Chiral anomalies in superfluid hydrodynamics

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YN, Yaron Oz - work in progress

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Hydrodynamics

- Description of matter in local equilibrium. Focuses on conserved currents - $T^{\mu\nu}$, J_a^μ.
- Constitutive relations - formulas for $T^{\mu\nu}$ and J_a^μ in terms of basic variables u^μ, T, μ_a.
- Gradient expansion.
  Zeroth order - ideal fluid. First order - transport terms.

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Examples of transport terms

- Shear viscosity - $T_\pi^{\mu\nu} = -2\eta\pi^{\mu\nu}$.
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Two approaches for finding transport terms

- Second law of thermodynamics - $\nabla_\mu s^\mu \geq 0$.
- Kubo formulas - 2-point correlators in thermal QFT.

Hydrodynamics and QFT

Field theory allows new features for currents

- Chirality - $\epsilon^{\mu\nu\rho\sigma}$.
- Anomalies - $\nabla_\mu J^\mu_a \neq 0$.
- Spontaneously broken currents - superfluidity.
- (Non-abelian currents.)

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- Nuclear and QCD fluids - neutron stars, early universe, heavy ion collisions...
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Task:

- Find the transport terms that follow from these new possibilities.

Anomalies in normal fluid

Son,Surowka 0906.5044, YN,Oz 1011.5107, Amado et.al. 1102.4577

Chiral magnetic effect

$$J_B^{a\mu} = B_b^\mu \left(C^{abc}\mu_c - \frac{n^a}{\epsilon+p}\frac{1}{2}C^{bdc}\mu_c\mu_d \right)$$

Chiral vortical effect

$$J_\omega^{a\mu} = \omega^\mu \left(C^{abc}\mu_b\mu_c - \frac{n^a}{\epsilon+p}\frac{2}{3}C^{bcd}\mu_b\mu_c\mu_d \right)$$

C_{abc} - JJJ anomaly coefficient. Omitted the terms from JTT anomaly.

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- $J_{B,\omega}^{a\mu} = j^{a\mu} - \frac{n^a}{\epsilon+p} j'^\mu$
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Nice structure emerges

- $\partial j^{a\mu}/\partial\mu_b = C^{abc}(B_c^\mu + 2\mu_c\omega^\mu)$
- $\partial j'^\mu/\partial\mu_a = C^{abc}\mu_b(B_c^\mu + 2\mu_c\omega^\mu)$

Superfluid hydrodynamics

Framework

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- Additional variable - the vacuum phase gradient ξ^a_μ.
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Transport terms in general case

- Work in progress, spurred by holography.

New chiral terms in superfluids

Bhattachary(y)a,Minwalla,Yarom 1105.3733

Results from second-law constraints

- Corrections to $T^{\mu\nu}$, J_a^μ and to the Josephson condition.
- The corrections involve $\epsilon^{\mu\nu\rho\sigma}u_\nu\xi_\rho$ - vanish in collinear limit.
- Proportional to $\pi_{\mu\nu}$ or to $E_\sigma^a - T\nabla_\sigma\frac{\mu_a}{T}$.
- Also, some unspeakable things.

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In particular - Chiral Electric Conductivity

$$J_E^{a\mu} = \chi^{abc} \epsilon^{\mu\nu\rho\sigma} u_\nu \xi_\rho^c \left(E_\sigma^b - T\nabla_\sigma \frac{\mu_b}{T} \right); \quad \chi_{abc} = \chi_{bac}$$

Identifying the anomaly in the new terms

Educated guess: $J_E^{a\mu}$ arises from the JJJ anomaly

Precise form of the coefficient strongly hinted at by the existing results:

$$J_E^{a\mu} = C^{cde}\left(\delta_d^a - \frac{n^a\mu_d}{h}\right)\left(\delta_e^b - \frac{n^b\mu_e}{h}\right)\epsilon^{\mu\nu\rho\sigma}u_\nu\xi_\rho^c\left(E_\sigma^b - T\nabla_\sigma\frac{\mu_b}{T}\right)$$

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Structural components of the guess

- Upgrade $\mu^a u_\mu$ in the anomalous normal-fluid result to ξ^a_μ.
- $B^\mu_a + 2\mu_a\omega^\mu$ and $E^a_\mu - \nabla_\mu\mu^a - \mu^a a_\mu \approx \left(\delta^a_b - \frac{\mu^a n_b}{h}\right)\left(E^b_\mu - T\nabla_\mu\frac{\mu^b}{T}\right)$ are the magnetic and electric fields for the gauge potential $A^a_\mu + \mu^a u_\mu$.

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