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UHECR bounds on Lorentz violation in the photon sector

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[1] S. Bernadotte & FRK, PRD **75**, 024028 (2007), hep-ph/0610216.

[2] C. Kaufhold & FRK, PRD **76**, 025024 (2007), arXiv:0704.3255 [hep-th].

[3] FRK & M. Risse, PRD **77**, 016002 (2008), arXiv:0709.2502v5 [hep-ph].

[4] FRK, arXiv:0710.3075 [hep-ph]. ← REVIEW

Introduction

1. INTRODUCTION

Fundamental question: *does space remain smooth as one probes smaller and smaller distances?*

A conservative limit on the typical length scale ℓ of any small-scale structure of space:

$$\text{LEP/Tevatron:} \quad \ell \lesssim 10^{-18}\,\text{m} \approx \hbar c / \big(200\,\text{GeV}\big)\,. \tag{1}$$

Yet, **astrophysics** provides us with very much higher energies.

Outline of this talk:

- phenomenology of a simple photon-propagation model;
- bounds from ultra-high-energy cosmic rays (UHECRs);
- theoretical implications.

Phenomenology

2. PHENOMENOLOGY

2.1 Model

Action for a Lorentz-violating deformation of quantum electrodynamics:

$$S_{\text{modQED}} = S_{\text{modM}} + S_{\text{standD}}\,, \tag{2}$$

with **modified-Maxwell term** [Chadha & Nielsen, NPB 217, 125 (1983)]:

$$S_{\text{modM}} = \int_{\mathbb{R}^4} \mathrm{d}^4x \left(-\tfrac{1}{4} \left(\eta^{\mu\rho}\eta^{\nu\sigma} + \kappa^{\mu\nu\rho\sigma}\right) F_{\mu\nu}(x)\, F_{\rho\sigma}(x) \right), \tag{3a}$$

and standard Dirac term for spin–$\frac{1}{2}$ particle with charge e and mass M:

$$S_{\text{standD}} = \int_{\mathbb{R}^4} \mathrm{d}^4x\, \overline{\psi}(x) \Big(\gamma^\mu \left(\mathrm{i}\,\partial_\mu - eA_\mu(x)\right) - M\Big)\psi(x)\,. \tag{3b}$$

Theory is gauge-invariant, CPT–even, and power-counting renormalizable.

Phenomenology

Here, $\kappa^{\mu\nu\rho\sigma}$ is a constant background tensor with the same symmetries as the Riemann curvature tensor and a double trace condition $\kappa^{\mu\nu}{}_{\mu\nu} = 0$, so that there are $20 - 1 = 19$ components.

As the 10 birefringent parameters are already tightly constrained [Kostelecky & Mewes, hep-ph/0205211], restrict the theory to the **nonbirefringent sector**:

$$\kappa^{\mu\nu\rho\sigma} = \tfrac{1}{2}\Big(\eta^{\mu\rho}\,\widetilde{\kappa}^{\nu\sigma} - \eta^{\mu\sigma}\,\widetilde{\kappa}^{\nu\rho} - \eta^{\nu\rho}\,\widetilde{\kappa}^{\mu\sigma} + \eta^{\nu\sigma}\,\widetilde{\kappa}^{\mu\rho}\Big), \tag{4}$$

for a symmetric and traceless matrix $\widetilde{\kappa}^{\mu\nu}$ with $10 - 1 = 9$ components.

Hence, there are **9 Lorentz-violating (LV) deformation parameters $\widetilde{\kappa}^{\mu\nu}$**.

Phenomenology (skip)

Rewrite these parameters $\widetilde{\kappa}^{\mu\nu}$ as follows:

$$\left(\widetilde{\kappa}^{\mu\nu}\right) \equiv \text{diag}\left(1, \tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3}\right) \overline{\kappa}^{00} + \left(\delta\widetilde{\kappa}^{\mu\nu}\right), \quad \delta\widetilde{\kappa}^{00} = 0, \tag{5}$$

with 1 independent parameter $\overline{\kappa}^{00}$ for the spatially isotropic part of $\widetilde{\kappa}^{\mu\nu}$ and 8 independent parameters $\delta\widetilde{\kappa}^{\mu\nu}$.

Express these parameters in terms of the so-called SME parameters:

$$\begin{pmatrix} \overline{\kappa}^{00} \\ \delta\widetilde{\kappa}^{01} \\ \delta\widetilde{\kappa}^{02} \\ \delta\widetilde{\kappa}^{03} \\ \delta\widetilde{\kappa}^{11} \\ \delta\widetilde{\kappa}^{12} \\ \delta\widetilde{\kappa}^{13} \\ \delta\widetilde{\kappa}^{22} \\ \delta\widetilde{\kappa}^{23} \end{pmatrix} \equiv \begin{pmatrix} (3/2)\,\widetilde{\kappa}_{\text{tr}} \\ -\left(\widetilde{\kappa}_{\text{o}+}\right)^{(23)} \\ -\left(\widetilde{\kappa}_{\text{o}+}\right)^{(31)} \\ -\left(\widetilde{\kappa}_{\text{o}+}\right)^{(12)} \\ -\left(\widetilde{\kappa}_{\text{e}-}\right)^{(11)} \\ -\left(\widetilde{\kappa}_{\text{e}-}\right)^{(12)} \\ -\left(\widetilde{\kappa}_{\text{e}-}\right)^{(13)} \\ -\left(\widetilde{\kappa}_{\text{e}-}\right)^{(22)} \\ -\left(\widetilde{\kappa}_{\text{e}-}\right)^{(23)} \end{pmatrix}. \tag{6}$$

Phenomenology

2.2 Possible spacetime origin

Calculations of standard photons and Dirac particles propagating in simple classical spacetime-foam models reproduce a restricted (isotropic) version of model (2):

$$\frac{4}{3}\,\overline{\kappa}^{00} = -\widetilde{\sigma}_2\,\widetilde{F} \equiv -\widetilde{\sigma}_2 \left(\frac{\widetilde{b}}{\widetilde{l}}\right)^4, \quad \delta\widetilde{\kappa}^{\mu\nu} = 0\,, \tag{7}$$

in terms of the quadratic coefficient of modified photon dispersion relation given below.

Phenomenology

For "defects" (size $\bar{b}$) embedded in Minkowski spacetime (separation $\bar{l}$), both quadratic and quartic photon terms have been calculated:

$$\omega_{\mathsf{p}}^2 \equiv M_{\mathsf{p}}^2\, c_{\mathsf{p}}^4/\hbar^2 + c_{\mathsf{p}}^2\, k^2 + \mathsf{O}(k^4)\,, \tag{8a}$$

$$\omega_{\gamma}^2 = \left[1 + \left(\widetilde{\sigma}_2\, \widetilde{F}\right)\right] c_{\mathsf{p}}^2\, k^2 + \left(\widetilde{\sigma}_4\, \widetilde{F}\, \widetilde{b}^2\right) c_{\mathsf{p}}^2\, k^4 + \mathsf{O}(k^6)\,, \tag{8b}$$

with wave number $k \equiv |\mathbf{k}|$,
effective defect on/off factors $\widetilde{\sigma}_2, \widetilde{\sigma}_4 \in \{-1, 0, +1\}$,
effective size $\widetilde{b}$, and effective excluded-volume factor $\widetilde{F} \equiv (\widetilde{b}/\widetilde{l})^4 \ll 1$.

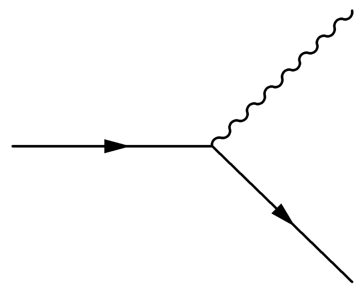
Specific results [1] for the relation between effective parameters (tilde) and fundamental spacetime parameters (bar):

$$\widetilde{b} = \beta\, \bar{b}\,,\ \ \widetilde{l} = \lambda\, \bar{l}\,,\ \ \widetilde{\sigma}_2 = -1\,,\ \ \widetilde{\sigma}_4 = 1\,, \tag{9}$$

with positive constants β and λ of order unity.

Phenomenology

2.3 Vacuum Cherenkov radiation



The decay process $p \to p + \gamma$ in model (2) has been studied classically by Altschul [hep-th/0609030] and quantum-mechanically by Kaufhold & FRK [2].

Radiated energy rate of a point particle with electric charge $Z_{\text{prim}}\, e$, mass $M_{\text{prim}} > 0$, momentum $\boldsymbol{q}$, and ultrarelativistic energy $E \sim c\,|\boldsymbol{q}|$:

$$\frac{\mathrm{d}W_{\text{modQED}}}{\mathrm{d}t} \sim Z_{\text{prim}}^2\, e^2\, \xi(\widehat{\mathbf{q}})\; E^2/\hbar\,\Big|_{E^2 \gg E_{\text{thresh}}^2}\,, \tag{10}$$

with (direction-dependent) coefficient $\xi \geq 0$ and **threshold energy**

$$E_{\text{thresh}}^2 = \frac{M_{\text{prim}}^2\, c^4}{R\big[\,(4/3)\,\overline{\kappa}^{00} + 2\,\delta\widetilde{\kappa}^{0j}\,\widehat{\mathbf{q}}^j + \delta\widetilde{\kappa}^{jk}\,\widehat{\mathbf{q}}^j\,\widehat{\mathbf{q}}^k\big]} + \mathrm{O}\left(M_{\text{prim}}^2\, c^4\right), \tag{11}$$

for ramp function $R[x] \equiv \big(x + |x|\big)/2$.

UHECR bounds

3. UHECR BOUNDS

3.1 Basic idea

An interesting observation [a,b]:

- if vacuum Cherenkov radiation has a **threshold** $E_{\text{thresh}}(\widetilde{b}, \widetilde{l}, \widetilde{\kappa})$, then UHECRs with $E > E_{\text{thresh}}$ cannot travel far, as they rapidly radiate away their energy;
- **observing** an UHECR of energy E implies that this energy is at or below threshold,

$$E \leq E_{\text{thresh}}(\widetilde{b}, \widetilde{l}, \widetilde{\kappa}), \tag{12}$$

which then gives **bounds** on combinations of $\widetilde{b}$, $\widetilde{l}$, and $\widetilde{\kappa}$.

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[a] Beall, PRD 1, 961 (1970).
[b] Coleman & Glashow, PLB 405, 249 (1997).

UHECR bounds

3.2 Bounds on LV photon parameters

Take the following **29 selected events**:

27 from Auger [arXiv:0712.2843],

1 from Fly's Eye [astro-ph/9410067],

1 from AGASA [PRL 73, 3491 (1994)].

These events are given by Table 1 on the next page, with in column

1. arrival time (year and Julian day);
2. primary energy E in EeV, where 1 EeV $\equiv 10^{18}$ eV;
3. arrival directions with right ascension and declination in degrees.

Uncertainties in the energies are of the order of 25 % and in the pointing directions of the order of 1 deg.

UHECR bounds

Table 1: UHECR events from Auger (2004–2007), Fly's Eye (1991), and AGASA (1993).

year	day	E	RA	DEC	year	day	E	RA	DEC
1991	288	320	85.2	48.0	2006	81	79	201.1	−55.3
1993	337	210	18.9	21.1	2006	185	83	350.0	9.6
2004	125	70	267.1	−11.4	2006	296	69	52.8	−4.5
2004	142	84	199.7	−34.9	2006	299	69	200.9	−45.3
2004	282	66	208.0	−60.3	2007	13	148	192.7	−21.0
2004	339	83	268.5	−61.0	2007	51	58	331.7	2.9
2004	343	63	224.5	−44.2	2007	69	70	200.2	−43.4
2005	54	84	17.4	−37.9	2007	84	64	143.2	−18.3
2005	63	71	331.2	−1.2	2007	145	78	47.7	−12.8
2005	81	58	199.1	−48.6	2007	186	64	219.3	−53.8
2005	295	57	332.9	−38.2	2007	193	90	325.5	−33.5
2005	306	59	315.3	−0.3	2007	221	71	212.7	−3.3
2005	306	84	114.6	−43.1	2007	234	80	185.4	−27.9
2006	35	85	53.6	−7.8	2007	235	69	105.9	−22.9
2006	55	59	267.7	−60.7					

UHECR bounds

With these 29 primary energies and directions, we obtain the following two–σ (95% CL) **Cherenkov bounds** on the nine isolated SME parameters of nonbirefringent modified-Maxwell theory [3]:

$$(ij) \in \{(23),(31),(12)\} \quad : \quad \left|(\widetilde{\kappa}_{\text{o}+})^{(ij)}\right| < 2 \times 10^{-18}\,, \tag{13a}$$

$$(kl) \in \{(11),(12),(13),(22),(23)\} \quad : \quad \left|(\widetilde{\kappa}_{\text{e}-})^{(kl)}\right| < 4 \times 10^{-18}\,, \tag{13b}$$

$$\widetilde{\kappa}_{\text{tr}} < 1.4 \times 10^{-19}\,, \tag{13c}$$

for the Sun-centered celestial equatorial coordinate system.

Here, we have set $M_{\text{prim}} = 56$ GeV$/c^2$ and, for (13c), used the 148 EeV Auger event which has a reliable energy calibration.

The Cherenkov bounds (13abc) only depend on the measured energies and flight directions of the charged cosmic-ray primaries at the top of the Earth atmosphere.

UHECR bounds

Current **laboratory bounds** (complete set of references in [3]):

direct bounds at the $\underline{10^{-12}}$ level for the three parity-odd nonisotropic parameters in $\widetilde{\kappa}_{o+}$;

direct bounds at the $\underline{10^{-14}}$ to 10^{-16} levels for the five parity-even nonisotropic parameters in $\widetilde{\kappa}_{e-}$;

direct bound at the $\underline{10^{-7}}$ level and indirect bound at the $\underline{10^{-8}}$ level for the single parity-even isotropic parameter $\widetilde{\kappa}_{tr}$.

Interestingly, the UHECR Cherenkov bounds are the strongest where the laboratory bounds are the weakest, they are truly complementary.

UHECR bounds

From the 148 EeV Auger event, we also get a bound on the general coefficient of the quartic photon term in (8b):

$$|\widetilde{F}\,\widetilde{b}^2| \lesssim \left(1.4 \times 10^{-35}\,\text{m}\right)^2 \,, \tag{14}$$

based on the analysis of Gagnon & Moore [hep-ph/0404196]
but scaling their result to $M_{\text{prim}} = 56\,\text{GeV}/c^2$ and $E_{\text{prim}} = 148\,\text{EeV}$. (#)

Taking $\widetilde{F} = 10^{-19}$ from (13c), this bound becomes

$$\widetilde{b} \lesssim 4 \times 10^{-26}\,\text{m}\,, \tag{15}$$

which is still a very small length.

(#) Bound (14) disagrees, by 16 orders of magnitude, with a claimed "quantum-gravity" effect in a gamma-ray flare from Mkn 501 as observed by the MAGIC telescope [arXiv:0708.2889]; see [4].

UHECR bounds

3.3 Discussion

Cherenkov-type bounds have been obtained for combinations of the effective defect size ($\widetilde{b}$) and separation ($\widetilde{l}$):

$$\widetilde{F} \equiv (\widetilde{b}/\widetilde{l})^4 \lesssim 10^{-19}\,, \tag{16a}$$

$$\widetilde{b} \lesssim 4 \times 10^{-26}\,\text{m} \approx \hbar\, c/\big(5 \times 10^9\,\text{GeV}\big)\,. \tag{16b}$$

Bound (16b) is already quite remarkable (cf. LEP/Tevatron/LHC) and, moreover, severely constrains (read: rules out) TeV–gravity models [cf. Arkani-Hamed, Dimopoulos, & Dvali, hep-ph/9803315]:

any such theory with, for example, a nonperturbative gravity scale $E_{\text{ADD}} = \hbar\, c/L_{\text{ADD}} \sim 5$ TeV needs to explain the origin of a very small numerical factor f in the quartic photon term (setting $c = 1$):

$$\omega_\gamma^2 = k^2 + f\, L_{\text{ADD}}^2\, k^4 + \mathsf{O}(k^6)\,, \quad |f| \lesssim 10^{-12}\,. \tag{17}$$

UHECR bounds

More generally, also the Lorentz-violating deformation parameters of modified-Maxwell theory are strongly bounded:

$$|\kappa^{\mu\nu\rho\sigma}| \lesssim 10^{-18}\,, \tag{18}$$

where, for the sake of argument, the "one-sided" Cherenkov bound on the isotropic parameter $\widetilde{\kappa}_{\text{tr}}$ has also been made "two-sided."

Bounds (16a) and (18) imply that:

a single-scale $(\widetilde{b} \sim \widetilde{l})$ classical spacetime foam is ruled out.

This conclusion holds, in fact, for arbitrarily small values of the defect separation $\widetilde{l}$, as long as a classical spacetime makes sense.

The surprising conclusion is that **Lorentz invariance remains valid** down towards smaller and smaller distances.

This conclusion would hold down to distances at which the **classical-quantum transition** occurs, possibly of order $l_{\text{Planck}} \approx 10^{-35}$ m ...

Outlook

4. OUTLOOK

Experimental result from astrophysics (UHECRs, in particular):

quantum spacetime foam must have "crystalized" to a classical spacetime manifold which is **remarkably smooth**, as quantified by the defect excluded-volume factor $\widetilde{F} \equiv (\widetilde{b}/\widetilde{l})^4 \lesssim 10^{-19} \ll 1$ and Lorentz-violating parameters $|\kappa^{\mu\nu\rho\sigma}| \lesssim 10^{-18} \ll 1$.

Obviously, these smoothness results are **null effects** and there is an analogy with the well-known Michelson–Morley experiment (1887): *theorists predict novel effects which are not seen by experiment.*

This suggests the need for **radically new concepts** (cf. SR in 1905).

For example, a self-tuning Lorentz-invariant vacuum variable [5] may play a crucial role for the flatness of spacetime by resolving the so-called cosmological constant problem. Work in progress . . .

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[5] FRK & G. Volovik, PRD **77**, 085015 (2008), arXiv:0711.3170 [gr-qc].

Conjectures – Fundamental length

A. TWO CONJECTURES

A.1 Fundamental length

In view of the conclusions from Sec. 3.3, the following question arises:

theoretically, are we really sure that quantum spacetime effects *only* show up at distances of the order of the Planck length?

Conjecture 1a: *quantum spacetime has a* ***fundamental length scale*** l, *which is conceptually different from the Planck length*,

$$l \stackrel{?}{\neq} l_{\text{Planck}} \equiv \sqrt{\hbar\, G/c^3} \approx 1.6 \times 10^{-35}\,\text{m}\,. \tag{19}$$

HEURISTICS: a quantum spacetime foam could arise from gravitational self-interactions which need not involve Newton's constant G describing the gravitational coupling of matter (similar to the case of a gas of instantons in Yang–Mills theory).

Conjectures – Fundamental length

Consider a generalized **quantum phase** for spacetime dynamics [6],

$$\mathcal{I}_{\text{grav}}^{\text{general}} = \frac{-1}{16\pi\, l^2} \int \mathrm{d}^4x\, \sqrt{|g|}\, (R + 2\lambda) + \frac{G/c^3}{l^2} \int \mathrm{d}^4x\, \sqrt{|g|}\, \mathcal{L}_{\text{matter}}^{\text{class.}}\,, \qquad (20)$$

which reproduces the classical Einstein equations but contains a new fundamental length l.

This suggests that, as far as spacetime is concerned, the role of Planck's constant $\hbar$ would be replaced by the squared length l^2, which might loosely be called the **quantum of area**.

Planck's constant $\hbar$ would continue to play a role in the description of the matter quantum fields.

But, with $\hbar$ and l^2 being **logically independent**, it is possible to consider the "limit" $\hbar \to 0$ while keeping l^2 fixed.

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[6] FRK, JETPL **86**, 73 (2007), gr-qc/0703009.

Conjectures – Fundamental length

Table 2: Fundamental dimensionful constants of nature, including the hypothetical quantum of area l^2.

quantum matter	classical relativity	quantum spacetime
$\hbar$	$c\,,\,G$	l^2

Here, we have considered only the 2nd and 3rd columns of Table 2 and leave the unified treatment of all columns to a future theory.

In that theory, "classical gravitation" may perhaps **emerge** from the combined quantum effects of matter and spacetime, giving the "large" Newton gravitational constant

$$G = f\, c^3\, l^2/\hbar\,, \tag{21}$$

as ratio of "small" quantum constants, with calculable numerical factor f.

Conjectures – Fundamental length

Return to the generalized action (20), possibly relevant for quantum spacetime as probed by classical matter.

Conjecture 1b: *the quantum spacetime length scale l is related to a nonvanishing cosmological constant or vacuum energy density.*

For the case of the early universe, with a vacuum energy density $\rho_{\text{vac}} \equiv E_{\text{vac}}^4$, it can be argued [6] that the following holds ($c = \hbar = 1$):

$$l \stackrel{?}{\sim} E_{\text{Planck}}/E_{\text{vac}}^2 \approx 2 \times 10^{-29}\,\text{m} \left(\frac{E_{\text{Planck}}}{10^{19}\,\text{GeV}}\right)\left(\frac{10^{16}\,\text{GeV}}{E_{\text{vac}}}\right)^2, \quad (22)$$

where the Planck energy scale is given by $E_{\text{Planck}} \equiv 1/l_{\text{Planck}}$ and the numerical value for E_{vac} has been identified with the “grand-unification” scale suggested by elementary particle physics.

Conjectures – Fundamental length

If (22) holds true with $l_{\text{Planck}}/l \sim 10^{-6}$, it is perhaps possible to have sufficiently rare defects left-over from the crystallization process of classical spacetime from the initial quantum spacetime foam.

With average spacetime defect size $\widetilde{b}$ set by l_{Planck} (matter related) and average defect separation $\widetilde{l}$ set by l (vacuum related), these spacetime defects would give the following excluded-volume factor in the modified photon dispersion relation (8b):

$$\widetilde{F} \equiv \left(\widetilde{b}/\widetilde{l}\right)^4 \stackrel{?}{\sim} 10^{-24}, \tag{23}$$

which is close to saturating the current UHECR bound,

$$\left(\widetilde{b}/\widetilde{l}\right)^4 \Big|^{\text{Fly's Eye}} \lesssim 3 \times 10^{-23}\,. \tag{24}$$

Conjectures – Cosmological constant

A.2 Cosmological constant

A different line of reasoning (motivated by "emerging symmetries" ideas) tries to explain the three cosmological constant problems:

1. why is $|\rho_{\text{vac}}| \ll (E_{\text{Planck}})^4$?
2. why is $\rho_{\text{vac}} \neq 0$?
3. why is now $\rho_{\text{vac}} \sim \rho_{\text{matter}}$?

Taking Lorentz-invariance seriously (cf. UHECR discussion in Sec. 3.3), a new idea on this famous problem is as follows [5]:

Conjecture 2: *The perfect quantum vacuum behaves as a self-sustained Lorentz-invariant medium with a new type of conserved charge.*

Argument is based solely on thermodynamics (cf. Einstein 1907) and has an analog in condensed-matter physics, the Larkin–Pikin effect (1969). Work in progress on the expanding (and accelerating!) universe [7].

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[5] FRK & G. Volovik, PRD **77**, 085015 (2008), arXiv:0711.3170 [gr-qc].

[7] FRK, arXiv:0803.0281 [gr-qc]; FRK & G. Volovik, in preparation.