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Electroweak baryon number violation

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1. INTRODUCTION

Conditions for baryogenesis [Sakharov, 1967]:

1. C and CP violation	Yes
2. Thermal nonequilibrium	Yes
3. Baryon number (B) violation	?

Strictly speaking, we know of only one physical theory that is expected to have B violation:

the electroweak Standard Model (EWSM).

[Side remark: the *ultimate* fate of black holes is uncertain.]

But the relevant physical processes of the EWSM are only known at

$$T \ll M_W \approx 10^2 \text{ GeV}$$

and their rate is negligible,

$$\Gamma \propto \exp[\,-4\,\pi\,\sin^2\theta_w\,/\,\alpha\,] \approx 0\ .$$

Clearly, we should study electroweak baryon number violation for the conditions of the early universe,

$$T \gtrsim 10^2 \text{ GeV} .$$

This is a difficult problem, but entirely well-posed.

In this talk, we focus on the fundamental physics, i.e., the microscopic processes.

That is, we must really deal with the fermions.

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2. OLD RESULTS

Consider $SU(2)$ Yang–Mills–Higgs theory with vanishing Yukawa couplings. Actually, forget about the Higgs, which should be reasonable above the EW phase transition.

Triangle anomaly in the AAA-diagram, provided the VVV-diagram is anomaly-free [ABJ69].

[Side remark: this is Feynman *perturbation* theory.]

The gauge vertices of the EWSM are V–A and must be nonanomalous (gauge invariance is needed for unitarity).

Instead, the $B+L$ current is anomalous [H76]:

$$\begin{aligned} \Delta(B-L) &= 0 \, , \\ \underbrace{\Delta(B+L)}_{\text{fermion charges}} &= \underbrace{2\, N_{\mathsf{fam}}}_{\text{constant}} \times \underbrace{\Delta N_{\mathsf{CS}}}_{\text{gauge field characteristic}} \; . \end{aligned}$$

In the $A_0 = 0$ gauge, have Chern–Simons number

$$N_{\mathsf{CS}}(t) = N_{\mathsf{CS}}[\vec{A}(\vec{x},t)]$$

and

$$\Delta N_{\mathsf{CS}} \equiv N_{\mathsf{CS}}(t_{\mathsf{out}}) - N_{\mathsf{CS}}(t_{\mathsf{in}}) \; .$$

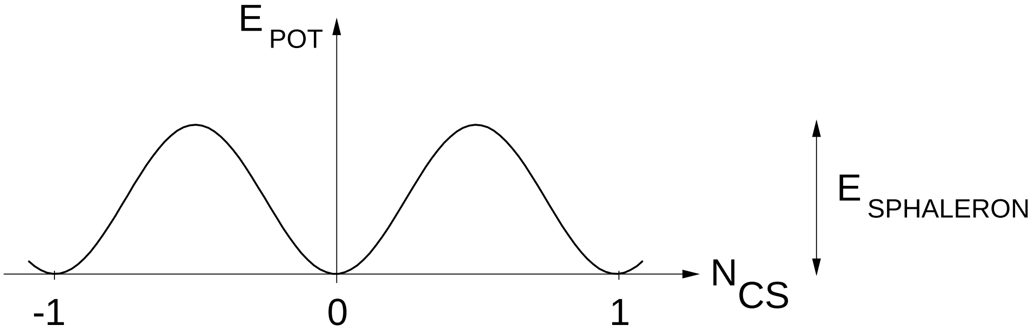

Figure 1: Potential energy surface over configuration space.

’t Hooft (1976) calculated the <u>tunneling</u> amplitude.

The BPST instanton, which is a finite action solution of the <u>imaginary-time</u> theory (Euclidean spacetime), gives

$$\Delta N_{\mathsf{CS}} = Q[A_{\mathsf{finite\ action}}] \in \mathbb{Z} \ ,$$

where the topological charge Q is the winding number of the map

$$S^3\Big|_{|x|=\infty} \to SU(2) \sim S^3 \ .$$

This holds <u>only</u> for transitions from near-vacuum to near-vacuum, i.e., at very low temperatures or energies.

As mentioned above, the rate is then effectively zero, but, at least, $\Delta(B+L)$ is <u>integer</u>.

3. BIG QUESTION

For real-time processes (e.g., in Minkowski spacetime), the topological charge Q is, in general, noninteger.

Hence, the question

$$\Delta(B+L) \propto \text{which gauge field characteristic}\,??$$

In the following, we consider pure $SU(2)$ Yang–Mills theory with a single isodoublet of left-handed fermions.

(The fermion number $B+L$ of the EWSM follows by multiplying with $2\,N_{\mathsf{fam}}$.)

Also, the gauge fields will be called dissipative if their energy density approaches zero uniformly as $t \to \pm\,\infty$.

4. NEW RESULTS

Start from the eigenvalue equation of the time-dependent Dirac Hamiltonian:

$$H(\vec{x},t)\,\Psi(\vec{x},t) = E(t)\,\Psi(\vec{x},t)\ .$$

Then, fermion number violation is related to the spectral flow $\mathcal{F}$.

Definition: $\mathcal{F}[\,t_f, t_i\,]$ is the number of eigenvalues of the Dirac Hamiltonian that cross zero from below minus the number of eigenvalues that cross zero from above, for the time interval $[\,t_i, t_f\,]$ with $t_i < t_f$.

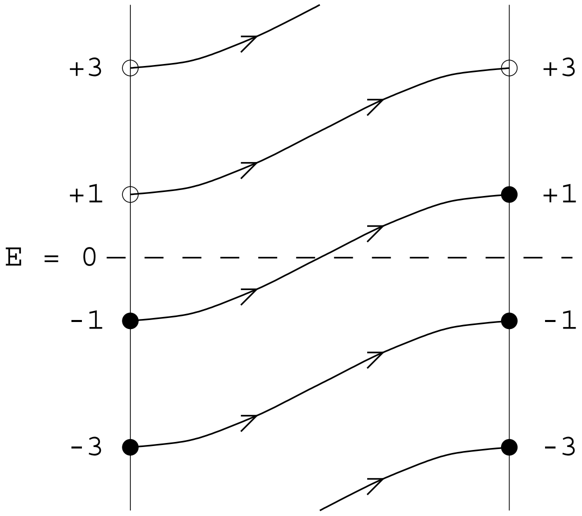

Figure 2: Spectral flow with $\mathcal{F}[\,t_f, t_i\,] = +1$.

Strongly dissipative gauge fields have [C80,GH95,K95]:

$$\mathcal{F} \;=\; \Delta N_{\mathsf{CS}}[A_{\text{associated vacuum}}] \equiv \Delta N_{\text{winding}} \;.$$

Now consider *spherically symmetric* gauge field solutions due to Lüscher and Schechter (1977). Three cases:

1. (low energy) $\Delta N_{\text{winding}} = 0$ and $\mathcal{F} = 0\,,$
2. (moderate energy) $\Delta N_{\text{winding}} = 1$ and $\mathcal{F} = 1\,,$
3. (high energy) $\Delta N_{\text{winding}} = 1$ and $\mathcal{F} = -1\,.$

$$\Rightarrow\; [\,\mathcal{F} \neq \Delta N_{\text{winding}}\,]_{\text{spherically symmetric fields}} \;.$$

In fact, there is another gauge field characteristic [KL01]:

$\Delta N_{\text{twist}} = 0$ for case 1 and 2 ,

$\Delta N_{\text{twist}} = -2$ for case 3 .

$$\Rightarrow\; [\,\mathcal{F} = \Delta N_{\text{winding}} + \Delta N_{\text{twist}}\,]_{\text{spherically symmetric fields}} \;.$$

For weakly dissipative gauge fields, one has thus

$$\Delta(B+L) =$$
$$2\,N_{\text{fam}} \times \Big(\Delta N_{\mathsf{CS}}\,[A_{\text{associated vacuum}}] + \underline{\text{extra terms}}\Big) \;.$$

But the “extra terms” are not known in general.

5. OUTLOOK

We know of only one physical theory with baryon number violation, the electroweak Standard Model.

Most discussions of electroweak baryogenesis have been based on the ’t Hooft selection rule $\Delta(B+L) \propto \Delta N_{\text{CS}}$.

But this relation has been found to be invalid for gauge field backgrounds that are weakly- (or non-)dissipative.

Such fields are, of course, relevant to the physics of the early universe.

At this moment, we have only a partial result for the correct selection rule, namely for spherically symmetric fields.

To generalize this result to arbitrary gauge fields will be difficult, but is absolutely necessary for a serious discussion of electroweak baryon number violation in the early universe.