

# Investigating Lorentz and CPT symmetry with antihydrogen

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# Outline

- Lorentz & CPT violation
  - the standard-model extension
  - signal properties
- Hydrogen and Antihydrogen
  - 1S to 2S
  - Hyperfine Zeeman
- Results in other systems
  - Neutral mesons, Atomic systems, Electrodynamics

# Lorentz and CPT symmetry

Nature appears symmetric under the transformations

**Lorentz**

Rotations  
Boosts

**CPT**

Charge conjugation  
Parity reversal  
Time reversal

CPT Theorem

Pauli, Lüders, Bell (1954)

**Premise:** local, point-particle, field theory, Lorentz invariant

**Conclusion:** CPT symmetric

Recently:

Greenberg hep-ph/0201258 (2002)

**Premise:** local quantum field theory

**Conclusion:** CPT violation implies Lorentz violation

# Approaches to testing CPT

|      | Phenomenological  | Theoretical Framework  |
|------|---|--|
| PROS | <ul style="list-style-type: none"><li>• Practical approach</li></ul>  | <ul style="list-style-type: none"><li>• Consistency with standard model</li><li>• Experimental breadth</li><li>• Predicts signal types</li></ul> |
| CONS | <ul style="list-style-type: none"><li>• Unclear theoretical basis</li><li>• Experiment dependent</li><li>• Limited predictive ability</li></ul> | <ul style="list-style-type: none"><li>• Challenging to find</li></ul>  |

**Standard-model extension:** a consistent, microscopic, general theoretical framework allowing Lorentz and CPT violation

Kostelecký, Potting, PRD **51** 3923 (1995)

Colladay, Kostelecký, PRD **55** 6760 (1997); PRD **58** 116002 (1998)

# Standard-Model Extension: Idea

Fundamental Theory

String Theory?



Spontaneous symmetry breaking?

Standard model

+ minuscule extension terms



QED, Dirac equation, ... + minuscule extension terms

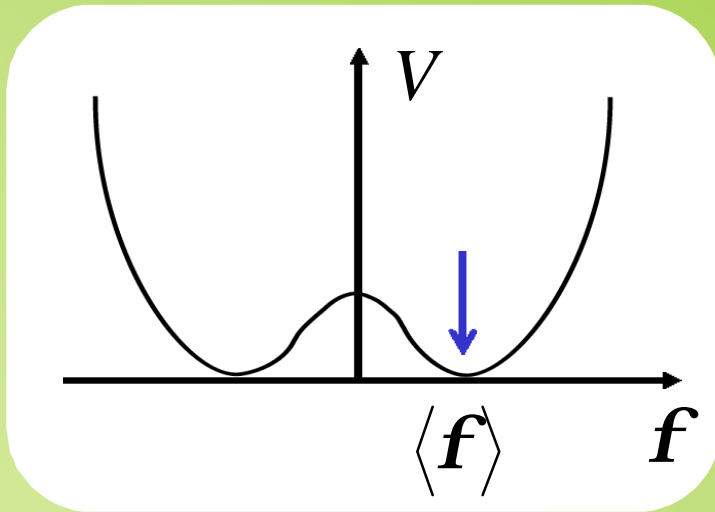
All break Lorentz symmetry;  
some also break CPT

Kostelecký, Potting, PRD **51** 3923 (1995)

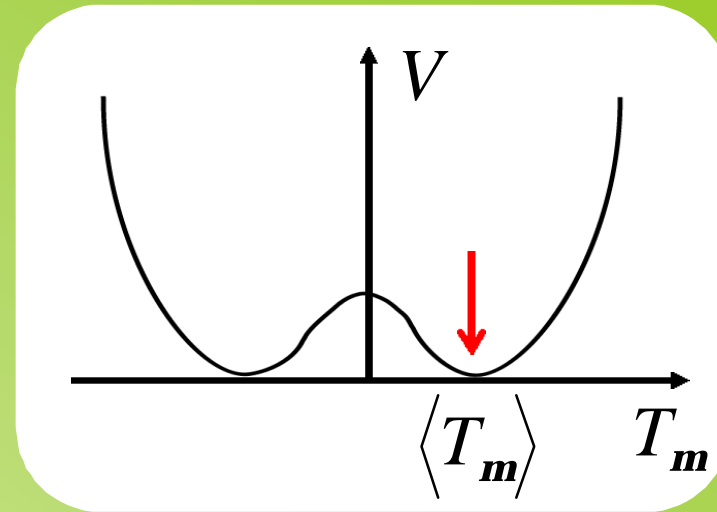
Colladay, Kostelecký, PRD **55** 6760 (1997); PRD **58** 116002 (1998)

# Lorentz violation through spontaneous symmetry breaking

$$L \supset I \bar{y} y$$

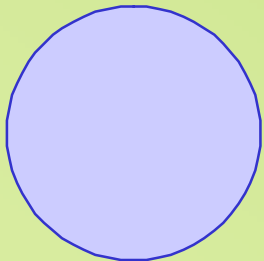


$$L \supset \hat{i} T_i \bar{\phi} \tilde{a}_5 \tilde{a}^i \phi$$



$$L \supset I \underbrace{\langle f \rangle}_m \bar{\phi} \phi$$

$$L \supset \mathbf{x} \underbrace{\langle T_m \rangle}_{b_m} \bar{y} \mathbf{g}_5 \mathbf{g}^m y$$



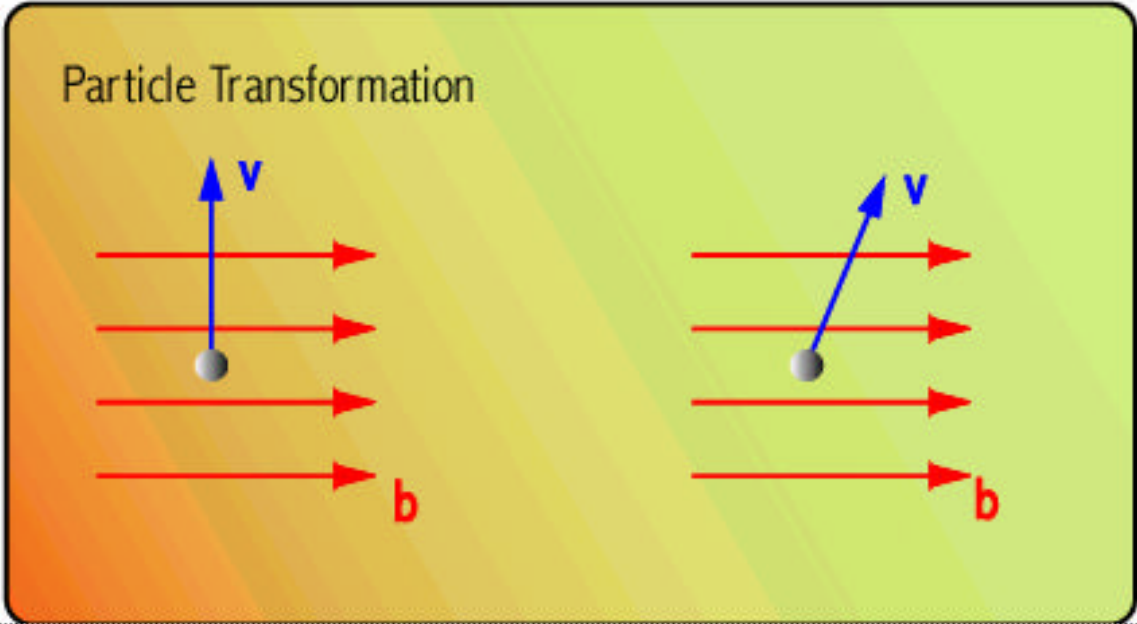
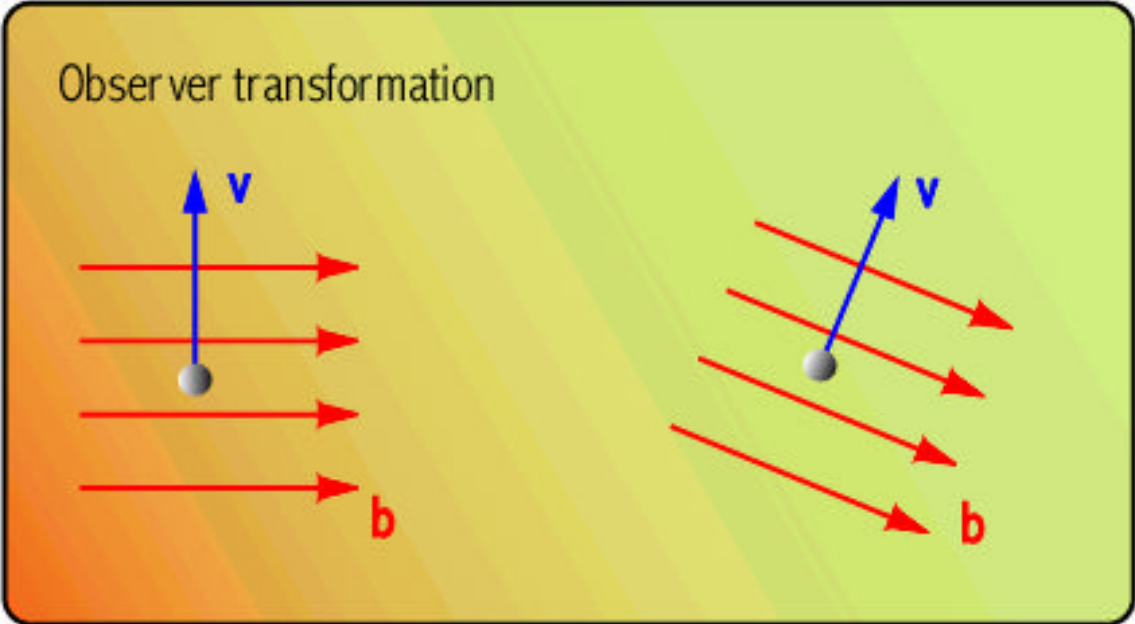
conventional



unconventional

Kostelecký, Samuel, PRD **39** 683 (1989); PRD **40** 1886 (1989)

# Lorentz Transformations



# Standard-model extension properties

| Conventional  | Unconventional                                   |
|---|--|
| Gauge structure<br>Power-counting renormalizability<br>Energy and momentum conservation<br>Quantization<br>Microcausality<br>Spin-statistics<br>Observer Lorentz covariance | Particle Lorentz non-covariance<br>CPT violation |

Kostelecký, Potting, PRD **51** 3923 (1995)

Colladay, Kostelecký, PRD **55** 6760 (1997); PRD **58** 116002 (1998)

## Special cases of standard-model extension include:

Realistic noncommutative field theories:

-- noncommutative coordinates:  $[x^\mu, x^\nu] = i\theta^{\mu\nu}$

--  $q$  plays role of prescribed background tensor in QFT

Carrol et al, PRL **87** 141601 (2001)

Isotropic high-energy limit:

-- preferred frame,

-- one parameter for each species: Coleman, Glashow PRD **59** 116008 (1999)

Others...



# Perturbed Hamiltonian

Four-component spinor  
for electron, positron,  
or (anti)proton

CPT violating

$$\left( i\gamma^\mu D_\mu - m - \underbrace{a_\mu \gamma^\mu - b_\mu \gamma_5 \gamma^\mu}_{\text{CPT violating}} - \frac{1}{2} H_{\mu\nu} \sigma^{\mu\nu} + ic_{\mu\nu} \gamma^\mu D^\nu + id_{\mu\nu} \gamma_5 \gamma^\mu D^\nu \right) \psi = 0$$

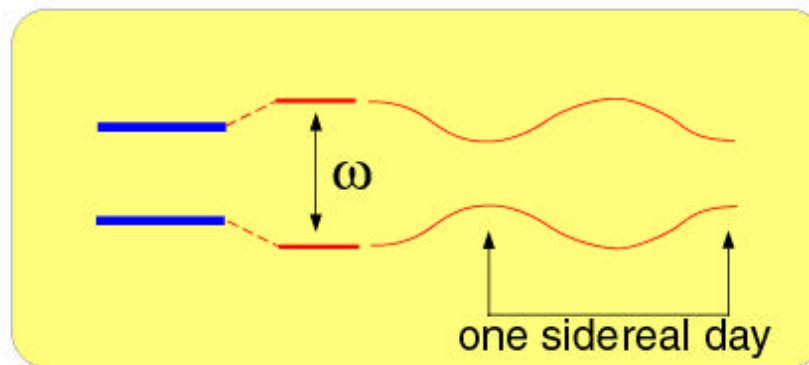
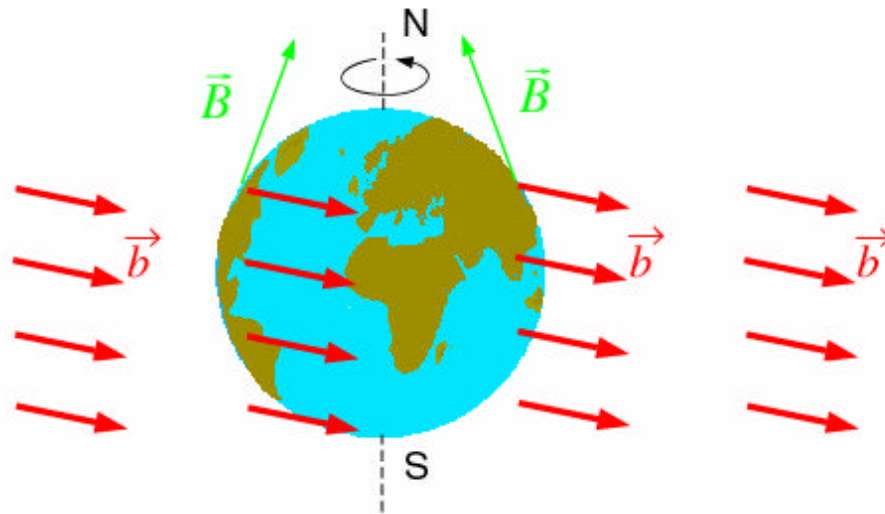
Regular Dirac  
equation

- Minuscule Lorentz-violating terms
- Different parameters for each particle sector
- Field redefinition needed for Hamiltonian

Bluhm, Kostelecký, Russell PRL **82** 2254 (1999), for example

# Sidereal variations

Kostelecký, PRL **80** 1818 (1998)



## Example: Spin-polarized Matter

Torsion-balance apparatus  
Spin-polarized test mass

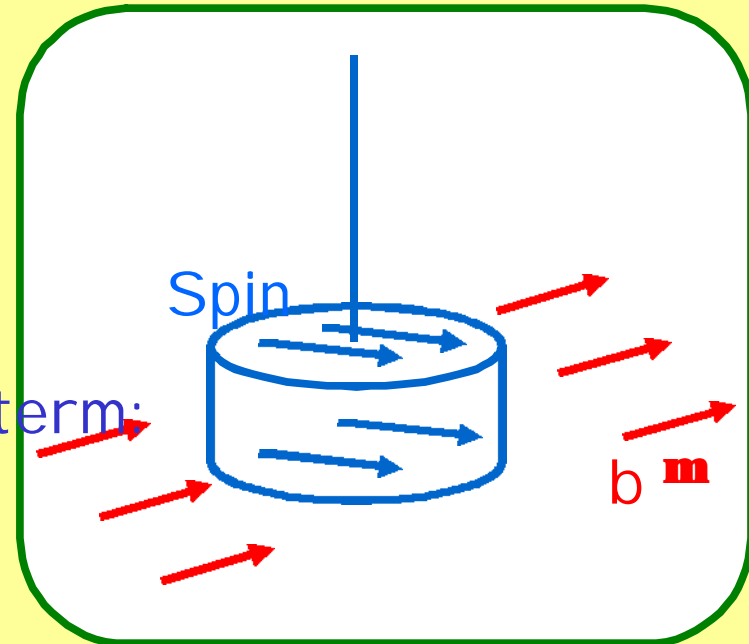
Probe standard-model extension term:

$$V = b \cdot \mathbf{S}$$

Analysis applies sidereal effects

Result:

$$\tilde{b}_J^e \lesssim 10^{-29} \text{ GeV}$$



Bluhm, Kostelecký,  
PRL **84** 1381 (2000)

Heckel, EotWash group,  
CPT '01 Proceedings

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- Hydrogen and Antihydrogen ←  
1S to 2S  
Hyperfine Zeeman
- Results in other systems  
Neutral mesons, Atomic systems, Electrodynamics

## Hydrogen and Antihydrogen

ATRAP – first talk this session, Gabrielse

ATHENA – second talk this session, Doser

cold antihydrogen:

- theoretical analysis possible for simple system
- clean bounds should be possible
- atomic clocks: no antiatoms, analysis complex, need models

## Antiprotonic atoms

ASACUSA – third talk this session, Widmann

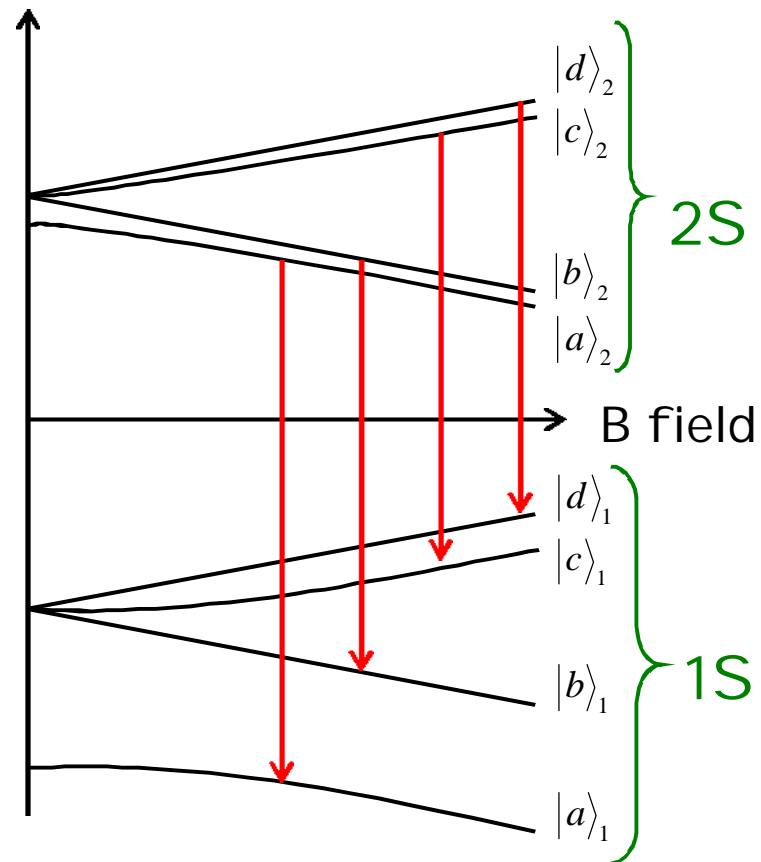
Antiprotonic He, for example:

- potential for tests involving antiproton/proton comparison

# 1S-2S transition in trapped antihydrogen and hydrogen

Allowed 1S-2S transitions

energy



Only the c and d states are trapped

$$|d\rangle_n = \left| \frac{1}{2}, \frac{1}{2} \right\rangle ,$$

$$|c\rangle_n = \sin \theta_n \left| -\frac{1}{2}, \frac{1}{2} \right\rangle + \cos \theta_n \left| \frac{1}{2}, -\frac{1}{2} \right\rangle$$

$$|b\rangle_n = \left| -\frac{1}{2}, -\frac{1}{2} \right\rangle ,$$

$$|a\rangle_n = \cos \theta_n \left| -\frac{1}{2}, \frac{1}{2} \right\rangle - \sin \theta_n \left| \frac{1}{2}, -\frac{1}{2} \right\rangle$$

$$\tan 2\theta_n \approx \frac{(51 \text{ mT})}{n^3 B}$$

What are the effects on the  $c \rightarrow c$  and  $d \rightarrow d$  transitions?

## Shifts in energy levels

Hydrogen with electron and proton angular momenta  $J$  and  $I$  :

$$\begin{aligned}\Delta E^H(m_J, m_I) \approx & (a_0^e + a_0^p - c_{00}^e m_e - c_{00}^p m_p) \\ & + (-b_3^e + d_{30}^e m_e + H_{12}^e) m_J / |m_J| \\ & + (-b_3^p + d_{30}^p m_p + H_{12}^p) m_I / |m_I|\end{aligned}$$

Antihydrogen: reverse signs of  $a$ ,  $d$  and  $H$  parameters

No zero-order effect in  $d \rightarrow d$  transition,  
since  $n=1$  and  $n=2$  states have identical spin:

$$|m_J = +1/2, m_I = +1/2\rangle$$

Bluhm, Kostelecký, Russell,  
PRL **82** 2254 (1999)

## Signal in $|c\rangle \rightarrow |c\rangle$ transition

Spin mixing is different in 1S and 2S  $\rightarrow$  unsuppressed signal

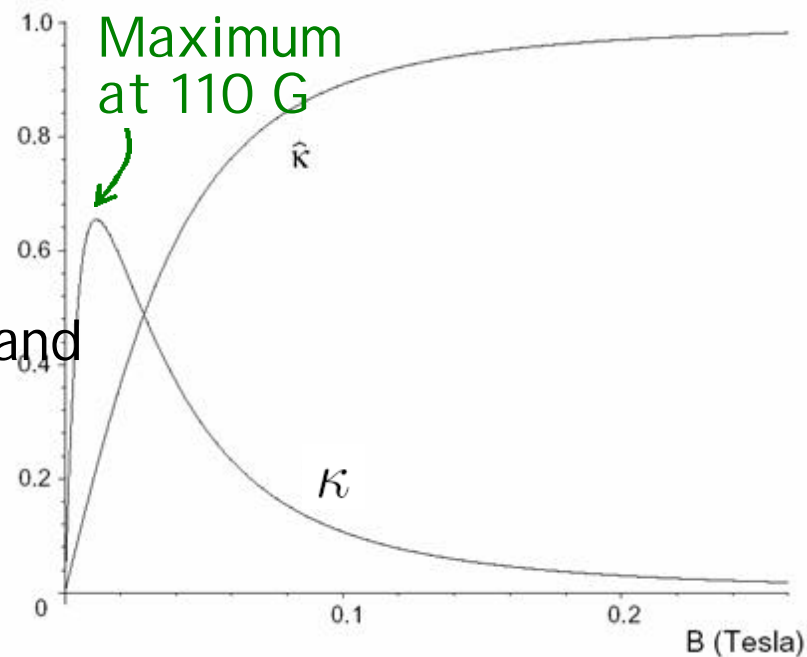
For  $c \rightarrow c$  in hydrogen:

$$\delta\nu_c^H \approx -\kappa(b_3^e - b_3^p - d_{30}^e m_e + d_{30}^p m_p - H_{12}^e + H_{12}^p)/2\pi$$

$$\kappa \equiv \cos 2\theta_2 - \cos 2\theta_1$$

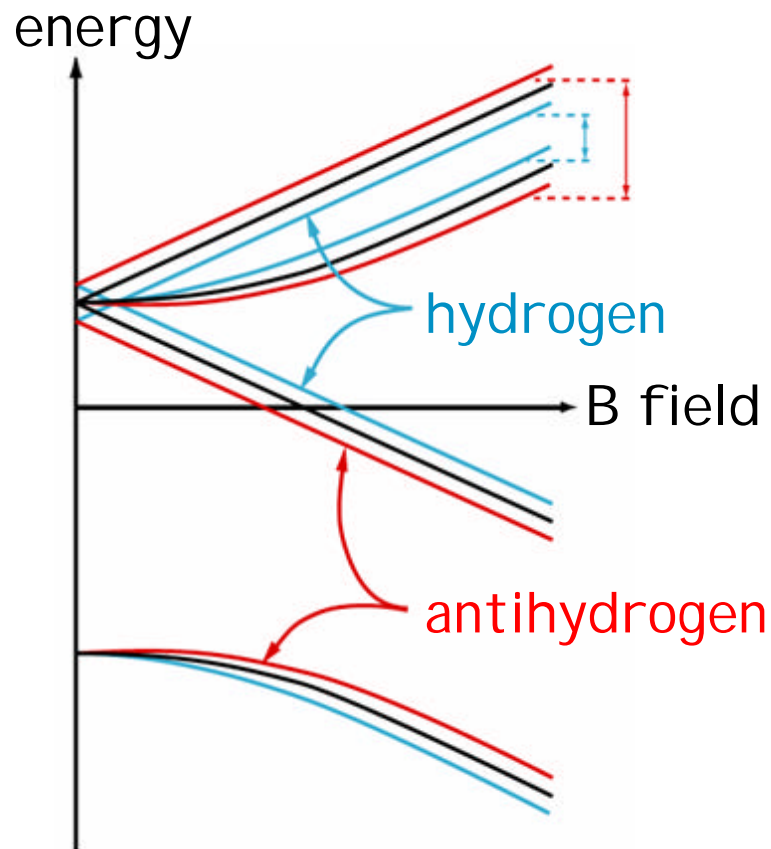
For  $c \rightarrow c$  in antihydrogen:  
reverse signs of b parameters and  
instantaneous comparison gives

$$\nu_c^H - \nu_c^{\bar{H}} \approx -\kappa(b_3^e - b_3^p)/\pi$$





# Hyperfine Zeeman transitions



Bluhm, Kostelecký, Russell,  
PRL **82** 2254 (1999)

Shifts are in opposite directions

$$\delta\nu_{c \rightarrow d}^H \approx (-b_3^p + d_{30}^p m_p + H_{12}^p)/\pi$$

$$\delta\nu_{c \rightarrow d}^{\bar{H}} \approx (b_3^p + d_{30}^p m_p + H_{12}^p)/\pi$$

Instantaneous comparison:

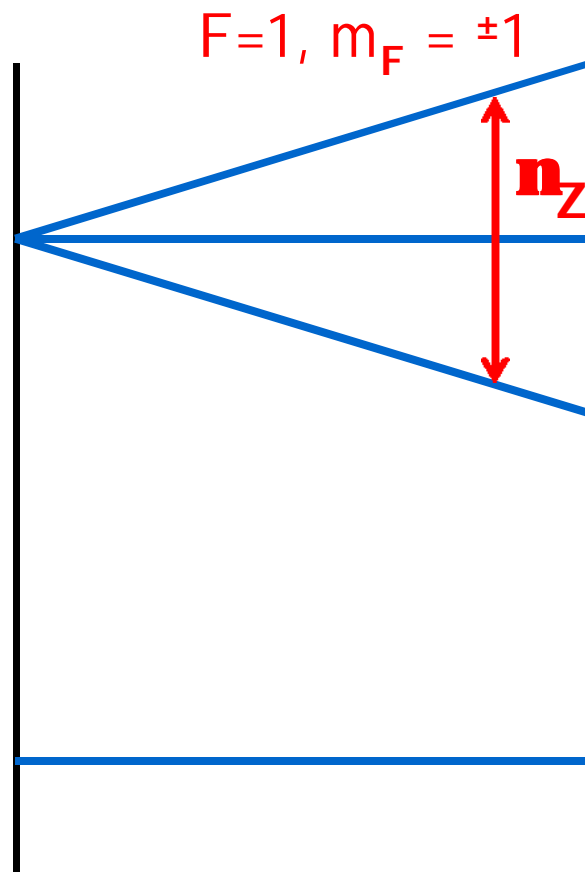
$$\nu_{c \rightarrow d}^H - \nu_{c \rightarrow d}^{\bar{H}} \approx -2b_3^p/\pi$$

With 1mHz resolution,

$$|b_3^p| \leq 10^{-27} \text{ GeV}$$

Cleaner than any competing  
experiment

# Hydrogen maser experiment



$$\left| \tilde{b}_3^p + \tilde{b}_3^e \right| \leq 2\pi\delta\nu_Z$$

$$dn_Z < 370 \text{ nHz}$$

Using electron bound,

$$\tilde{b}_J^p \leq 2 \times 10^{-27} \text{ GeV}$$

→ Best proton bound

Phillips et al, PRD **63** 111101 R (2001)

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# Neutral Mesons

Relevant mesons:  $K$ ,  $D$ ,  $B_d$ ,  $B_s$  and antiparticles

Time evolution via  $2 \times 2$  effective hamiltonian  $\Lambda : i\partial_t \Psi = \Lambda \Psi$

CPT symmetry  $\Leftrightarrow \mathbf{x} = 0$  where  $\mathbf{x} \approx \Lambda_{11} - \Lambda_{22}$

Standard - model extension:

$$\mathbf{x} \approx \mathbf{b}^m \Delta a_m$$

Decay probabilities have  
velocity  $\hat{a}$  dependence  
sidereal dependence

Assumption of constant  $\hat{a}$  is incompatible with QFT

Kostelecký, PRL **80** 1818 (1998);  
PRD **61** 016002 (1999);  
PRD **64** 076001 (2001)

# Neutral Mesons: status of bounds on $a_{\mu}$

| Coefficient              | K system              | D system                 | $B_d$ system | $B_s$ system |
|--------------------------|-----------------------|--------------------------|--------------|--------------|
| $\tilde{a} / \text{GeV}$ | $\approx 10^{-20}$    | $\approx 10^{-15}$       | ?            | ?            |
| $a_{\perp} / \text{GeV}$ | $\approx 10^{-21}$    | $\approx 10^{-15}$       | ?            | ?            |
| Experiments              | E773 reanalysis, KTeV | E831 (FOCUS) preliminary |              |              |

CPT '01 Proceedings:  
Nguyen, KTeV Collab

CPT '01 Proceedings:  
Gardner, FOCUS Collab

Existing/future  $B_d$  data  
could yield bounds (BaBar,  
BELLE, BTeV...)

## Clock-comparison bounds

$^9\text{Be}^+$  and H maser  
 $^{201}\text{Hg}$  and  $^{199}\text{Hg}$   
 $^{21}\text{Ne}$  and  $^3\text{He}$   
 $^{199}\text{Hg}$  and  $^{133}\text{Cs}$   
 $^3\text{He}$  and  $^{129}\text{Xe}$

Sidereal, GeV

Kostelecký, Lane  
 PRD **60** 116010 (1999)

|   | tilde coefficient | Prestage<br><i>et al.</i> | Lamoreaux<br><i>et al.</i> | Chupp<br><i>et al.</i> | Berglund<br><i>et al.</i> | Bear<br><i>et al.</i> |
|---|-------------------|---------------------------|----------------------------|------------------------|---------------------------|-----------------------|
| p | $b_J$             | *                         | *                          | —                      | -27                       | *                     |
| p | $d_J$             | *                         | *                          | —                      | -25                       | *                     |
| p | $g_{D,J}$         | *                         | *                          | —                      | -25                       | *                     |
| p | $c_{Q,J}$         | *                         | —                          | —                      | —                         | —                     |
| p | $g_{Q,J}$         | *                         | —                          | —                      | —                         | —                     |
| p | $c_-$             | *                         | *                          | *                      | —                         | —                     |
| p | $g_-$             | *                         | *                          | *                      | —                         | —                     |
| p | $c_{XY}$          | *                         | *                          | *                      | —                         | —                     |
| p | $g_{XY}$          | *                         | *                          | *                      | —                         | —                     |
| n | $b_J$             | -27                       | -29                        | —                      | -30                       | -31                   |
| n | $d_J$             | -25                       | -26                        | —                      | -28                       | -29                   |
| n | $g_{D,J}$         | -25                       | -27                        | —                      | -28                       | -29                   |
| n | $c_{Q,J}$         | -25                       | —                          | —                      | —                         | —                     |
| n | $g_{Q,J}$         | *                         | —                          | —                      | —                         | —                     |
| n | $c_-$             | -25                       | -27                        | -27                    | —                         | —                     |
| n | $g_-$             | *                         | *                          | *                      | —                         | —                     |
| n | $c_{XY}$          | -25                       | -27                        | -27                    | —                         | —                     |
| n | $g_{XY}$          | *                         | *                          | *                      | —                         | —                     |
| e | $b_J$             | —                         | —                          | —                      | -27                       | —                     |
| e | $d_J$             | —                         | —                          | —                      | -22                       | —                     |
| e | $g_{D,J}$         | —                         | —                          | —                      | -22                       | —                     |
| e | $c_{Q,J}$         | —                         | —                          | —                      | —                         | —                     |
| e | $g_{Q,J}$         | —                         | —                          | —                      | —                         | —                     |
| e | $c_-$             | —                         | —                          | —                      | —                         | —                     |
| e | $g_-$             | —                         | —                          | —                      | —                         | —                     |
| e | $c_{XY}$          | —                         | —                          | —                      | —                         | —                     |
| e | $g_{XY}$          | —                         | —                          | —                      | —                         | —                     |

## $^{129}\text{Xe}/^3\text{He}$ Maser

Best test for neutron parameters in standard-model extension

$$\tilde{b}_{\perp}^n \equiv \sqrt{(\tilde{b}_X^n)^2 + (\tilde{b}_Y^n)^2} = (4.0 \pm 3.3) \times 10^{-31} \text{ GeV}$$

Bear et al, PRL **85** 5038 (2000)

# Space-based clock-comparison tests

ACES, PARCS, RACE, SUMO:

Cs, Rb, and H masers planned for International Space Station;

Also Superconducting microwave oscillator experiment

Advantages of satellite platform

Orbit inclination and precession gives access to many more coefficient

About 50 to 60 coefficients for each experiment

Greater speeds and rotation rates accessible in space

Boost factor  $v = 10^{-4} c$

Earth period 24 hours; Space station 92 minutes

Possible technological advantages

Bluhm, Kostelecký, Lane, Russell, PRL **88** 090801 (2002)



# Penning-trap bounds

Results in context of standard-model extension

| Type                              | Particles                | Result     | Reference                                  |
|-----------------------------------|--------------------------|------------|--|
| Instantaneous anomaly frequency   | Electron, positron       | $10^{-21}$ | Dehmelt et al, PRL <b>83</b> 4694 (1999)   |
| Instantaneous cyclotron frequency | Hydrogen ion, antiproton | $10^{-26}$ | Gabrielse et al, PRL <b>82</b> 3198 (1999) |
| Sidereal anomaly frequency        | Electron                 | $10^{-21}$ | Mittleman et al, PRL <b>83</b> 2116 (1999) |

Bluhm, Kostelecký, Russell, PRL **79** 1432 (1997);  
PRD **57** 3932 (1998)

# Muons

Muonium spectroscopy of hyperfine Zeeman levels

$$\sqrt{(\tilde{b}_X^\mu)^2 + (\tilde{b}_Y^\mu)^2} \leq 2 \times 10^{-23} \text{ GeV}$$

## CERN and BNL g-2 Experiments

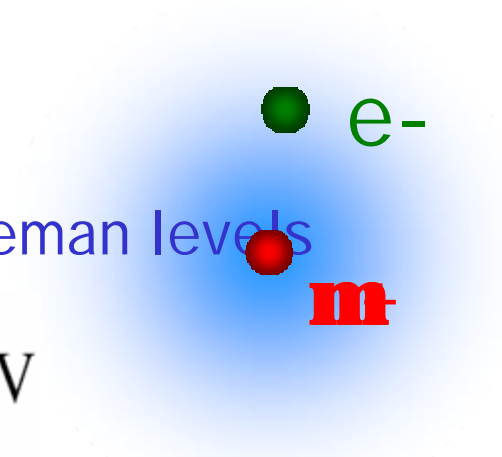
Anomaly-frequency comparisons

$$b_Z^{\mathbf{m}} < 10^{-23} \text{ GeV}$$

Sidereal anomaly-frequency variations (estimated)

$$b_J^{\mathbf{m}} < 10^{-24} \text{ GeV}$$

Bluhm, Kostelecký, Lane, PRL **84** 1098 (2000);  
Hughes et al, PRL **87** 111804 (2001);  
Deile et al, CPT '01 Proceedings



# Electrodynamics

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}(k_F)_{\kappa\lambda\mu\nu}F^{\kappa\lambda}F^{\mu\nu}$$

Standard-model extension predicts birefringence  
→ Sensitive tests using astrophysical sources

Bounds from pulse spreading:

$$|k^a| < 3 \times 10^{-16}$$

Bounds from differential polarization ro

$$|k^a| < 2 \times 10^{-32}$$

*"Best test of  
Special Relativity"*

Kostelecký, Mewes, PRL **87** 251304 (2001);  
hep-ph/0205211

# Summary

**Standard-model extension: a viable theoretical framework that allows Lorentz and CPT violation**

## **Trapped Antihydrogen**

**Unsuppressed signal occurs in the 1S-2S transition**

**Another occurs in the hyperfine Zeeman transition**

**Comparisons of hydrogen and antihydrogen would produce clean bound on an untested parameter combination**

**Bounds exist in other systems —**

**neutral mesons, atomic systems, electrodynamics**