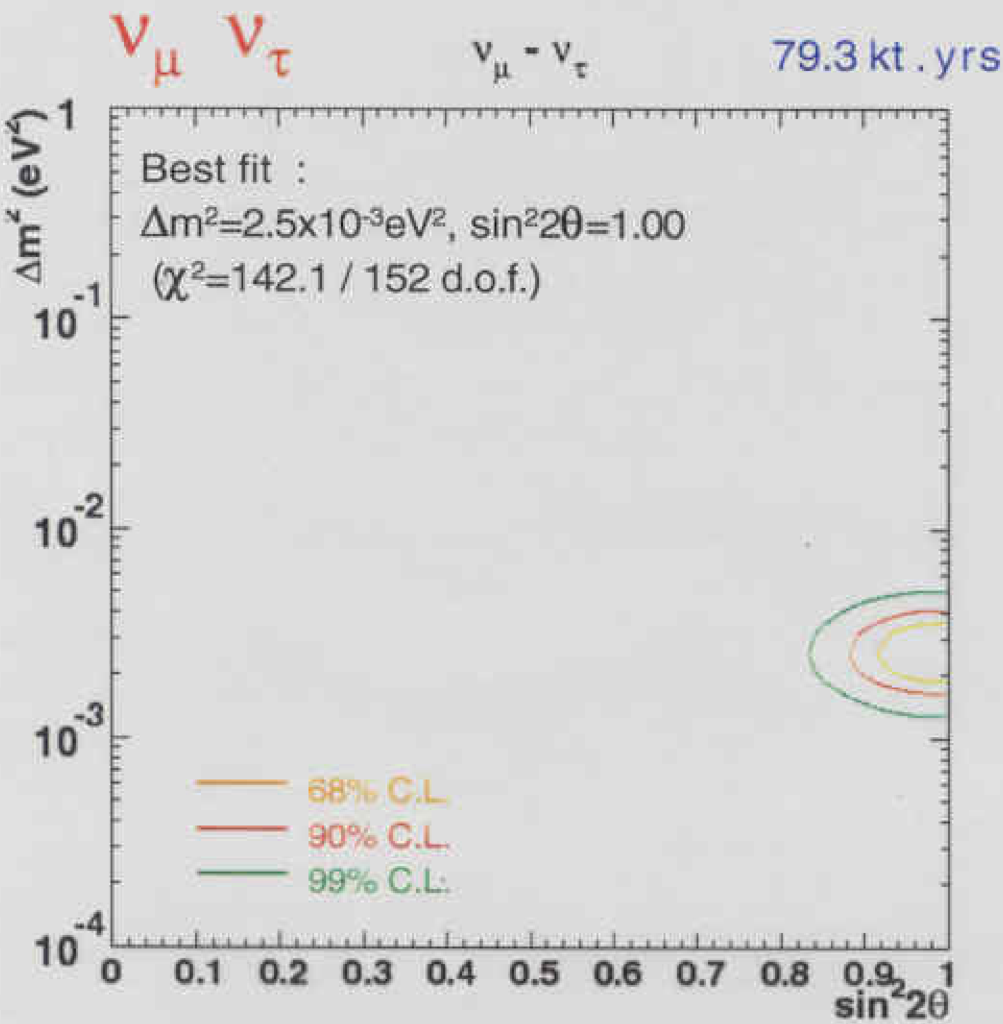
21 JUNE, 2002

Allowed region

(FC + PC + UP-thru + UP-stop)



SK combined result

$\Delta m^2 = (1.7 \sim 4) \times 10^{-3} eV^2$

$\sin^2 2\theta > 0.89$ (90% C.L.)

SIGN (Δm^2) - UNDETERMINED

3-ν MIXING : $m_1 < m_2 < m_3$ - NH

OR $m_3 < m_1 < m_2$ - IH

SNO

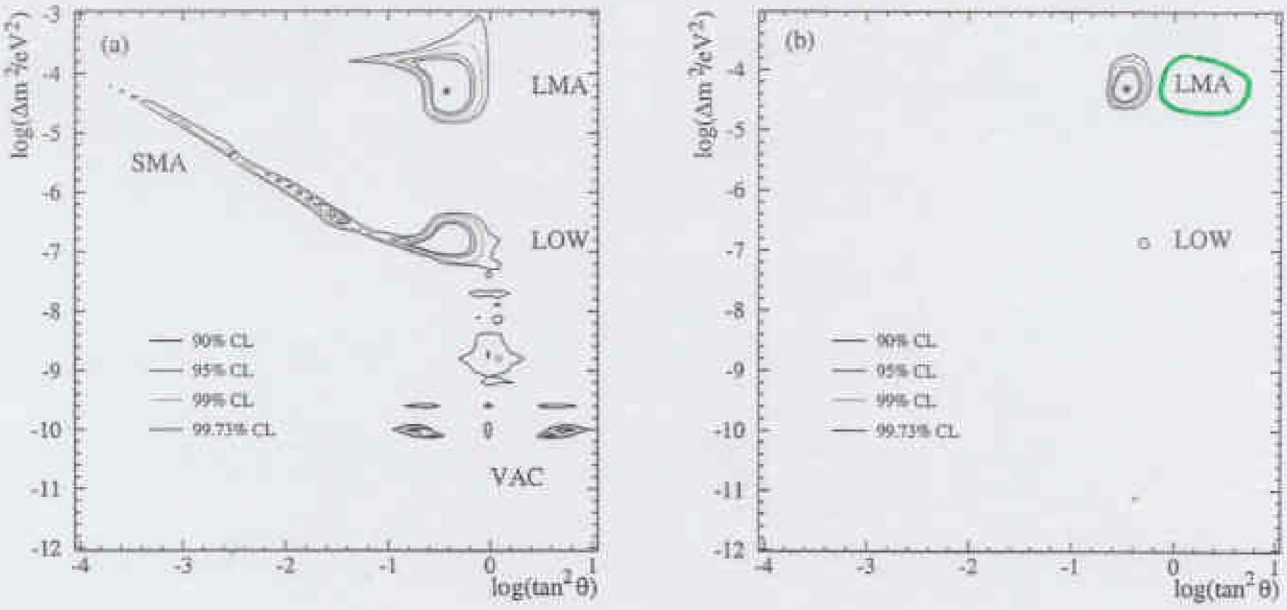


FIG. 4: Allowed regions of the MSW plane determined by a χ^2 fit to (a) SNO day and night energy spectra and (b) with additional experimental and solar model data. The star indicates the best fit. See text for details.

SNO web site: http://sno.phy.queensu.ca
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$\Delta m^2_\odot [eV^2] > 0$ $\sin^2 2\theta_\odot$

B.F.V.: 5×10^{-5} 0.75

95% C.L. $(2.5 - 10) \times 10^{-5}$ $0.56 - 0.89$

$\cos 2\theta_\odot \geq 0.26$ AT 99.73% C.L.

EVIDENCES FOR ν-OSCILLATIONS:

- ν_{ATM} : SK

  UP-DOWN ASYMMETRY
  (ZENITH ANGLE DEPENDENCE)
  MULTI-GEV μ-LIKE SAMPLE

  DOMINANT
  $\overset{(-)}{\nu}_\mu \longrightarrow \overset{(-)}{\nu}_\tau$

  K2K, MINOS, CNGS.

- $\nu_\odot$:

  HOMESTAKE, KAMIOKANDE,
  SAGE, GALLEX/GNO,
  SUPER-KAMIOKANDE,
  SNO

  DOMINANT
  $\nu_e \longrightarrow \nu_{\mu,\tau}$

  KAMLAND, BOREXINO, ...

- LSND

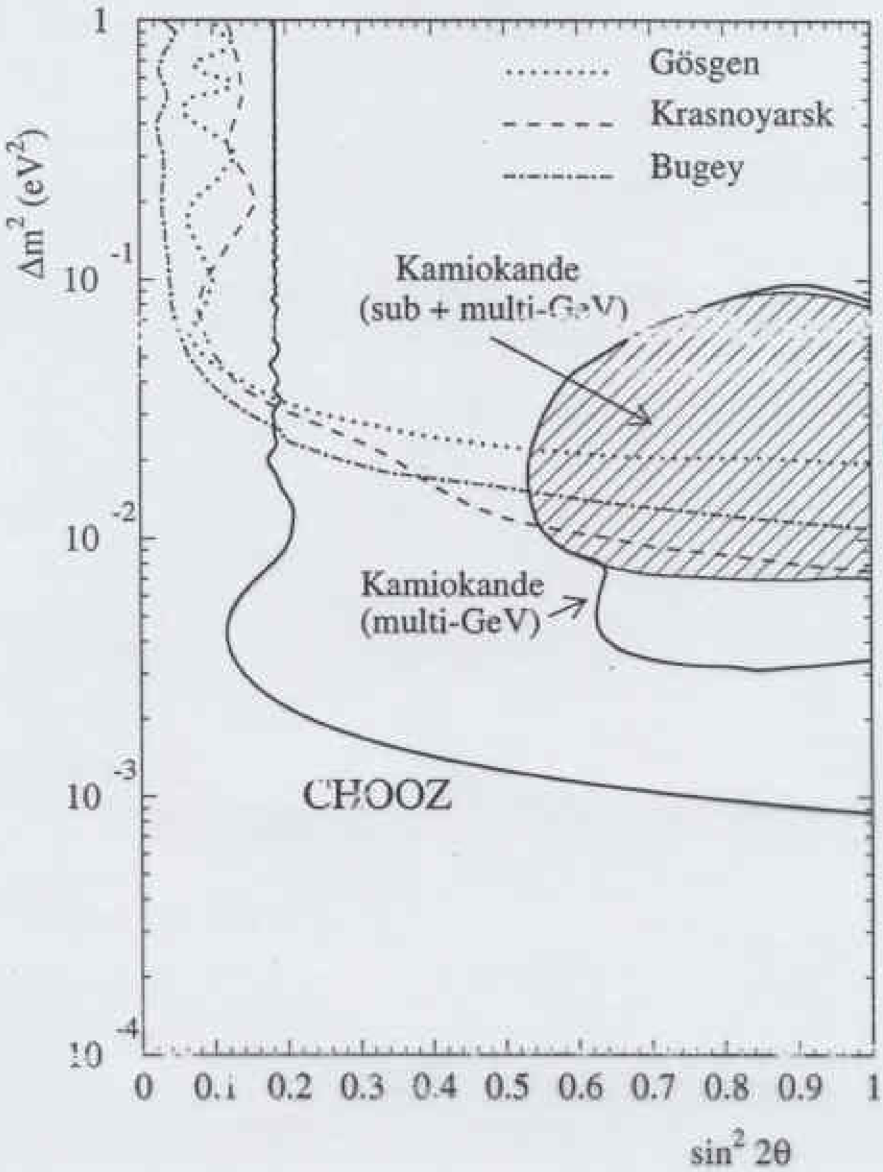
  $\bar{\nu}_\mu \longrightarrow \bar{\nu}_e$ MINIBOONE

$$\nu_{\ell L} = \sum_{j=1} U_{\ell j}\, \nu_{jL}, \qquad \ell = e, \mu, \tau$$

ν-FACTORIES : 3-ν MIXING, LMA MSW
$L \sim (3000 - 7000)$ km.

CHOOZ : $\bar{\nu}_e \to \bar{\nu}_e$

$\sim 1\,km$
$\bar{E}_{\bar{\nu}} \sim 2\,MeV$



$\overset{(-)}{\nu}_\mu \rightleftarrows \overset{(-)}{\nu}_e$

Figure 6: The 90% C.L. exclusion plot for CHOOZ, compared with previous experimental limits and with the KAMIOKANDE allowed region.

STANDARD PARAMETRIZATION:

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13}e^{i\alpha_{21}/2} & U_{e3}\, e^{i(\frac{\alpha_{31}}{2}+\delta)} \\ -s_{12}c_{23}-c_{12}s_{23}U^*_{e3} & (c_{12}c_{23}-s_{12}s_{23}U^*_{e3})e^{i\frac{\alpha_{21}}{2}} & s_{23}c_{13}e^{i(\alpha_{31}/2+\delta)} \\ s_{12}s_{23}-c_{12}c_{23}U^*_{e3} & (-c_{12}s_{23}-s_{12}c_{23}U^*_{e3})e^{i\frac{\alpha_{21}}{2}} & c_{23}c_{13}e^{i(\alpha_{31}/2+\delta)} \end{pmatrix}$$

$c_{ij} \equiv \cos\theta_{ij}$, $s_{ij} \equiv \sin\theta_{ij}$, $0 \le \theta_{12}, \theta_{13}, \theta_{23} \le \pi/2$

$U_{e3} = s_{13}e^{-i\delta}$, $\delta \in [0, 2\pi]$ - DIRAC CP-VIOLATING PHASE

α_{21}, α_{31} - MAJORANA CP-VIOLATING PHASES

IF ν_j ARE MAJORANA PARTICLES, S.M. BILENKY, J. Hosek, S.T.P. '80

CP - SYMMETRY CAN BE VIOLATED EVEN IN THE CASE OF $n=2$ FAMILIES OF LEPTONS:

$$n^M_{CP} = \frac{n(n+1)}{2} - n = \frac{n(n-1)}{2}$$

$\nu_{\odot}$, ν_{ATM}, CHOOZ DATA:

$$\theta_{\odot} = \theta_{12} \sim \pi/6, \quad \theta_{ATM} = \theta_{23} \sim \pi/4, \quad \theta_{CHOOZ} \equiv \theta = \theta_{13} < \frac{\pi}{12} \div \frac{\pi}{15}$$

$$U_{PMNS} = \begin{pmatrix} \sim \frac{1}{2} & \sim \frac{\sqrt{3}}{2} & \varepsilon \\ U_{\mu 1} & U_{\mu 2} & \sim \frac{1}{\sqrt{2}} \\ U_{\tau 1} & U_{\tau 2} & \sim \frac{1}{\sqrt{2}} \end{pmatrix}$$

IF CP-VIOLATION DUE TO U_{PMNS}, OBSERVABLE MANIFESTATIONS POSSIBLE IN

$$\overset{(-)}{\nu}_l \rightleftarrows \overset{(-)}{\nu}_{l'}, \quad l \neq l' = e, \mu, \tau$$

CP-INVARIANCE: CABIBBO '78

$$P(\nu_l \to \nu_{l'}; E, L) = P(\bar{\nu}_l \to \bar{\nu}_{l'}; E, L), \; l \neq l'$$

CPT-INVARIANCE: $P(\nu_l \to \nu_{l'}) = P(\bar{\nu}_{l'} \to \bar{\nu}_l)$

$l = l'$: $P(\nu_l \to \nu_l) = P(\bar{\nu}_l \to \bar{\nu}_l)$

T-INVARIANCE: $P(\nu_l \to \nu_{l'}) = P(\nu_{l'} \to \nu_l)$

3-ν MIXING:

$$A_T^{(l,l')} \equiv P(\nu_l \to \nu_{l'}) - P(\nu_{l'} \to \nu_l)$$

$$A_T^{(e,\mu)} = A_T^{(\mu,\tau)} = -A_T^{(e,\tau)}$$ KRASTEV, S.T.P. '88

IN VACUUM: $A_{T\,VAC}^{(e,\mu)} = J_{CP}^{VAC} F^{VAC}$

$$J_{CP}^{VAC} = Im\left\{U_{\mu 2} U_{e3}^* U_{\mu 3}^* U_{e2}\right\} =$$

$$= \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos\theta_{13} \sin\delta$$

$$F^{VAC} = \sin \frac{\Delta m_{21}^2}{2E} L + \sin \frac{\Delta m_{13}^2}{2E} L + \sin \frac{\Delta m_{32}^2}{2E} L$$

$\overset{(-)}{\nu}_\ell \rightleftarrows \overset{(-)}{\nu}_{\ell'}$ IN MATTER:

MATTER EFFECTS VIOLATE

LANGACKER ET AL., '87

CP $\quad P(\nu_\ell \to \nu_{\ell'}) \neq P(\bar\nu_\ell \to \bar\nu_{\ell'})$

CPT $\quad P(\nu_\ell \to \nu_{\ell'}) \neq P(\bar\nu_{\ell'} \to \bar\nu_\ell)$

CAN CONSERVE T (EARTH)

$$P(\nu_\ell \to \nu_{\ell'}) \overset{?}{=} P(\nu_{\ell'} \to \nu_\ell)$$

IN MATTER WITH CONSTANT DENSITY (EARTH MANTLE)

$$J_{CP}^{MATTER} = J_{CP}^{VAC}\,\tilde{F}$$

KRASTEV, S.T.P. '88
HARRISON, SCOTT, '00
XING, '01

IF ν_j – MAJORANA PARTICLES,

BILENKY ET AL. '80
DOI ET AL. '81

U_{PMNS} – CONTAINS 3ν MIXING
$\begin{cases} \delta - \text{DIRAC} \\ \alpha_{21}, \alpha_{31} - \text{MAJORANA} \\ \text{PHYSICAL CP-VIOLATING PHASES} \end{cases}$

ν – OSCILLATIONS $\overset{(-)}{\nu}_l \rightleftarrows \overset{(-)}{\nu}_{l'}$, $l, l' = e, \mu, \tau$

BILENKY, Hosek, STP '80

- ARE NOT SENSITIVE TO THE NATURE OF ν_j;
- PROVIDE INFORMATION ON $\Delta m^2_{jk} = m_j^2 - m_k^2$, $j > k = 1,2,3$ BUT NOT ON THE ABSOLUTE VALUES OF NEUTRINO MASSES m_j.

HOW CAN ONE OBTAIN INFORMATION ON

- THE NATURE OF ν_j ?
- m_1, m_2, m_3, i.e., ON THE ν-MASS SPECTRUM ?
- ON THE CP-VIOLATION IN THE LEPTON SECTOR, INDUCED BY THE MAJORANA CP-VIOLATING PHASES ?

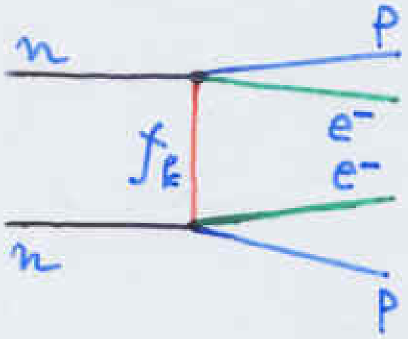
THE MAJORANA NATURE OF ν_j CAN MANIFEST ITSELF IN THE EXISTENCE OF PROCESSES

$\Delta L = \pm 2$: $K^+ \rightarrow \pi^- \mu^+ \mu^+$
$\mu^- + (A,Z) \rightarrow \mu^+ + (A, Z-2), \ldots$

THE MOST SENSITIVE PROCESS – $(\beta\beta)_{0\nu}$ – decay OF CERTAIN EVEN-EVEN NUCLEI :

$(\beta\beta)_{0\nu}$ – decay:

$$(A,Z) \to (A, Z+2) + e^- + e^-$$

$\sum_k$ 

$$\nu_{eL}(x) = \sum_k U_{ek} \underbrace{\nu_{kL}(x)}_{m_{\nu_k}}$$

$$\mathcal{H}_W^{\beta} = \frac{G_F}{\sqrt{2}}\, 2\, (\bar{e}_L(x)\gamma_\alpha \nu_{eL}(x))\, j_\alpha^{h}(x) + h.c.$$

$$S^{(2)} = -\frac{(-i)^2}{2}\, 4 \left(\frac{G_F}{\sqrt{2}}\right)^2 \int dx_1\, dx_2 \times T^{h}_{\alpha\beta}(x_1, x_2) \times$$

$$\times N\left[\bar{e}_L(x_1)\gamma_\alpha\, \nu_{eL}(x_1)\, \nu_{eL}^T(x_2)\, \gamma_\beta^T\, \bar{e}_L^T(x_2)\right]$$

$$A(\beta\beta)_{0\nu} \sim \langle m_\nu \rangle ,$$

$$\langle m_\nu \rangle \underset{\substack{m_k \lesssim \text{few MeV} \\ CP\text{-inv.}}}{=} \sum_k |U_{ek}|^2 \frac{1}{i}\, \underbrace{\eta^{CP}(\nu_k)}\, m_k$$

Data: ^{76}Ge

$$\boxed{|\langle m_\nu \rangle| < (1 \div 2)\ \text{eV}}$$

The Majorana nature of the massive neutrinos can manifest itself in the existence of $L \neq 0$, $\Delta L = 2$, processes. The process most sensitive to the existence of massive Majorana neutrinos (coupled to the electron) is the neutrinoless double β ($(\beta\beta)_{0\nu}-$) decay of certain even-even nuclei

$$(A, Z) \to (A, Z+2) + e^- + e^-. \tag{1}$$

If the $(\beta\beta)_{0\nu}-$ decay is generated *only by the left-handed (LH) charged current weak interaction through the exchange of virtual massive Majorana neutrinos*, $A((\beta\beta)_{0\nu})$ is proportional to the so-called "effective Majorana mass"

$$A((\beta\beta)_{0\nu}) \quad \sim <m> \equiv \sum_{j=1} U_{ej}^2 \, m_j, \qquad m_j \lesssim \text{ few MeV}, \tag{2}$$

where m_j is the mass of the Majorana neutrino ν_j and U_{ej} is the element of neutrino (lepton) mixing matrix U.

A large number of experiments are searching for $(\beta\beta)_{0\nu}-$decay of different nuclei at present. No indications that this process takes place were found.
^{76}Ge Heidelberg-Moscow experiment:

$$|<m>| \; < 0.35 \text{ eV}, \quad 90\% \text{ C.L.} \tag{3}$$

Taking into account a factor of 3 uncertainty associated with the calculation of the relevant nuclear matrix element

$$|<m>| \; < (0.35 \div 1.05) \text{ eV}, \quad 90\% \text{ C.L.}. \tag{4}$$

The IGEX collaboration has obtained [30]:

$$|<m>| \; < (0.33 \div 1.35) \text{ eV}, \quad 90\% \text{ C.L.} \tag{5}$$

Considerably higher sensitivity to the value of $|<m>|$ is planned to be reached in several $(\beta\beta)_{0\nu}-$decay experiments of a new generation:

- the NEMO3 experiment scheduled to start in 2001, will search for $(\beta\beta)_{0\nu}-$decay of ^{100}Mo and ^{82}Se; will reach a sensitivity to $|<m>| \; \cong 0.1$ eV.

- CUORE: a similar sensitivity is planned to be reached with the cryogenic detector CUORE; will search for the $(\beta\beta)_{0\nu}-$decay of ^{130}Te.

- GENIUS: sensitivity to $|<m>| \; \cong 10^{-2}$ eV, is planned to be achieved utilizing one ton of enriched ^{76}Ge.

- EXO: proposal to study the $(\beta\beta)_{0\nu}-$decay of ^{136}Xe in a background-free experiment with detection of the two e^- and the $^+$Ba atom in the final state; the estimated sensitivity of this experiment is $|<m>| \; \cong (1-5) \times 10^{-2}$ eV.

As is well known, the explanation of the atmospheric and solar neutrino data in terms of neutrino oscillations requires the existence of 3-neutrino mixing in the weak charged lepton current:

$$\nu_{l\mathrm{L}} = \sum_{j=1}^{3} U_{lj}\nu_{j\mathrm{L}}, \tag{5}$$

where ν_{lL}, $l = e, \mu, \tau$, are the three left-handed flavour neutrino fields, $\nu_{j\mathrm{L}}$ is the left-handed field of the neutrino ν_j having a mass m_j and U is a 3×3 unitary mixing matrix - the Pontecorvo-Maki-Nakagawa-Sakata neutrino (lepton) mixing matrix. If ν_j are Majorana neutrinos,

$$C(\bar{\nu}_j)^T = \nu_j, \; j = 1, 2, 3,$$

C is the charge conjugation matrix, and for $m_j \lesssim$ few MeV,

$$|<m>| = \left|m_1 U_{\mathrm{e}1}^2 + m_2 U_{\mathrm{e}2}^2 + m_3 U_{\mathrm{e}3}^2\right| \tag{6}$$

$$= \left|m_1|U_{\mathrm{e}1}|^2 + m_2|U_{\mathrm{e}2}|^2\, e^{i\alpha_{21}} + m_3|U_{\mathrm{e}3}|^2\, e^{i\alpha_{31}}\right| \tag{7}$$

where

$$U_{ej} = |U_{ej}|\, e^{\frac{i\alpha_j}{2}}$$

$$\alpha_{21} \equiv (\alpha_2 - \alpha_1), \quad \alpha_{31} \equiv (\alpha_3 - \alpha_1)$$

are two CP-violating phases. If CP-invariance holds, one has

$$\alpha_{21} = k\pi, \quad \alpha_{31} = k'\pi, \qquad k, k' = 0, 1, 2, ...$$

L. WOLFENSTEIN '81
B. KAYSER '84
BILENKY, NEDELCHEVA, S.T.P. '84

In this case

$$\eta_{21} \equiv e^{i\alpha_{21}} = \pm 1, \quad \eta_{31} \equiv e^{i\alpha_{31}} = \pm 1, \tag{8}$$

represent the relative CP-parities of the neutrinos ν_1 and ν_2, and ν_1 and ν_3, respectively.

We can numerate (without loss of generality) the neutrino masses in such a way that $m_1 < m_2 < m_3$. If we denote by $\theta_\odot$ and θ respectively the mixing angles constrained by the solar neutrino data and the data from the CHOOZ experiment, then depending on the type of the neutrino mass spectrum one has either

$$|U_{\mathrm{e}1}| = \cos\theta_\odot\sqrt{1 - |U_{\mathrm{e}3}|^2}, \quad |U_{\mathrm{e}2}| = \sin\theta_\odot\sqrt{1 - |U_{\mathrm{e}3}|^2}, \quad |U_{\mathrm{e}3}|^2 = \sin^2\theta, \tag{9}$$

or

$$|U_{\mathrm{e}2}| = \cos\theta_\odot\sqrt{1 - |U_{\mathrm{e}1}|^2}, \quad |U_{\mathrm{e}3}| = \sin\theta_\odot\sqrt{1 - |U_{\mathrm{e}1}|^2}, \quad |U_{\mathrm{e}1}|^2 = \sin^2\theta. \tag{10}$$

Relations (9) are valid for the hierarchical neutrino mass spectrum, while those in eq. (10) are realized for the neutrino mass spectrum with inverted hierarchy.

The neutrino oscillation experiments provide information on $\Delta m^2_{jk} = m^2_j - m^2_k$ $(j > k)$. In the case of 3-neutrino mixing as an independent set of three neutrino mass parameters one can choose

$$m_1, \quad \sqrt{\Delta m^2_{21}}, \quad \sqrt{\Delta m^2_{32}}.$$

Then :

$$m_2 = \sqrt{m_1^2 + \Delta m^2_{21}}\ , \tag{11}$$

$$m_3 = \sqrt{m_1^2 + \Delta m^2_{21} + \Delta m^2_{32}}\ . \tag{12}$$

The Δm^2 inferred from the atmospheric neutrino data,

$$\Delta m^2_{\text{atm}} = \Delta m^2_{31} = \Delta m^2_{21} + \Delta m^2_{32}\ , \tag{13}$$

while for the one deduced from the solar neutrino data, $\Delta m^2_{\odot}$, we have two possibilities:

$$\Delta m^2_{\odot} \equiv \Delta m^2_{32} \quad \text{or} \quad \Delta m^2_{\odot} \equiv \Delta m^2_{21}\ . \tag{14}$$

Depending on the relative magnitudes of m_1, $\sqrt{\Delta m^2_{21}}$ and $\sqrt{\Delta m^2_{32}}$, one recovers the different possible types of neutrino mass spectrum:

1. if $m_1 \ll \sqrt{\Delta m^2_{21}} \ll \sqrt{\Delta m^2_{32}}$, one has $m_1 \ll m_2 \ll m_3$, i.e., hierarchical (H) neutrino mass spectrum;
2. $m_1 \ll \sqrt{\Delta m^2_{32}} \ll \sqrt{\Delta m^2_{21}}$ implies $m_1 \ll m_2 \simeq m_3$, i.e., neutrino mass spectrum with inverted hierarchy (IH);
3. for $\sqrt{\Delta m^2_{21}}, \sqrt{\Delta m^2_{32}} \ll m_1$, we have $m_1 \simeq m_2 \simeq m_3$, i.e., quasi-degenerate (QD) neutrino mass spectrum;
4. if $\sqrt{\Delta m^2_{21}} \ll \sqrt{\Delta m^2_{32}} \sim O(m_1)$, one finds $m_1 \simeq m_2 < m_3$, i.e., spectrum with "partial mass hierarchy"; interpolates between H and QD.
5. for $\sqrt{\Delta m^2_{32}} \ll \sqrt{\Delta m^2_{21}} \sim O(m_1)$, we have $m_1 < m_2 \simeq m_3$, i.e., spectrum with "partial inverted mass hierarchy"; interpolates between IH and QD.

Given the values of $\Delta m^2_{\odot}$, $\theta_{\odot}$, Δm^2_{atm} and of θ,

$$|<m>| = |<m>|\ (m_1, \alpha_{21}, \alpha_{31}; S), \qquad S = H,\ IH$$

The knowledge of m_1 would allow to determine the neutrino mass spectrum.

GIVEN THE VALUES OF Δm^2_{atm} AND $\Delta m^2_{\odot}$, INFERRED FROM THE DATA, ONE HAS:

- $m_1 \ll 0.02\,eV$ - HIERARCHICAL OR INVERTED HIERARCHY
- $0.02 \lesssim m_1 \lesssim 0.2\,eV$ - PH OR PIH
- $m_1 > 0.2\,eV$ - QD

ν - MASS SPECTRUM

The Troitzk [67] and Mainz [68] ^{3}H β-decay experiments, studying the electron spectrum, provide information on the electron (anti-)neutrino mass m_{ν_e}. The data contain features which require further investigation (e.g., a peak in the end-point region which varies with time [67]). The upper bounds given by the authors (at 95% C.L.) read:

$$m_{\nu_e} < \overset{2.2}{2.5}\ \text{eV} \quad [67], \quad m_{\nu_e} < \overset{2.2}{2.9}\ \text{eV} \quad [68]. \tag{15}$$

There are prospects to increase the sensitivity of the ^{3}H β-decay experiments,

$$\text{KATRIN}: \qquad m_{\nu_e} \sim (0.3 - 0.4)\ \text{eV}.$$

Cosmological and astrophysical data provide information on the sum of the neutrino masses. The current upper bound reads (see, e.g., [70] and the references quoted therein):

$$\sum_j m_j \lesssim 5.5\ \text{eV}\ . \tag{16}$$

The future experiments MAP and PLANCK can be sensitive to [71]

$$\sum_j m_j \cong 0.4\ \text{eV}\ . \tag{17}$$

ν-OSCILLATION DATA:

$$0.03\,\text{eV} \leq \sum_{j=1}^{3} m_j < \overset{6.6}{7.5}\,\text{eV}$$

4 Hierarchical Neutrino Mass Spectrum

The hierarchical neutrino mass spectrum is characterized by

$$m_1 \ll m_2 \ll m_3. \tag{42}$$

This type of neutrino mass spectrum is predicted by the standard versions of the see-saw mechanism of neutrino mass generation. The pattern corresponds to

$$m_1 \ll \sqrt{\Delta m^2_{21}} \ll \sqrt{\Delta m^2_{32}}\,. \tag{43}$$

Using (42) and (43) it is possible to make the identification

$$\begin{aligned}
\Delta m^2_{\odot} \equiv \Delta m^2_{21}, \quad \Delta m^2_{\rm atm} \equiv \Delta m^2_{32},\\
|U_{\rm e1}|^2 = \cos^2\theta_{\odot}(1 - |U_{\rm e3}|^2),\\
|U_{\rm e2}|^2 = \sin^2\theta_{\odot}(1 - |U_{\rm e3}|^2),\\
|U_{\rm e3}|^2 \equiv \sin^2\theta < 0.09 \qquad \text{(CHOOZ)}.
\end{aligned} \tag{44}$$

We will suppose that $\Delta m^2_{\rm atm}$ lies in the interval (5) or (13), $\Delta m^2_{\odot}$ and $\theta_{\odot}$ take values in the regions given in Tables 1 and 2, and that $|U_{\rm e3}|^2$ satisfies the CHOOZ upper bound. Equations (42) and (44) further imply:

$$m_2 \simeq \sqrt{\Delta m^2_{\odot}}\,, \qquad m_3 \simeq \sqrt{\Delta m^2_{\rm atm}}\,. \tag{45}$$

One finds

$$|\!<\!m\!>\!| \simeq \left|\sqrt{\Delta m^2_{\odot}}\,(1 - |U_{\rm e3}|^2)\sin^2\theta_{\odot} + \sqrt{\Delta m^2_{\rm atm}}\,|U_{\rm e3}|^2 e^{i(\alpha_3-\alpha_2)}\right| \tag{46}$$

where we have neglected the contribution of the term $\sim m_1$. Although in this case one of three massive Majorana neutrinos effectively "decouples" and does not give a contribution to $|\!<\!m\!>\!|$, the value of $|\!<\!m\!>\!|$ still depends on the Majorana CP-violating phase $\alpha_{32} = \alpha_3 - \alpha_2$. This reflects the fact that in contrast to the case of massive Dirac neutrinos (or quarks), CP-violation can take place in the mixing of only two massive Majorana neutrinos.

5 Inverted Mass Hierarchy Spectrum

The inverted mass hierarchy spectrum is characterized by

$$m_1 \ll m_2 \simeq m_3. \tag{66}$$

The identification with the neutrino oscillation parameters probed in the solar and atmospheric neutrino experiments and in CHOOZ reads

$$\begin{aligned}
\Delta m^2_{\odot} &\equiv \Delta m^2_{32}, \\
\Delta m^2_{\rm atm} &\equiv \Delta m^2_{31} \simeq \Delta m^2_{21}, \\
|U_{e1}|^2 &= \sin^2\theta < 0.09 \quad \text{(CHOOZ)}, \\
|U_{e2}|^2 &= \cos^2\theta_{\odot}(1 - |U_{e1}|^2), \\
|U_{e3}|^2 &= \sin^2\theta_{\odot}(1 - |U_{e1}|^2).
\end{aligned} \tag{67}$$

We also have:

$$m_2 \simeq m_3 \simeq \sqrt{\Delta m^2_{\rm atm}}. \tag{68}$$

The inverted mass hierarchy spectrum can also be defined by the inequalities

$$m_1 \ll (<) \sqrt{\Delta m^2_{32}} \ll \sqrt{\Delta m^2_{21}}. \tag{69}$$

The term $m_1|U^2_{e1}|$ in the expression for $|{<}m{>}|$ can be neglected, since $m_1 \ll m_{2,3}$ and $|U^2_{e1}| \ll 1$. This approximation would be valid as long as the sum of the two other terms in eq. (21) exceeds $\sim 5.0 \times 10^{-3}$ eV. Under the assumption that $m_1|U^2_{e1}|$ gives a negligible contribution to $|{<}m{>}|$ we have

$$|{<}m{>}| \simeq \left| |U_{e2}|^2 m_2 + |U_{e3}|^2 m_3 e^{i(\alpha_3-\alpha_2)} \right| \tag{70}$$

$$= m_{2,3}\sqrt{1 - 4|U_{e2}|^2|U_{e3}|^2 \sin^2\left(\frac{\alpha_3-\alpha_2}{2}\right)} \tag{71}$$

$$= \sqrt{\Delta m^2_{\rm atm}}\,(1 - |U_{e1}|^2)\sqrt{1 - \sin^2 2\theta_{\odot} \sin^2\left(\frac{\alpha_3-\alpha_2}{2}\right)}, \tag{72}$$

Even though one of the three massive Majorana neutrinos "decouples", the value of $|{<}m{>}|$ depends on the Majorana CP-violating phase $(\alpha_3 - \alpha_2)$.

Obviously, $|{<}m{>}|$ satisfies

$$\sqrt{\Delta m^2_{\rm atm}}\,(1 - |U_{e1}|^2)|\cos 2\theta_{\odot}| \leq |{<}m{>}| \leq \sqrt{\Delta m^2_{\rm atm}}\,(1 - |U_{e1}|^2). \tag{73}$$

The upper and the lower limits correspond respectively to the CP-conserving cases $\phi_2 = \phi_3$ ($\alpha_3 - \alpha_2 = 0$, or $\alpha_{21} = \alpha_{31} = 0, \pm\pi$) and $\phi_2 = -\phi_3$ ($\alpha_3 - \alpha_2 = \pm\pi$, or $\alpha_{21} = \alpha_{31} + \pi = 0, \pm\pi$).

The expression for $|<m>|$ permits to relate the value of $\sin^2(\alpha_3 - \alpha_2)/2$ to the experimentally measured quantities $|<m>|$, $\Delta m^2_{\rm atm}$ and $\sin^2 2\theta_\odot$:

$$\sin^2 \frac{\alpha_3 - \alpha_2}{2} = \left(1 - \frac{|<m>|^{\,2}}{\Delta m^2_{\rm atm}\,(1 - \,|U_{\rm e1}|^2\,)^2}\right) \frac{1}{\sin^2 2\theta_\odot}. \qquad (75)$$

A more precise determination of $\Delta m^2_{\rm atm}$ and $\theta_\odot$ and a sufficiently accurate measurement of $|<m>|$ could allow to get information about the value of $(\alpha_3 - \alpha_2)$, provided the neutrino mass spectrum is of the inverted hierarchy type.

6 The Case of Three Quasi-Degenerate Neutrinos

The neutrinos $\nu_{1,2,3}$ are quasi-degenerate in mass if

$$m_1 \simeq m_2 \simeq m_3 \equiv m, \tag{76}$$

and

$$m \gg \sqrt{\Delta m^2_{\rm atm}}\,. \tag{77}$$

When (76) holds but $m \sim O(\sqrt{\Delta m^2_{\rm atm}}\,)$, we have a partial hierarchy or partial inverted hierarchy between the neutrino masses.

As for the hierarchical neutrino mass spectrum we have:

$$\begin{aligned}
\Delta m^2_{\odot} \equiv \Delta m^2_{21}, \quad \Delta m^2_{\rm atm} &\equiv \Delta m^2_{31}, \\
|U_{e1}|^2 &= \cos^2\theta_{\odot}(1 - |U_{e3}|^2\,), \\
|U_{e2}|^2 &= \sin^2\theta_{\odot}(1 - |U_{e3}|^2\,), \\
|U_{e3}|^2 &= \sin^2\theta < 0.09 \quad \text{(CHOOZ)}.
\end{aligned} \tag{78}$$

Equation (78) allows to express eqs. (76) and (77) in the compact form

$$\sqrt{\Delta m^2_{21}} \ll (<)\ \sqrt{\Delta m^2_{32}} \ll m_1. \tag{79}$$

The mass scale m effectively coincides with the electron (anti-)neutrino mass m_{ν_e} measured in the ^{3}H β-decay experiments:

$$m = m_{\nu_e}. \tag{80}$$

Thus, the experimental upper bounds in eq. (15) lead to $m < 2.5$ eV.

The QD neutrino mass spectrum under discussion is actually realized for values of the neutrino mass m, which is measured in the ^{3}H $\beta-$decay experiments, $m = m_{\nu_e} \gtrsim (0.2-0.3)$ eV. The new ^{3}H $\beta-$decay experiment KATRIN is planned to have a record sensitivity of 0.35 eV to the neutrino mass m_{ν_e}. The realization of this project could be crucial for the test of the possibility of three quasi-degenerate neutrinos.

The effective Majorana mass $|<m>|$ can be approximated by

$$|<m>| \simeq m \left|\cos^2\theta_{\odot}(1 - |U_{e3}|^2\,)e^{i\alpha_1} + \sin^2\theta_{\odot}(1 - |U_{e3}|^2\,)e^{i\alpha_2} + |U_{e3}|^2 e^{i\alpha_3}\right|. \tag{81}$$

S. Pascoli, S.T.P., L. Wolfenstein, '01

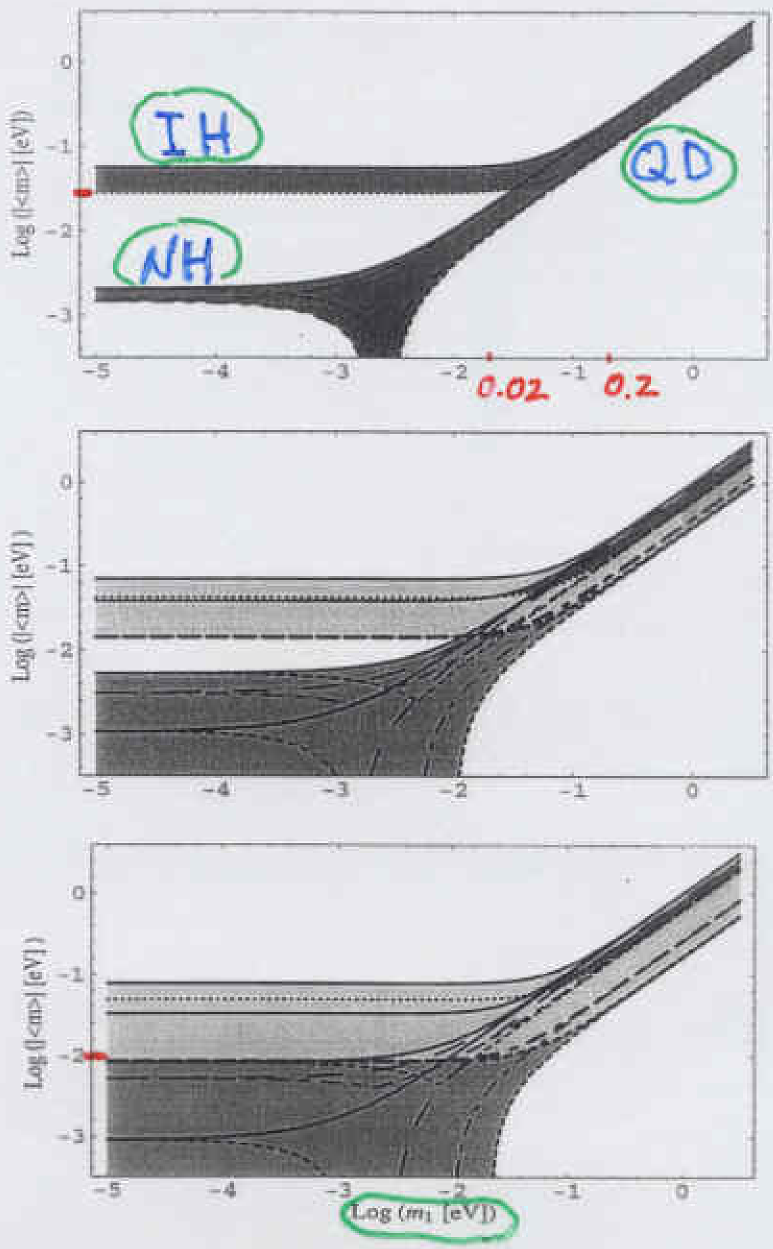
QUESTIONS:

Q1: SUPPOSE $\Delta m^2_{\odot}$, Δm^2_{atm}, $\theta_{\odot}$, θ ARE DETERMINED WITH HIGH PRECISION.
CAN ONE PREDICT THE VALUE OF $|<m>|$?

Q2: SUPPOSE $|<m>|$ IS MEASURED AND $|<m>| \neq 0$, $|<m>| \gtrsim 5\cdot 10^{-3}$ eV OR 10^{-2} eV
WHAT CAN ONE DEDUCE FROM THE MEASUREMENT (m_1, SPECTRUM, ...) ?

Q3: ARE THERE CASES WHERE A VALUE OF $|<m>|$ WOULD IMPLY CP-VIOLATION ?

PASCOLI, S.T.P.'02



B.F.V.

90% C.L.

99.73% C.L.

Figure 3: The dependence of $|<m>|$ on m_1 in the case of the LMA solution, for $\Delta m^2_{\odot} = \Delta m^2_{21}$ and $\Delta m^2_{\odot} = \Delta m^2_{32}$, and for the best fit values (upper panel) and the 90% C.L. allowed values (middle panel) of the neutrino oscillation parameters found in refs. [1, 41]. The lower panel is obtained by using the 99.73% C.L. allowed values of $\Delta m^2_{\odot}$ and $\cos 2\theta_{\odot}$ from [1] and the 99% C.L. allowed values of $\Delta m^2_{\rm atm}$ and $\sin^2\theta$ from [41] (the latter article does not include results at 99.73% C.L.). In the case of CP-conservation, the allowed values of $|<m>|$ are constrained to lie: for i) $\Delta m^2_{\odot} = \Delta m^2_{21}$ and the middle and lower panels (upper panel) - in the medium-grey and light-grey regions *a)* between the two lower thick solid lines (on the lower thick solid line) if $\eta_{21} = \eta_{31} = 1$, *b)* between the two long-dashed lines and the axes (on the long-dashed line) if $\eta_{21} = -\eta_{31} = 1$, *c)* between the two thick dash-dotted lines and the axes (on the dash-dotted lines) if $\eta_{21} = -\eta_{31} = -1$, *d)* between the three thick short-dashed lines and the axes (on the short-dashed lines) if $\eta_{21} = \eta_{31} = -1$; and for ii) $\Delta m^2_{\odot} = \Delta m^2_{32}$ and the middle and lower panels (upper panel) - in the light-grey regions *a)* between the two upper thick solid lines (on the upper thick solid line) if $\eta_{21} = \eta_{31} = \pm 1$, *b)* between the dotted and the doubly-thick short-dashed lines (on the dotted line) if $\eta_{21} = -\eta_{31} = -1$, *c)* between the dotted and the doubly-thick dash-dotted lines (on the dotted line) if $\eta_{21} = -\eta_{31} = +1$. In the case of CP-violation, the allowed regions for $|<m>|$ cover all the grey regions. Values of $|<m>|$ in the dark grey regions signal CP-violation.

V. BARGER ET AL.:

"NO-GO FOR CP-VIOLATION IN $(\beta\beta)_{0\nu}$-DECAY"

TOO PESSIMISTIC... (PASCOLI, S.T.P.)

CONSIDER, FOR EXAMPLE,

IH SPECTRUM ($m_1 < 0.02\,eV$), $|U_{e1}|^2$-NEGLIGIBLE

$$\sqrt{\Delta m^2_{atm}}\cos 2\theta_{\odot} \leq |<m>| \leq \sqrt{\Delta m^2_{atm}}$$

"JUST CP-VIOLATING" REGION:

$$(|<m>|)_{MAX} < \sqrt{(\Delta m^2_{atm})_{MIN}}$$

$$(|<m>|)_{MIN} > \sqrt{(\Delta m^2_{atm})_{MAX}}\,(\cos 2\theta_{\odot})_{MAX}$$

$$|<m>| = \xi\left[(|<m>|_{EXP})_{MIN} \pm \Delta\right]$$

OBTAINED FOR THE MAXIMAL VALUE OF THE CORRESPONDING $M_{NUCLEAR}$

$\xi \geq 1.$

NECESSARY CONDITION FOR ESTABLISHING CP-VIOLATION:

$$\xi < \frac{\sqrt{(\Delta m^2_{atm})_{MIN}}}{\sqrt{(\Delta m^2_{atm})_{MAX}}\,(\cos 2\theta_{\odot})_{MAX} + 2\Delta} \cong \frac{1}{(\cos 2\theta_{\odot})_{MAX}}$$

PASCOLI, S.T.P. '02

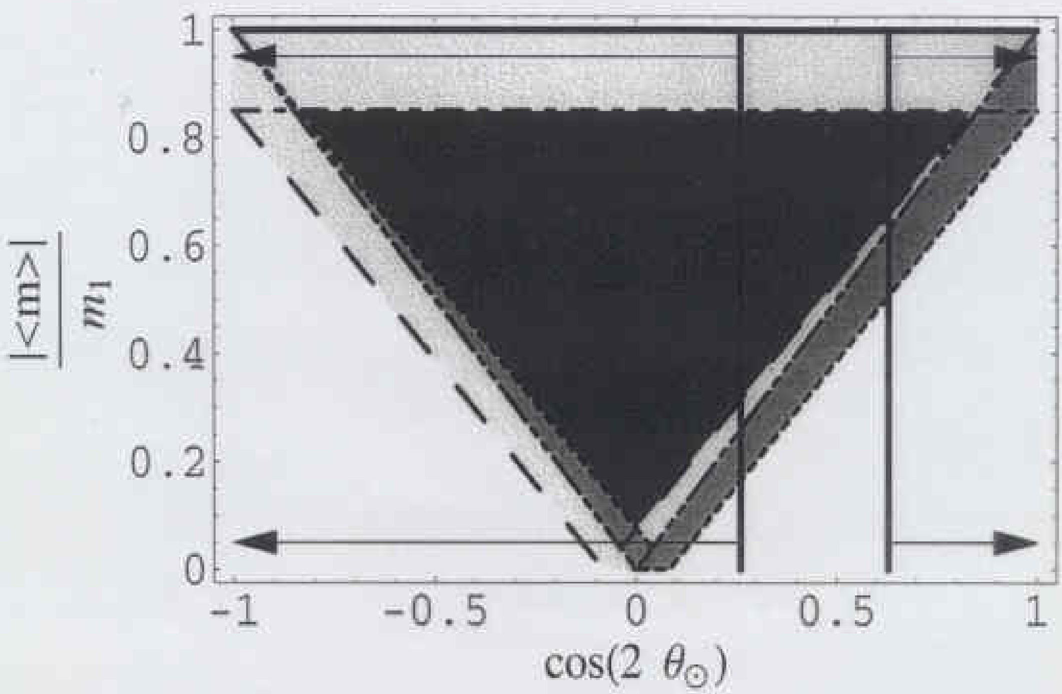


Figure 2: The dependence of $|\langle m \rangle| / m_1$ on $\cos 2\theta_\odot$ for the quasi-degenerate neutrino mass spectrum ($m_1 > 0.2$ eV, $m_1 \simeq m_2 \simeq m_3$). If CP-invariance holds, the values of $|\langle m \rangle| / m_1$ lie: i) for $\eta_{21} = \eta_{31} = 1$ - on the line $|\langle m \rangle| / m_1 = 1$, ii) for $\eta_{21} = -\eta_{31} = 1$ - in the region between the thick horizontal solid and dash-dotted lines (in light grey and medium grey colors), iii) for $\eta_{21} = -\eta_{31} = -1$ - in the light grey polygon with long-dashed and long-dashed-double-dotted line contours and iv) for $\eta_{21} = \eta_{31} = -1$ - in the medium grey polygon with the short-dashed and long-dashed-double-dotted line contours. The "just-CP-violation" region is denoted by dark-grey color. The values of $\cos 2\theta_\odot$ between the doubly thick solid lines correspond to the lower and upper limits of the LMA solution regions found in ref. [1] at 99.73% C.L.

CONCLUSIONS.

FUTURE $(\beta\beta)_{0\nu}$ - DECAY EXPERIMENTS CAN PROVIDE INFORMATION ON

- THE NEUTRINO MASS SPECTRUM
- m_1 (SMA MSW) $\Rightarrow m_1, m_2, m_3$ OR
- $m_1 = [m_1^{min}, m_1^{max}]$ (LMA, LOW-QVO)
- CP - VIOLATION IN THE LEPTON SECTOR (LMA, LOW-QVO, $\Delta m^2_{\odot} = \Delta m^2_{32}, |U_{e1}|^2 \leq 5 \cdot 10^{-2}$)

COMBINED WITH DATA FROM THE ^3H β-DECAY EXPERIMENTS (KATRIN), A POSITIVE RESULT FROM $(\beta\beta)_{0\nu}$ - DECAY EXPERIMENTS, i.e.,

$$|<m>| \gtrsim 0.02\, eV, \quad m_{\bar{\nu}_e} \geq 0.35\, eV$$

WOULD ALLOW TO DETERMINE

m_1, m_2, m_3 AND TO ANSWER THE QUESTION

WHETHER CP - SYMMETRY IS VIOLATED IN THE LEPTON SECTOR OR NOT.

VERY IMORTANT TO KNOW

$$min\,(|\cos 2\theta_{\odot}|),\ |U_{e3}|^2\ (|U_{e1}|^2)$$

$$\Delta m^2_{\odot} = \Delta m^2_{21} \text{ or } \Delta m^2_{\odot} = \Delta m^2_{32}.$$