Theory of Rare Kaon Decays

Eduardo de Rafael
CPT, CNRS–Luminy, Marseille

- BLOIS Conference on CP Violation, May 1989

  Chiral Perturbation Theory (χPT)
  is the appropriate framework to describe Kaon Physics
  Blois'89 $\Rightarrow K_L \to \pi^0\gamma\gamma$ (χPT–spectrum *before exp.*)

- From BLOIS'89 to BLOIS'02

  – χPT relates many Rare Kaon Decays
  – New Phenomenological Analyses
  – HOWEVER there are coupling constants of the higher order effective chiral Lagrangian which remain unknown. They limit, at present, the predictive power.
  – *Predictions* rely on MODELS (uncleanly related to QCD)

- Future after BLOIS'02 ?

  Large–N_c QCD is the appropriate framework to make progress in the field of Kaon Physics

Talk at the Blois Conference 2002

Theoretical Framework for K–Physics

$SU(3)_C \times SU(2)_L \times U(1)_W$

Quarks–Gluons; Leptons–W-Z-γ, H(?)

$\Downarrow$ *OPE – Renorm. Group*

$$\mathcal{L}_{\mathrm{QCD}} + \frac{G_F}{\sqrt{2}} V_{\mathrm{CKM}} \sum_i C_i(\mu) Q_i + \sum_l F_l\left(\frac{m_q^2}{M_W^2}\right)(\bar{s}_L \gamma^\mu d_L)\bar{l}_L \gamma_\mu l_L + \mathrm{h.c.}$$

u, d, s – Gluons; Leptons; γ

Four–Quark Operators $\langle Q_i \rangle$ — $(\bar{s}_L \gamma^\mu d_L)$ Currents of QCD

$\Downarrow$? — $\Downarrow$

Large–N_c QCD

$\Downarrow$ — $\Downarrow$

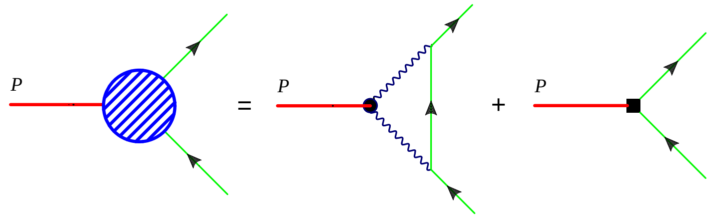
Effective χPT Field Theory of Goldstone Modes

π, K, η; Leptons; γ

$K_S \to \gamma\gamma$, $K_L \to \pi^0\gamma\gamma$ — $K^+ \to \pi^+\nu\bar{\nu}$, $K_L \to \pi^0\nu\bar{\nu}$

$K_L \to \mu^+\mu^-$, $K_L \to \pi^0 l^+ l^-$

$\pi^0 \to e^+e^-$ and $\eta \to \mu^+\mu^-$



- The Chiral Loop is Log Divergent *Savage–Luke–Wise '92*

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm WZ} + i\frac{3}{32}\left(\frac{\alpha}{\pi}\right)^2 \underbrace{\left\{\chi_1 {\rm tr}\left[U^\dagger Q_R D_\mu U Q_L - \cdots\right] + \chi_2 {\rm tr}[\cdots]\right\}}_{\frac{\chi}{4f_\pi}\partial_\mu\left(\pi^0 + \frac{1}{\sqrt{3}}\eta_8\right)\quad \chi = -\frac{1}{4}(\chi_1+\chi_2)} \bar{l}\gamma^\mu\gamma_5 l$$

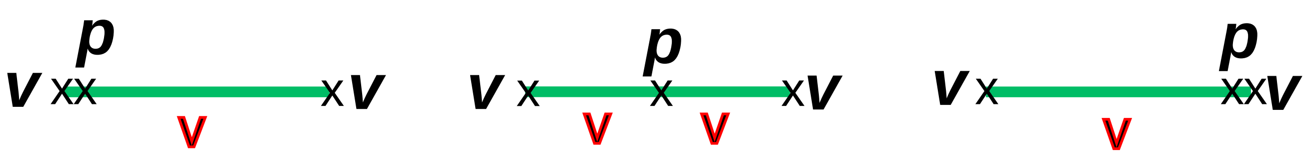
- Phenomenology *Gómez Dumm–Pich '98*

$$\frac{\Gamma\left(P\to\bar{l}l\right)}{\Gamma\left(P\to\gamma\gamma\right)} = 2\beta\left(\frac{\alpha}{\pi}\right)^2\left(\frac{m_l^2}{M_P^2}\right)\left|\mathcal{A}(P\to\bar{l}l)\right|^2 ; \quad {\rm Im}\mathcal{A} = \frac{\pi}{2\beta}\log\left(\frac{1-\beta}{1+\beta}\right)$$

$${\rm Re}\mathcal{A} = f(\beta) + \frac{N_c}{3}\left[\frac{3}{2}\log\left(\frac{m_l^2}{\mu^2}\right) - \frac{5}{2}\right] + \chi(\mu)$$

- Calculation of $\chi(\mu)$ *Knecht–Peris–Perrottet–de Rafael '99*

  – The Relevant Green's Function is VPV

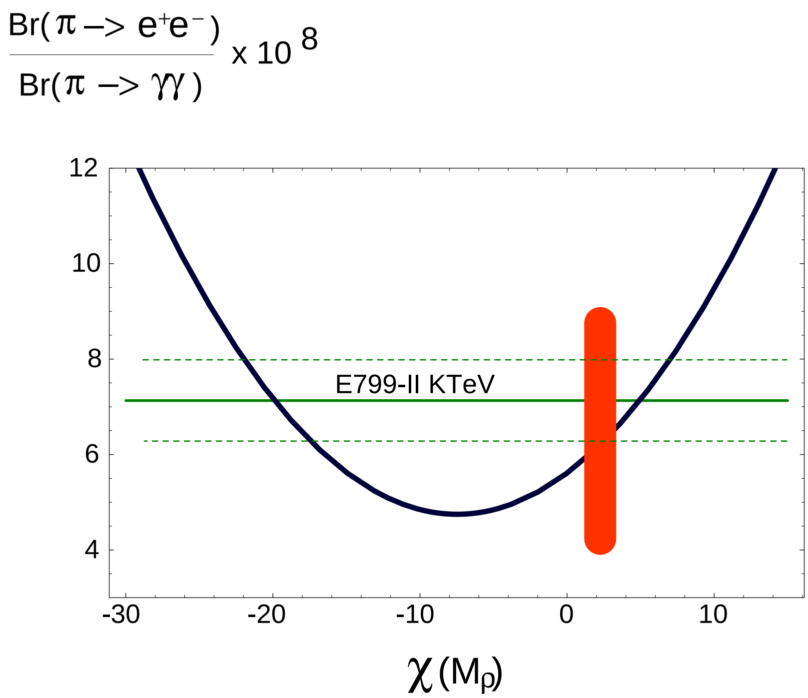


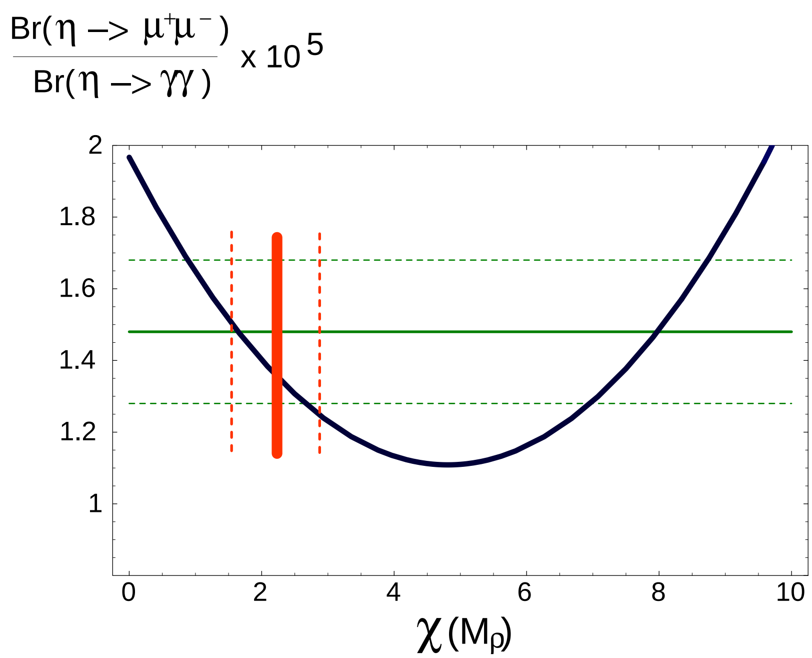
$$\chi(\mu) = \frac{8\pi^2(4\pi\mu^2)^{\epsilon/2}}{\Gamma(2-\epsilon/2)}\left(1-\frac{1}{6}\epsilon/2\right)\int_0^\infty \frac{dQ^2}{(Q^2)^{1+\epsilon/2}} W_{VPV}(Q^2)$$

  – The *Minimal Hadronic Approximation* here is a V–pole and a V–double pole

$$\chi(\mu) = -\frac{N_c}{2}\left[\log\left(\frac{M_V^2}{\mu^2}\right) - \frac{11}{12}\right] - \frac{4\pi^2 f_\pi^2}{M_V^2}$$

Branching Ratios Predictions





Lepton Flavour Violating Modes

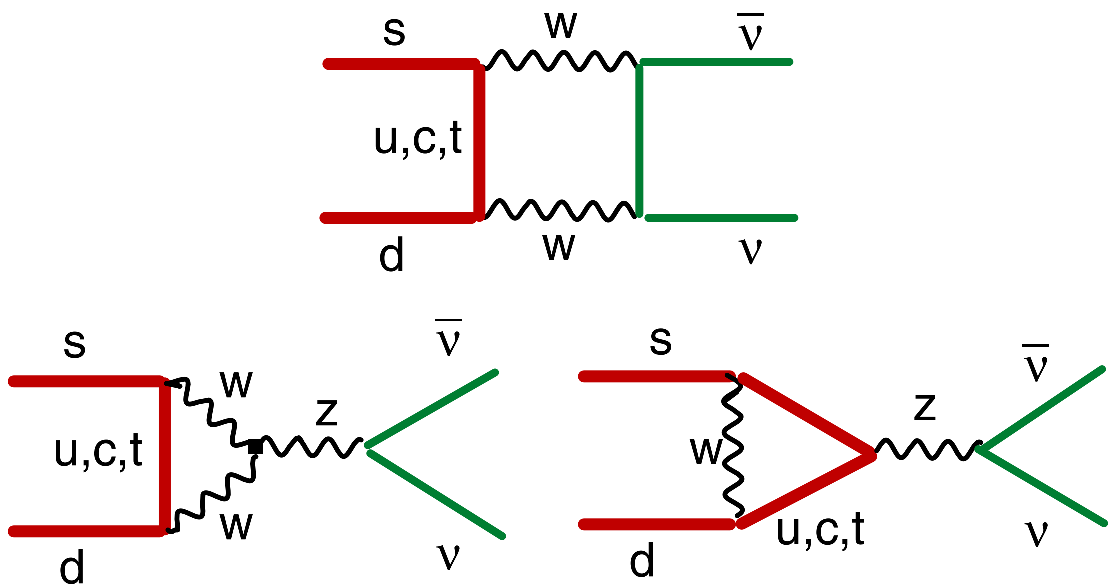
Evidence for any of them ⇒ Physics beyond the SM

Table 1: *Littenberg's* compilation

Decay	Branching Ratio	Experiment
$K_L \to \mu e$	$< 4.7 \times 10^{-12}$	E871
$K^+ \to \pi^+ \mu^+ e^-$	$< 2.8 \times 10^{-11}$	E865
$K_L \to \pi^0 \mu^\pm e^\mp$	$< 4.4 \times 10^{-10}$	KTeV
$K^+ \to \pi^- \mu^+ \mu^+$	$< 3 \times 10^{-9}$	E865
$K^+ \to \pi^- e^+ e^+$	$< 6.4 \times 10^{-10}$	E865
$K^+ \to \pi^+ \mu^- e^+$	$< 5.2 \times 10^{-10}$	E865
$K^+ \to \pi^- \mu^+ e^+$	$< 5.0 \times 10^{-10}$	E865
$K_L \to e^\pm e^\pm \mu^\mp \mu^\mp$	$< 6.1 \times 10^{-9}$	E799-I

$K^+ \to \pi^+ \bar{\nu}\nu$ and $K_L \to \pi^0 \bar{\nu}\nu$

- Short–Distance Feynman Graphs



- Branching Ratios

$$\mathrm{Br}(K^+ \to \pi^+ \bar{\nu}\nu)|_{\text{BNL–E787}} = \left(1.57^{+1.75}_{-0.82}\right) \times 10^{-10}$$

$$\mathrm{Br}(K^+ \to \pi^+ \bar{\nu}\nu)|_{\text{Buchalla-Buras '99}} = (0.72 \pm 0.21) \times 10^{-10}$$

$$\mathrm{Br}(K_L \to \pi^0 \bar{\nu}\nu)|_{\text{KTeV}} < 5.9 \times 10^{-7} \quad \left(< 1.6 \times 10^{-9}\right)$$

Grossman-Nir '97; Isidori-D'Ambrosio '01

$$\mathrm{Br}(K_L \to \pi^0 \bar{\nu}\nu)|_{\text{SM}} = (2.6 \pm 1.2) \times 10^{-11}$$

$K^+ \to \pi^+ \bar\nu\nu$ and $K_L \to \pi^0 \bar\nu\nu$

$$\left(\lambda_c = V_{cs}^* V_{cd}\,, \quad \lambda_t = V_{ts}^* V_{td}\,, \quad x_i = \frac{m_i^2}{M_W^2} \quad i = u, c, t\right)$$

- GIM Structure

$$\sum_{i=u,c,t} \lambda_i F(x_i) = \underbrace{\lambda_c}_{10^{-1}} [F(x_c) - F(x_u)] + \underbrace{\lambda_t}_{10^{-4}} [F(x_t) - F(x_u)]$$

- DYNAMICAL Scales

$$F(x_u) \sim \frac{\chi^2_{\rm QCD}}{M_W^2} \sim 10^{-5} \ll F(x_c) \sim x_c \log \frac{1}{x_c} \sim 10^{-3} \ll F(x_t) \sim 1$$

- Physical Amplitudes

$$\begin{aligned} A(K^+ \to \pi^+ \bar\nu\nu) \quad &\sim \quad \lambda_c F_c + \lambda_t F_t \\ A(K_L \to \pi^0 \bar\nu\nu) \quad &\sim \quad \underbrace{{\rm Im}\lambda_c}_{10^{-4}} \underbrace{F_c}_{10^{-3}} + \underbrace{{\rm Im}\lambda_t}_{10^{-4}} F_t \end{aligned}$$

- Branching Ratios

$$\mathrm{Br}(K^+ \to \pi^+ \bar\nu\nu)|_{\rm BNL\text{-}E787} = \left(1.57\,^{+1.75}_{-0.82}\right) \times 10^{-10}$$

$$\mathrm{Br}(K^+ \to \pi^+ \bar\nu\nu)|_{\rm Buchalla\text{-}Buras\ '99} = (0.72 \pm 0.21) \times 10^{-10}$$

$$\mathrm{Br}(K_L \to \pi^0 \bar\nu\nu)|_{\rm KTeV} < 5.9 \times 10^{-7} \quad \left(< 1.6 \times 10^{-9}\right)$$

Grossman-Nir '97; Isidori-D'Ambrosio '01

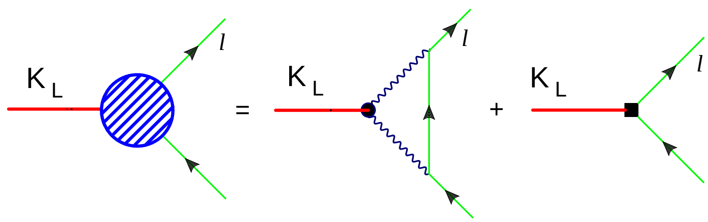
$K_L \to \mu^+\mu^-$

- AGS E871

  Br$(K_L \to \mu^+\mu^-) = (7.18 \pm 0.17) \times 10^{-9}$

  Br$(K_L \to e^+e^-) = \left(8.7 \begin{array}{c} +5.7 \\ -4.1 \end{array}\right) \times 10^{-12}$ ♣

- Phenomenology *Gómez Dumm–Pich '98*



$$\mathcal{R}_{\bar{l}l} \equiv \frac{\Gamma\left(K_L \to \bar{l}l\right)}{\Gamma\left(K_L \to \gamma\gamma\right)} = 2\beta \left(\frac{\alpha}{\pi}\right)^2 \left(\frac{m_l^2}{m_K^2}\right) \left|\mathcal{A}(K_L \to \bar{l}l)\right|^2 \; ; \quad \mathrm{Im}\mathcal{A} = \frac{\pi}{2\beta} \log\left(\frac{1-\beta}{1+\beta}\right)$$

  – Useful *Parameterization* of the *Long Distance* Contributions

$$\mathcal{H}_{\rm eff} = \mathcal{C}\left\{\frac{\alpha N_c}{12\pi f_\pi}\epsilon_{\mu\nu\alpha\beta}F^{\mu\nu}A^\alpha(\partial^\beta K_L) + \left(\frac{\alpha}{\pi}\right)^2 \frac{\chi_{\,\rm LD}(\mu)}{4 f_\pi}(\partial_\mu K_L)\,\bar{l}\gamma^\mu\gamma_5 l\right\}$$

  – Then, *Long Distance* Real Amplitude

$$\mathrm{Re}\mathcal{A}_{LD} = \mathcal{A}(\beta,\mu) + \chi_{\,\rm LD}(\mu)$$

$K_L \to \mu^+\mu^-$ Short–Distance Contribution

$\left(\lambda_c = V_{cs}^* V_{cd}\,, \quad \lambda_t = V_{ts}^* V_{td}\,, \quad x_t = \frac{m_t^2}{M_W^2}\right)$

Buchalla–Buras '99

$$\mathcal{H}_{\rm eff}^{\rm SD} = \cdots - \frac{G_{\rm F}}{\sqrt{2}}\frac{\alpha}{\pi}\frac{2}{\sin^2\Theta_W}\left[\lambda_c Y_{NL} + \lambda_t Y(x_t)\right]\left(\bar{s}\gamma_\mu\frac{1-\gamma_5}{2}d\right)\bar{l}\gamma^\mu\frac{1-\gamma_5}{2}l + {\rm h.c.}$$

- Recall the LD–parameterization

$$\mathcal{H}_{\rm eff}^{\rm LD} = \mathcal{C}\left\{\frac{\alpha N_c}{12\pi f_\pi}\epsilon_{\mu\nu\alpha\beta}F^{\mu\nu}A^\alpha(\partial^\beta K_L) + \left(\frac{\alpha}{\pi}\right)^2\frac{\chi_{\rm LD}(\mu)}{4f_\pi}(\partial_\mu K_L)\bar{l}\gamma^\mu\gamma_5 l\right\}$$

- The effect of $\mathcal{H}_{\rm eff}^{\rm SD}$ is to induce a shift of the local $\chi_{\rm LD}(\mu)$–coupling:

$$\chi_{\rm LD}(\mu) \Rightarrow \chi_{\rm LD}(\mu) - \chi_{\rm SD} \equiv \chi_{\rm eff} \quad (\mu = M_\rho)$$

with
$$\frac{G_{\rm F}}{\sqrt{2}}\frac{f_\pi^2}{\sin^2\Theta_W}\left[{\rm Re}\lambda_c Y_{NL} + {\rm Re}\lambda_t Y(x_t)\right] = -\frac{\alpha}{\pi}\,\mathcal{C} \times \chi_{\rm SD}$$

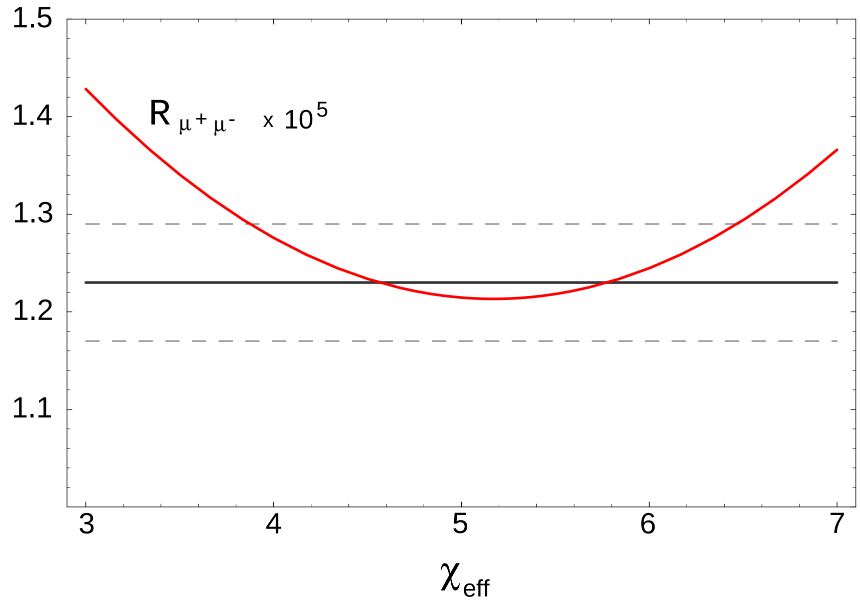
The Decay Ratio $\mathcal{R}_{\bar{l}l}$ only depends on the parameter $\chi_{\rm eff}$ (see next Figure)

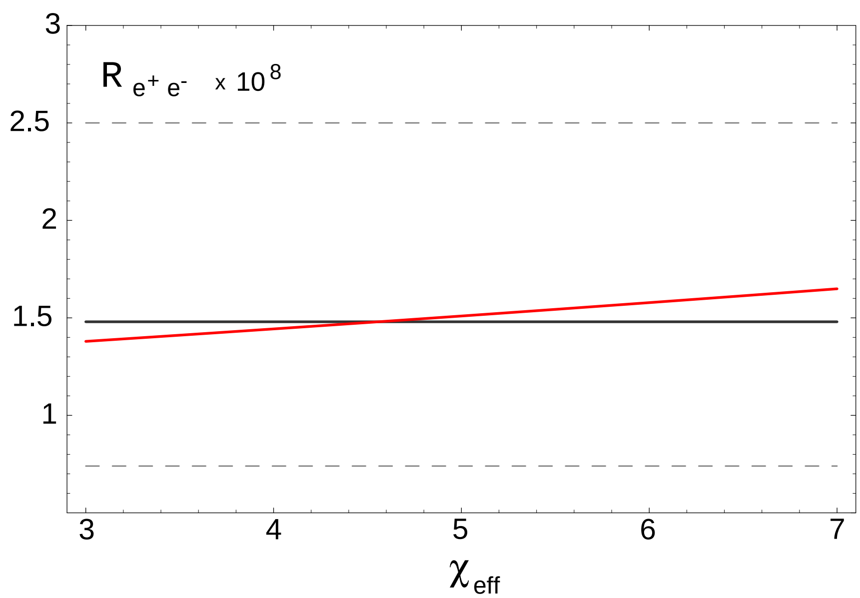
- HOWEVER, the constant $\mathcal{C}$ is only known in *absolute value* from:

$$\Gamma(K_L \to \gamma\gamma) = \left(\frac{\alpha}{\pi}\right)^2\frac{m_K^3}{64\pi f_\pi^2}|\mathcal{C}|^2$$

AND, the constant $\chi_{\rm LD}$ is *a priori* unknown

Branching Ratios versus $\chi_{\rm eff}$





Notice that

$$3.9 \leq \chi_{\rm eff} \leq 6.5 \qquad \text{and} \qquad {\rm Re}\mathcal{A} = 0 \Leftrightarrow \chi_{\rm eff} = 5.2$$

Large–N_c Calculation of $\chi_{\,\mathrm{LD}}(\mu)$

David Greynat-de Rafael '02 (preliminary)

- **Crucial Observation:**
  The *Residue* of the leading OPE behaviour of the Green's function

$$\langle 0 \mid T\left\{ J_\mu(x) J_\nu(0) P(y) \mathcal{L}^{\Delta S=1}(z) \right\} \mid 0 \rangle$$

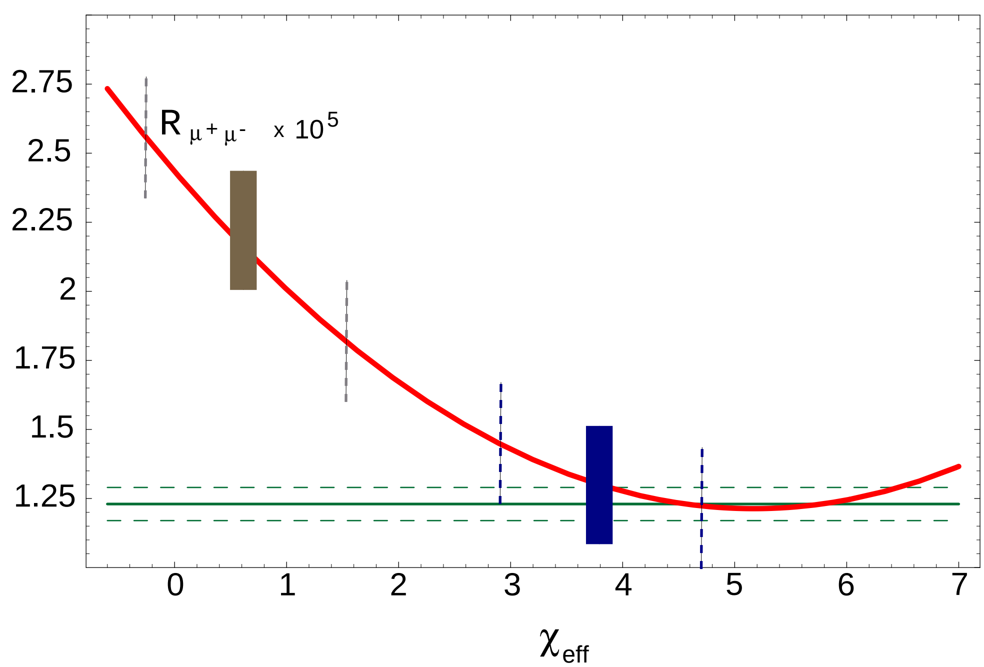
and the on–shell $\mathcal{A}(K_L \to \gamma\gamma)$ amplitude, to first non–trivial order in the chiral expansion, are *proportional* to each other, but for a *calculable* term

$$\chi_{\,\mathrm{LD}}(\mu) = \frac{5}{12}N_c - 2\pi^2\left[\frac{N_c}{4\pi^2}\log\left(\frac{M_V^2}{\mu^2}\right) - \frac{N_c}{4\pi^2} + 2\frac{F_0^2}{M_V^2}\left(1+\frac{\tilde{\mathcal{C}}}{\mathcal{C}}\right)\right]$$

where

$$\tilde{\mathcal{C}} = \frac{G_{\,\mathrm{F}}}{\sqrt{2}} V_{ud}V_{us}^{*}\, g_8\; 2F_0^2 \frac{M_K^2}{M_K^2 - m_\pi^2} \times \left(\frac{8}{3}\frac{M_K^2 - m_\pi^2}{F_0^2}\, 4L_5\right)$$

- **Predicted Branching Ratios (Two Solutions)**



Conclusions and Outlook

- Large–N_c QCD provides a very useful framework to formulate calculations of the Low–Energy Constants of the effective Chiral Lagrangian (both Strong and Electroweak).

- Predictions (S. Peris talk at this Conference)

  Calculation of the B_k–Factor in the chiral limit
  Calculation of Q_7 and Q_8 Weak Matrix Elements
  Hadronic Light–by–Light Contribution to $g_\mu - 2$
  Hadronic Electroweak Contribution to $g_\mu - 2$

- Extensions to Rare K–Decays, as discussed here for $K_L \to \bar{\mu}\mu$, are possible

- Good Prospects for Rare K–Physics
  K–Decays in particular

"IFF"

Experimental Projets

specially at BNL, are Pursued