Is the ‘effective conductivity’ a useful parameter?

Abstract

The magnetospheres of Jupiter and Saturn exhibit significant rotation because angular momentum is transferred from the planet via ion-neutral collisions in the upper atmosphere. The standard viewpoint of this process holds that the angular momentum so extracted from the upper atmosphere is replaced by upwards viscous transfer of angular momentum from the lower atmosphere. The efficiency of this process is normally described by a parameter known as the ‘effective conductivity’.

However, thermospheric modelling suggests that this conventional model may be incomplete, and that angular momentum may instead be supplied by the thermospheric flow. This poster proposes a simple model of this latter process, compares the predictions of the old and new models, and thus shows that the ‘effective conductivity’ has limited usefulness as a parameterisation of the neutral atmosphere.

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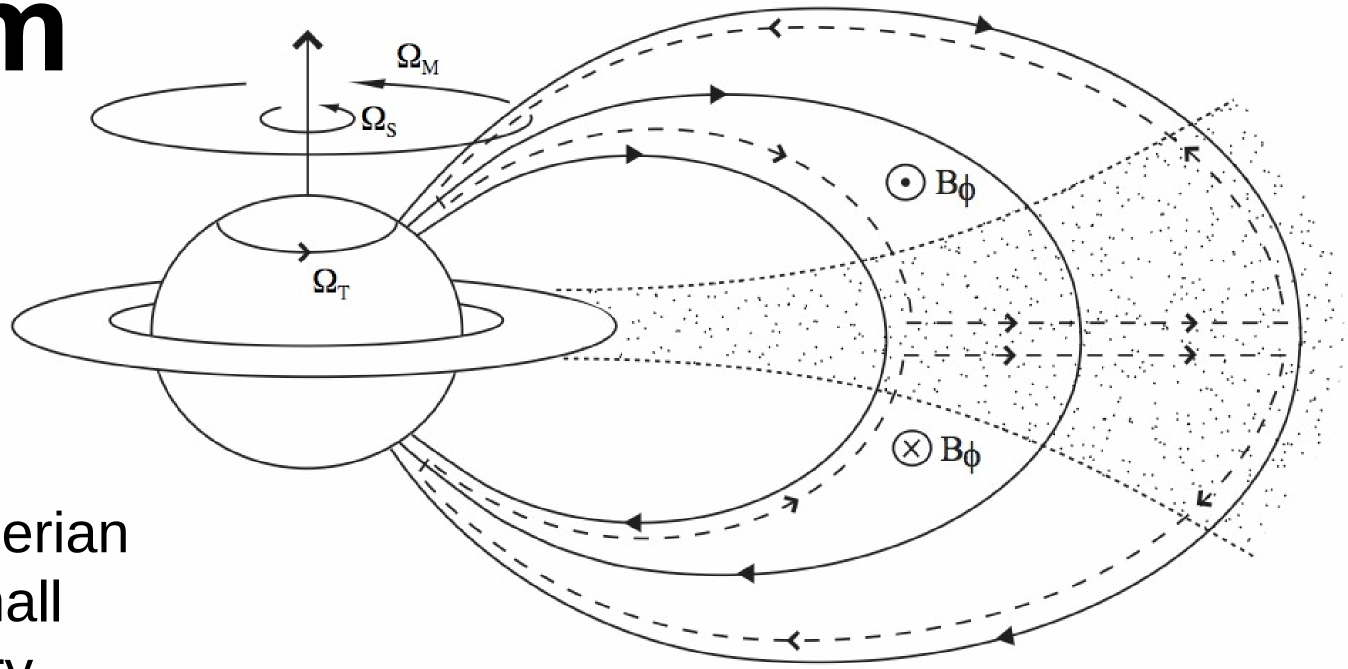
The Problem



The magnetosphere is loaded with plasma originating from the rings and moons.

This plasma enters the magnetosphere at the Keplerian orbital velocity, which is small compared with the planetary rotation velocity.

Friction between the magnetosphere and the thermosphere then tends to spin up the plasma by extracting angular momentum from the thermosphere.

How is the angular momentum extracted from the thermosphere replaced?

Ω_S deep atmosphere

Ω_T thermosphere

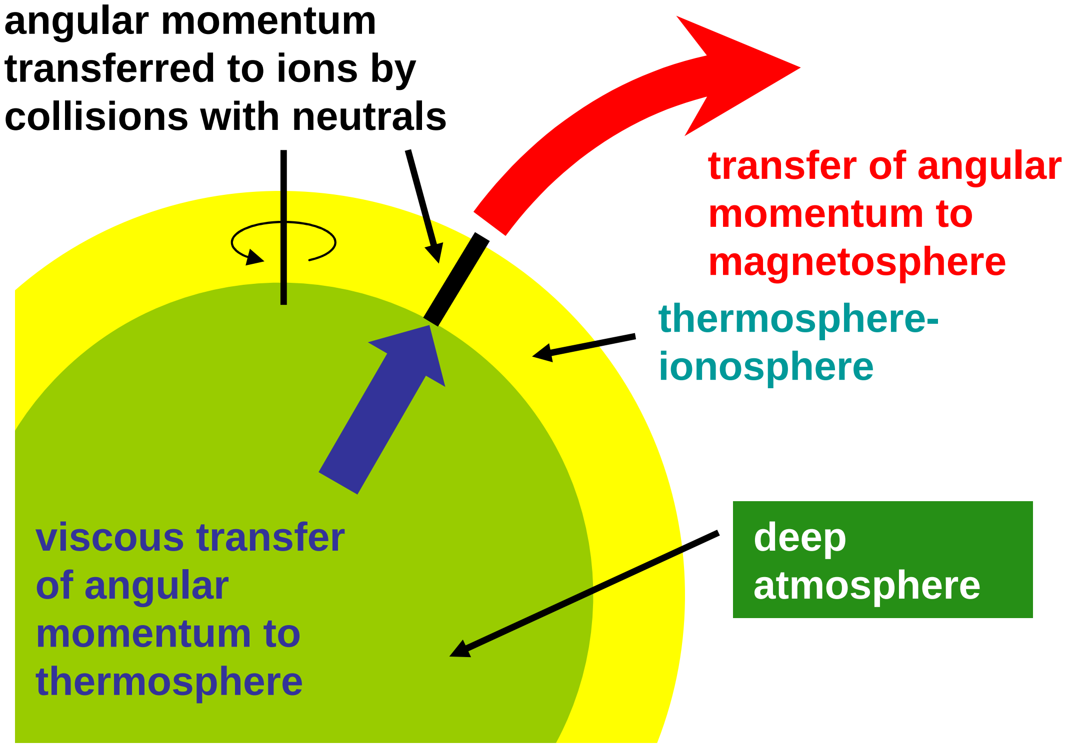
Ω_M magnetosphere

usually (but not always):

$$\Omega_S > \Omega_T > \Omega_M$$

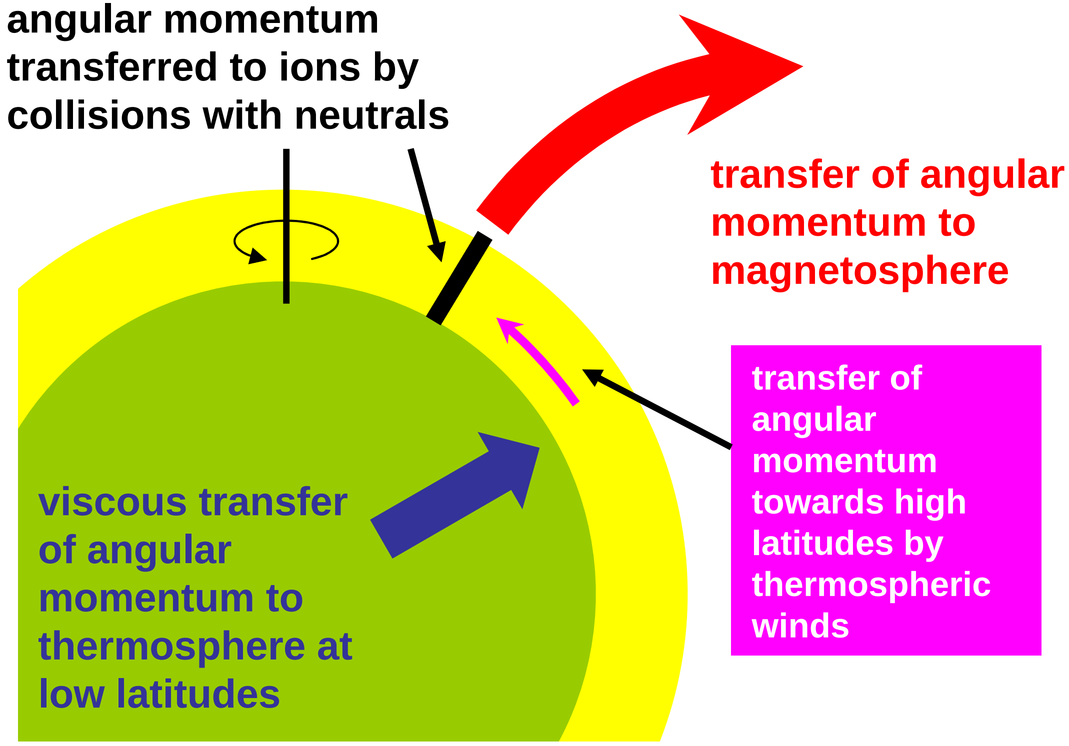
Model 1 (Huang and Hill, 1989)

Angular momentum is extracted from the thermosphere by ion drag. This induces a vertical shear in the neutral wind velocity that allows vertical viscous transfer of angular momentum from the deep atmosphere. In steady state, ion drag is balanced by viscous forces.



Model 2 (this study)

Angular momentum is extracted from the thermosphere by ion drag. The resulting sub-corotational winds are driven polewards by the Coriolis force, inducing a flow that transports angular momentum from lower latitudes. In steady state, ion drag is balanced by meridional advection.



Model 1 - Theory

We calculate the vertical profile of neutral angular velocity required in the thermosphere to sustain the necessary viscous supply of angular momentum

upwards magnetic field

vertical shear in neutral angular velocity

local Pedersen conductivity

$$\overbrace{\sigma_P B^2 \rho^2 (\Omega_M - \omega_T)}^{\text{a.m. extracted by ion drag}} + \overbrace{\rho^2 \frac{\partial}{\partial z}\left(\eta \frac{\partial \omega_T}{\partial z}\right)}^{\text{a.m supplied by upwards viscous transfer}} = 0$$

plasma angular velocity

local neutral angular velocity

off-axis distance

viscosity

altitude

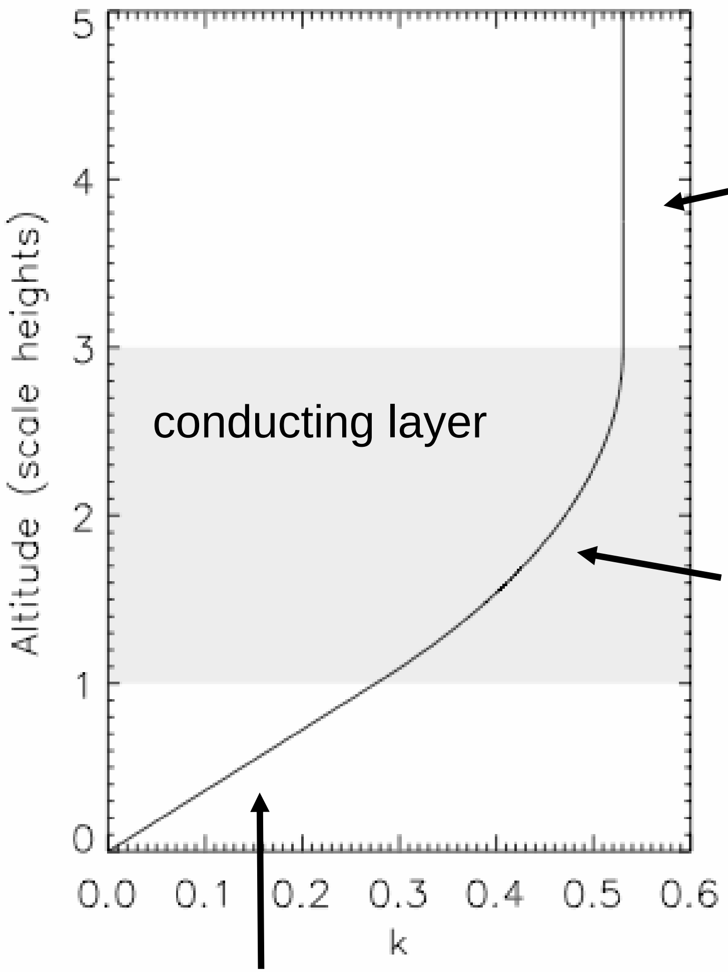
We solve this equation at all altitudes in the thermosphere to find a profile of neutral wind velocity. Solutions are linear in the plasma corotation lag, so we can rewrite the equation as shown to the right. The constant *k* is the neutral corotation lag normalised to the plasma corotation lag. **Our solutions for *k* will thus be independent of the plasma angular velocity.**

Normalised version

$$\sigma_P B^2 (1 - k) + \frac{\partial}{\partial z}\left(\eta \frac{\partial k}{\partial z}\right) = 0$$

$$k = \frac{\Omega_S - \omega_T}{\Omega_S - \Omega_M}$$

Model 1 - Solution



above conducting layer there is no shear/no a.m. flux

shear across conducting layer allows upwards viscous transfer of a.m.

below conducting layer there is constant shear/constant a.m. flux

The figure shows a solution for *k*, as a function of altitude, for an idealised isothermal atmosphere with a layer of uniform Pedersen conductivity (shaded).

For magnetospheric studies, we can summarise the behaviour by integrating or averaging quantities across the conducting layer: a height-integrated conductivity Σ_P, a height-averaged neutral angular velocity Ω_T, and a height-averaged normalised neutral corotation lag *K*.

For the example shown, *K* ~ 0.45.

$$\Omega_T = \frac{1}{\Sigma_P} \int \sigma_P \omega_T dz$$

$$\Sigma_P = \int \sigma_P dz \qquad K = \frac{\Omega_S - \Omega_T}{\Omega_S - \Omega_M}$$

Model 1 - Consequences

The current in the ionosphere, J, goes as the conductivity and the plasma-neutral velocity difference.

$$J \sim \Sigma_P(\Omega_M - \Omega_T)$$

We can eliminate the neutral rotation velocity by introducing the parameter *K*.

$$J \sim \Sigma_P(1 - K)(\Omega_M - \Omega_S)$$

Because *K* is **independent of the plasma velocity**, we can subsume it within a constant 'effective conductivity'.

$$J \sim \Sigma_P^*(\Omega_M - \Omega_S)$$

The Effective Conductivity

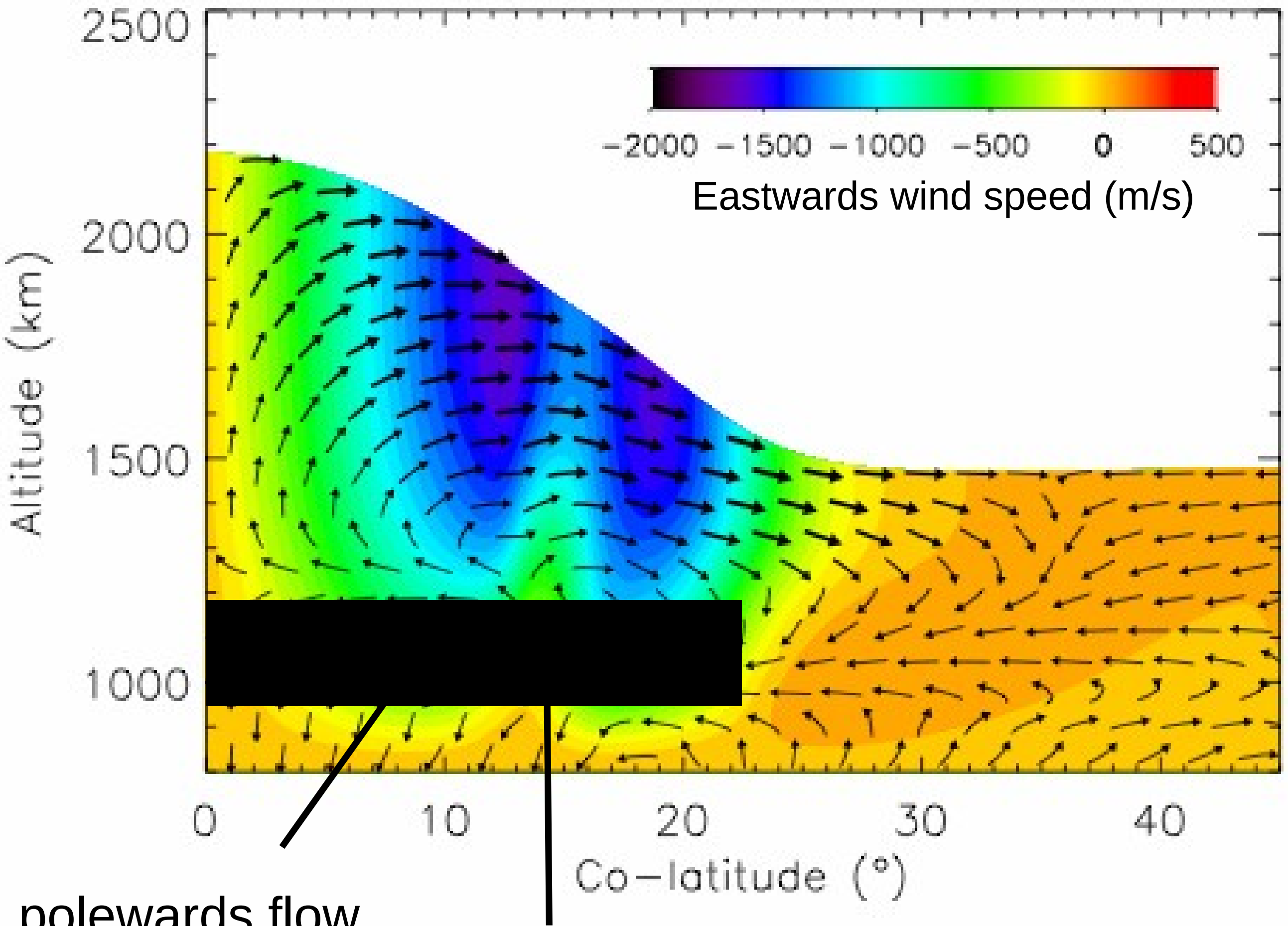
$$\Sigma_P^* = \Sigma_P(1 - K)$$

In this model the thermosphere always responds **linearly** to the magnetosphere. The parameter *K* behaves like a 'spring constant'. This behaviour is identical to a reduction in the conductivity by the factor *(1-K)*. The behaviour of the upper atmosphere is thus encapsulated entirely by the single parameter Σ_P*.

The thermosphere is treated as a **passive system** that responds in a very simple way to the magnetosphere. In this model, the magnetosphere can structure the thermosphere, but the thermosphere cannot structure the magnetosphere.

Model 2 - Motivation

Arrows: vertical and meridional winds; Colours: zonal winds



polewards flow in conducting layer supplies angular momentum

region subject to sub-corotational drag (shaded)

Numerical modelling of the thermosphere (see Figure) shows that meridional flow may be a more important source of angular momentum than viscosity. In the conducting layer the sub-corotational winds induced by ion drag are driven polewards by the Coriolis force. This polewards flow supplies angular momentum directly, and is also spun up by Coriolis as it moves towards the pole.

Can we construct a simple model of this flow that provides more insight than a numerical model?

Model 2 - Theory

We assume axial symmetry in the polar regions; we then suppose that the conducting layer is coupled to a constant polewards flow of mass. It is the inertia of this mass flow that supplies angular momentum.

azimuth integrated $\dot{m}$
polewards mass flow

$$\boxed{\underbrace{\Sigma_P B^2 \rho^2 (\Omega_M - \Omega_T)}_{\text{a.m. extracted by ion drag}} + \underbrace{\frac{\dot{m}}{2\pi\rho}\frac{d}{d\rho}(\Omega_T \rho^2)}_{\text{a.m. supplied by divergence of polewards flow}} = 0}$$

The efficiency of Model 1 depends only on the **vertical structure** of the atmosphere. We solve for *K* **once**, and this then applies at **every latitude**, independent of the behaviour of adjacent latitudes.

The efficiency of Model 2 depends on the **horizontal structure** of the atmosphere. To solve for the neutral wind Ω_T we require a model of the variation of Ω_M with latitude. Thus, in Model 2, the behaviours of **adjacent latitudes are coupled together**.

Model 2 - Theory

Close to the pole we can assume that

$$\rho \simeq R\theta$$

The model then simplifies to:

$$\frac{1}{\theta}\frac{d}{d\theta}(\Omega_T\theta^2) + \left(\frac{\theta}{\theta_0}\right)^2(\Omega_M - \Omega_T) = 0$$

$$\boxed{\theta_0 = \sqrt{\frac{\dot{m}}{2\pi\Sigma_P B^2 R^2}}}$$

The efficiency with which a.m. is supplied then depends on the parameter θ_0, which represents the latitude range over which inertia may support the flow against ion drag.

Assuming parameters for Saturn (Σ_P~1mho, B~60,000nT, R~60,000km) we find that θ_0~2º requires a total mass flow rate of 100 ton/s.

This corresponds to a polewards flow velocity of ~10m/s (a typical flow speed predicted by modelling) at a pressure of ~10nbar (close to the location of the conducting layer). Thus it is plausible that the polewards flow will supply sufficient angular momentum to balance ion drag.

In practice, we treat θ_0 as a free parameter in our modelling.

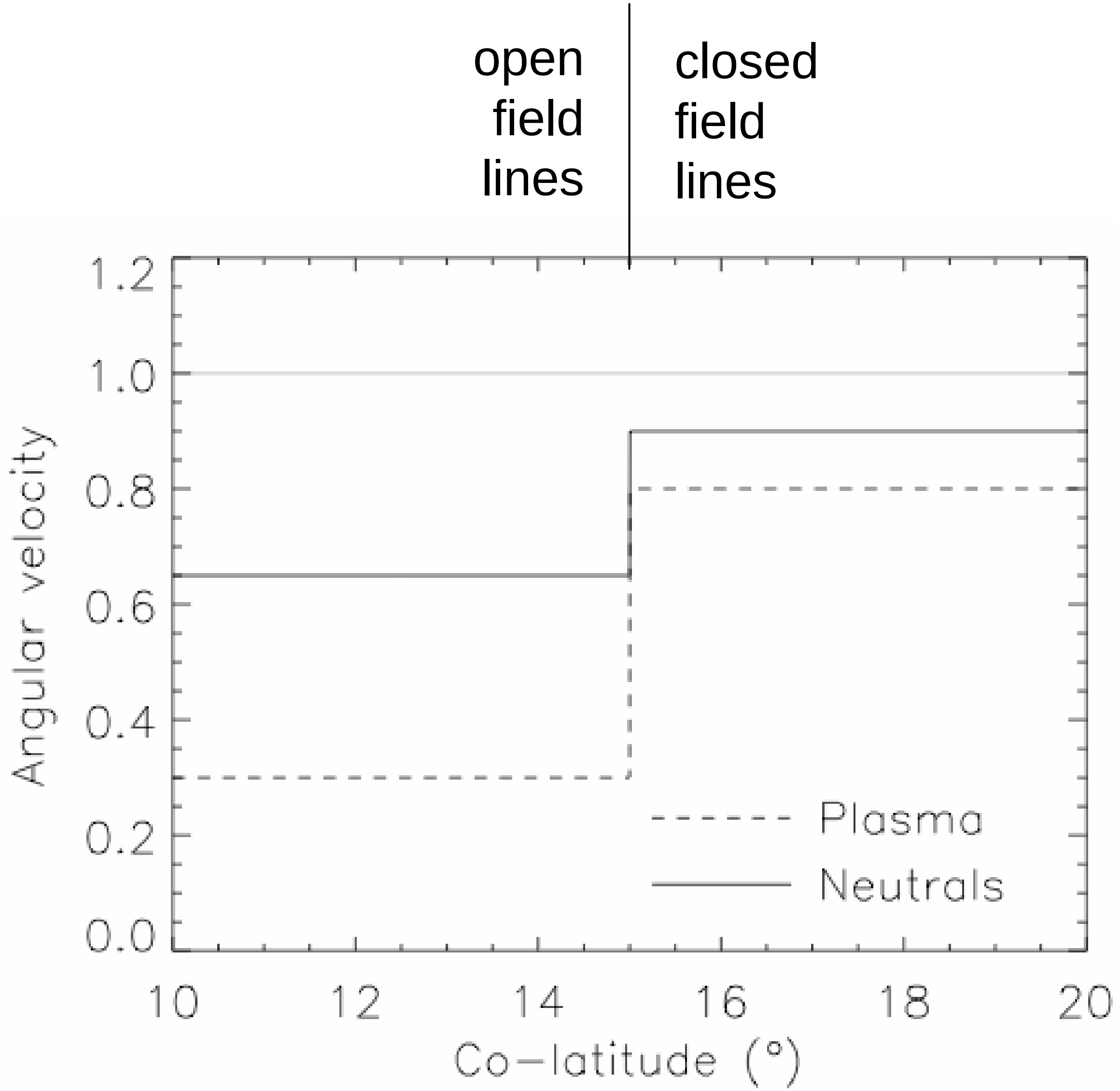
Case Study A

We now investigate the predictions of the two models at a shear in the plasma flow, such as may be associated with current sheets that generate auroral emissions (Cowley et al., 2004).

Predictions of Model 1

Assuming K = 0.5, Model 1 predicts that the neutral angular velocity follows exactly the shear in the plasma velocity.

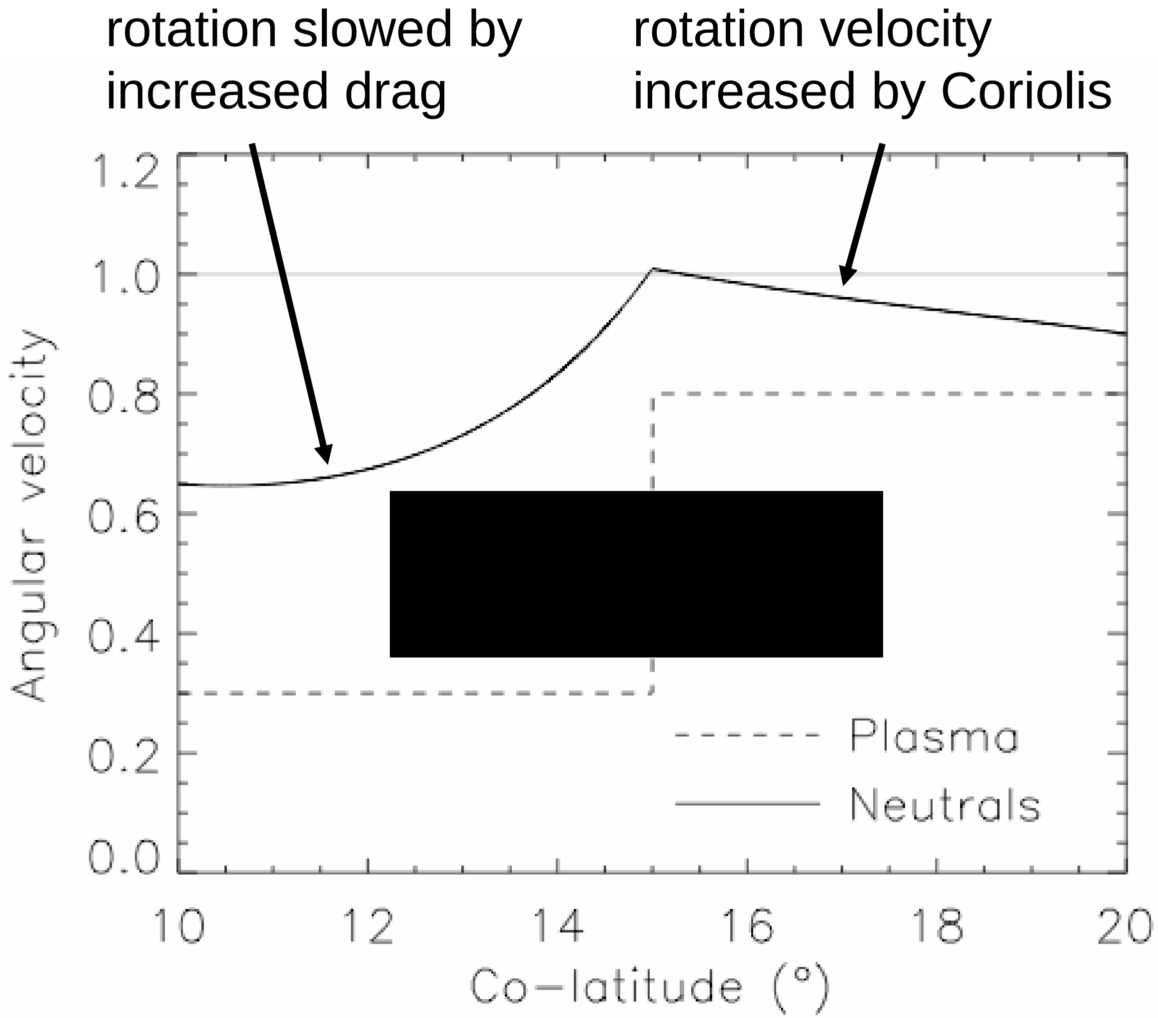
For a true conductivity of 1mho, the upwards field-aligned current at the boundary is ~0.039A/m. This is similar to the values calculated by e.g. Jackman & Cowley (2006).



Predictions of Model 2

We adjust the parameter θ_0 to find solutions which match the predictions of Model 1 at 10º and 20º co-latitude. In this case we require θ_0~5.4º.

For a true conductivity of 1mho, the upwards field-aligned current at the boundary is ~0.076A/m.



Comparison

1. Model 2 produces a profile of neutral velocity that is smeared in latitude relative to that predicted by Model 1. This is a consequence of the inertia of the polewards flowing gas. A signature of the high plasma velocities on closed field lines is advected across the open-closed field line boundary by the neutral flow.

2. Model 2 predicts a current sheet with approximately twice the intensity of that predicted by Model 1. This is because there is no sharp gradient in the neutral velocity at the plasma flow boundary, so the change in the relative velocities of plasma and neutrals is greater. This increases the change in the horizontal current, and thus increases the intensity of the current sheet.

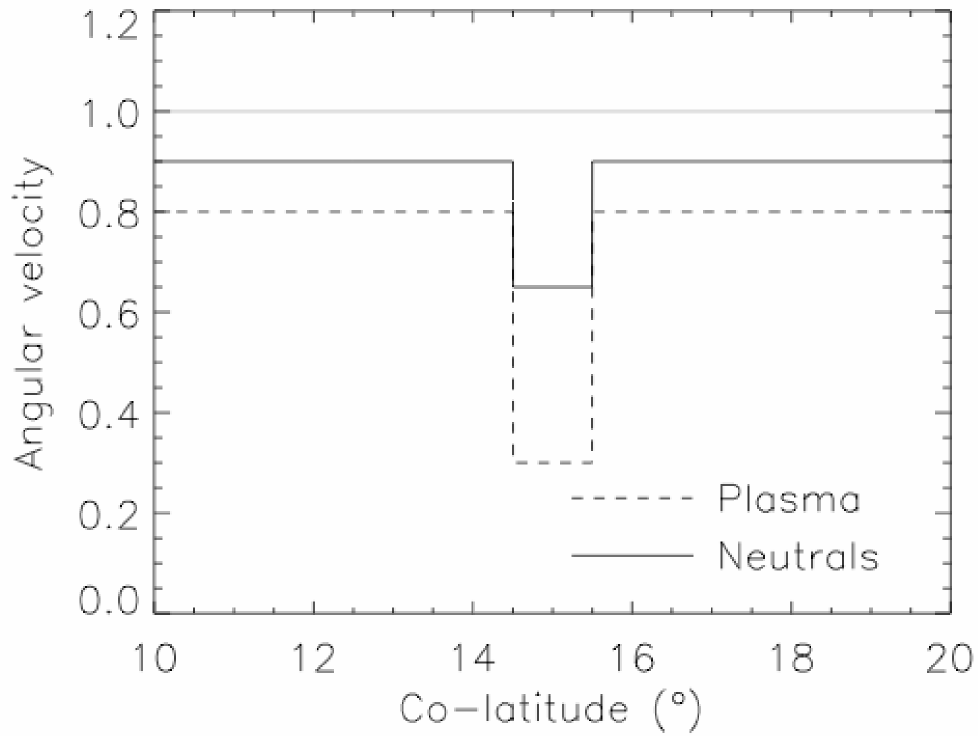
Case Study B

We now investigate the predictions of the two models at a dip in the plasma flow, such as may be associated with mass-loading by moons (Cowley and Bunce, 2003). For simplicity and clarity, we represent the dip using an idealised 'top-hat', with regions of constant flow velocity on either side.

Predictions of Model 1

Again, we assume K = 0.5.

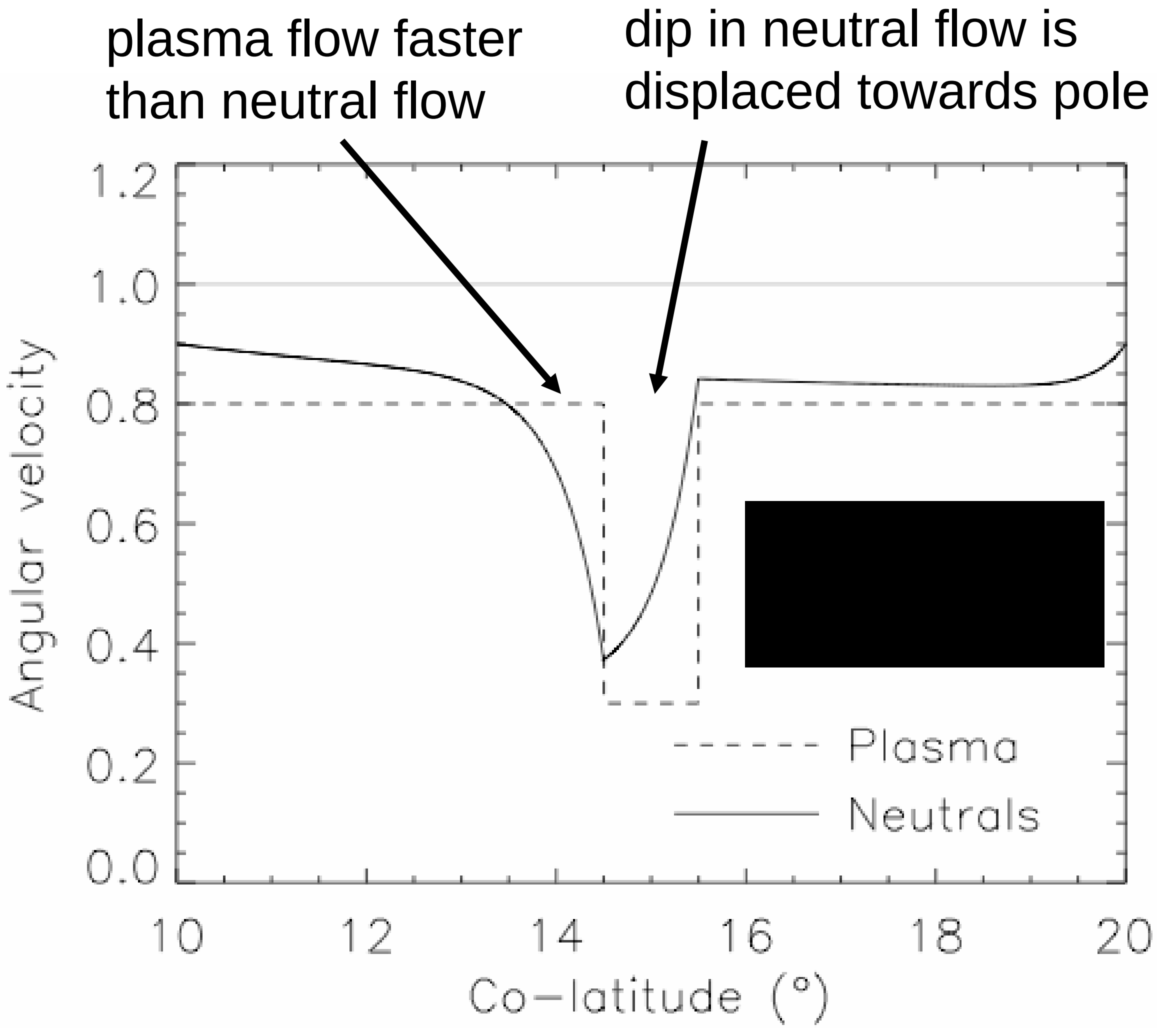
The response of the neutrals follows the dip in the plasma flow; the dip only influences latitudes to which it directly maps.



Predictions of Model 2

In this case we require θ_0~2.5º to match Model 1 at the boundaries.

The dip in the neutral flow is displaced towards the pole; it does not recover instantaneously from the dip, and there is a small region in which the plasma flow is faster than the neutral flow.



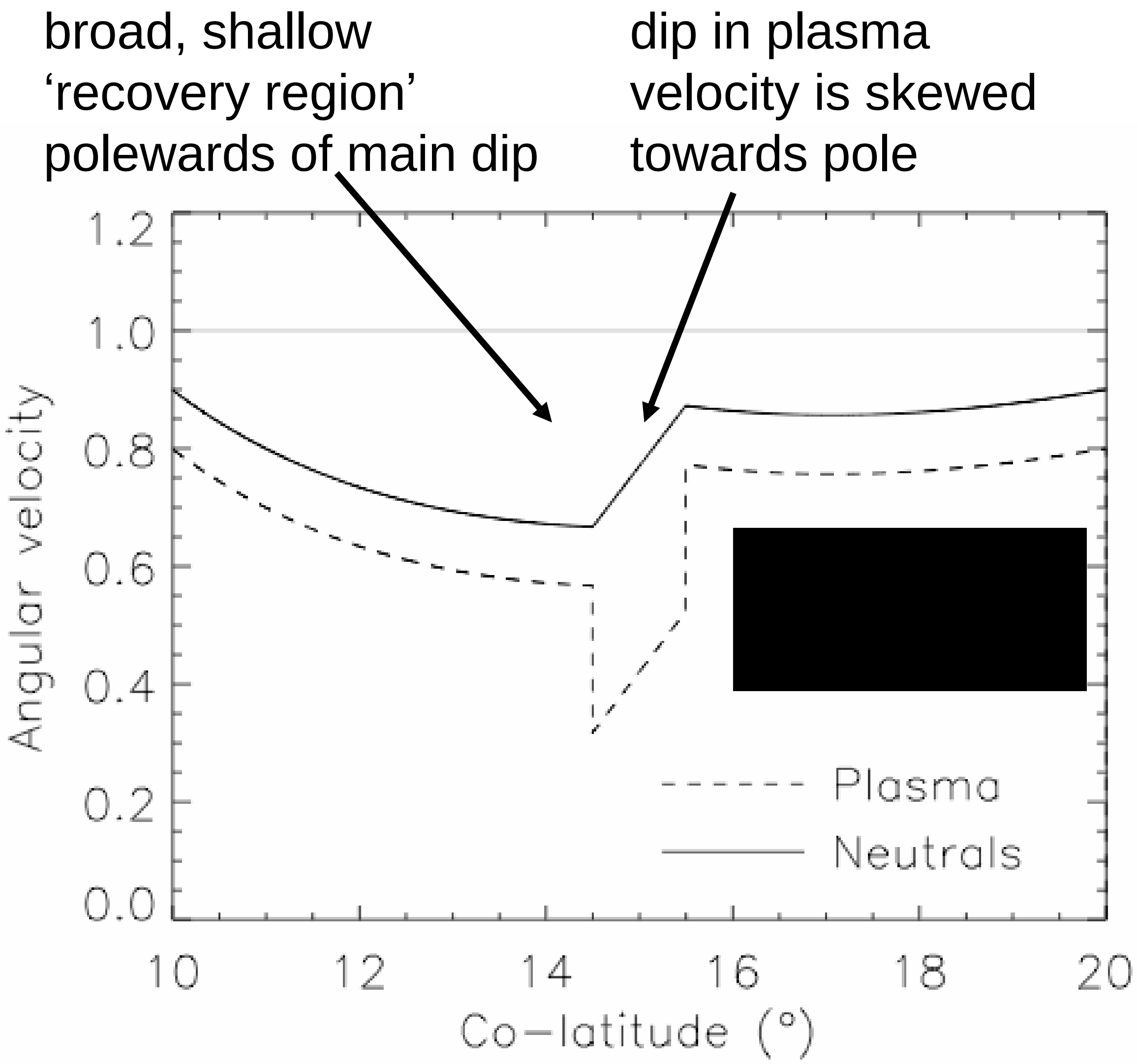
It does not normally make sense for the plasma flow to exceed the neutral flow. The plasma only rotates due to the torque exerted by the neutrals; thus, it cannot normally rotate more quickly than the neutrals. This motivates us to modify Model 2 such that the lag of the plasma flow to the neutral flow is fixed, not the plasma flow itself....

Model 3: identical to Model 2, but we fix $\Omega_M - \Omega_T$ to be the same as that predicted by Model 1. This is equivalent to fixing the torque due to ion drag to be the same as that predicted by Model 1. **The plasma velocity must now always lag the neutral velocity.**

Predictions of Model 3

In this case we require θ_0~4.1º to match Model 1 at the boundaries.

The main plasma flow dip is now skewed towards the pole, and a signature of the dip has been advected polewards due to the inertia of the neutral flow. Thus the effects of the dip are no longer localised.



Conclusions: is Model 1 useful?

1. We have shown that **Model 1**, which represents the behaviour of the upper atmosphere with a **single parameter** - the ‘effective conductivity’ - **does not** accurately represent the response of the neutral atmosphere in all circumstances.

2. Numerical modelling indicates that advection is probably a more important process for supplying angular momentum than viscosity. Thus we expect **Model 2** to be **more realistic**, and Model 1 will only be useful if it produces **qualitatively similar** results to Model 2.

3. **This is not the case.** The ‘effective conductivity’ model produces **qualitatively different** results close to shears and dips in the magnetospheric plasma flow. The ‘effective conductivity’ is thus expected to be useful only in relatively **uninteresting** regions of the system. Close to structure in the plasma flow, it is probably preferable either to use the true conductivity - and admit the neglect of the neutral atmosphere as an error - or to employ a more realistic model of the supply of angular momentum, such as our Model 2.

4. **Questions/further work**
- are there limited circumstances in which **Model 1** dominates?
- what determines θ_0?
- how does it vary with latitude?
- does the flow **self-regulate** to provide just enough a.m.?
- do the predictions of Model 2 match the output of **numerical models**?
- to what extent are dips ‘advected’ by the flow in **realistic** models of the magnetospheric plasma flow?