Comments on CPT

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Source: "CPT Violation Does Not Lead to Violation of Lorentz Invarience and Vice Versa" by M.Chaichian, A.Dolgov, VN, A.Tureanu.

- We present a class of interacting nonlocal quantum field theories, in which the CPT invariance is violated while the Lorentz invariance is not !!
- We rule out a previous claim in the literature that the CPT violation implies the violation of Lorentz invariance!!
- Violation of Lorentz invariance does not lead to the CPT violation!!

Introduction

- Lorentz symmetry and the CPT invariance are two of the most respectable symmetries in Nature
- The individual symmetries, C, P and T, have been observed to be violated.
- Combined product, CPT, remarkably remains still as an exact symmetry.

- Prehistory of *CPT*
  J.Schwinger
- First Proof of *CPT*
  Lüders and Pauli (Bell?) within the Hamiltonian formulation of quantum field theory with local and Lorentz invariant interaction.
- General Proof of *CPT*
  Jost within the axiomatic formulation of quantum field theory. The "local commutativity" condition was relaxed to "weak local commutativity".
- Lorentz symmetry has been an essential ingredient of the proof, both in the Hamiltonian QFT and in the axiomatic QFT.

- Violation of Lorentz symmetry and CPT was considered in literature for decades. A long list of references includes Coleman,Glashow, Okun,Colladay,Kostelecky, Cohen,Lehner ...
- Relation between the CPT and Lorentz invariance.
  Does the violation of any of symmetry automatically imply the violation of the other one?
- This issue has recently become a topical one due to the growing phenomenological importance of CPT violating scenarios in neutrino physics and in cosmology.

- Different masses for neutrino and antineutrino. First phenomenological consideration by Murayama and Yanagida(2001).(See *MINOS* data (2010)).
- CPT-violating quantum field theory with a mass difference between neutrino and antineutrino
  First by Barenboim et al (2001) and later by Greenberg (2002) .
- Greenberg conclusion: CPT violation implies violation of Lorentz invariance.
- This result was given as a "theorem". The dispute on the validity of the theorem is the subject of this talk.

CPT-violating free field model

- Bose commutation relations for particle $a(p), a^+(p^{'})$ with mass m;
- Bose commutation relations for antiparticle $b(p), b^+(p^{'})$ with masse $\tilde{m}$
- Hamiltonian as a sum over free oscillators

Greenberg arguments

- Propagator is not Lorentz covariant, unless the masses of particle and antiparticle coincide.
- Theory is nonlocal and acausal: the $\Delta(x, y)$-function, i.e. the commutator of two fields, does not vanish for space-like separation, unless the two masses are the same, thus violating the Lorentz invariance.
- These arguments support a general “theorem” that interacting fields that violate CPT symmetry necessarily violate Lorentz invariance.

Not a QFT

- I would like to point out that such theory can not be considered as a quantum field theory.
- There atre no equation of motion. Conjugate momenta do not exist and, as a result, there are no canonical equal-time commutation relations
- "Free fields" separated by a space-like distance do not commute. They do not anticommute as well.
- One has no rule whether to apply commutation or anticommutation relations in quantizing the fields!
- There is no concept of spin to start with altogether.

CPT-violating, Lorentz invariant non-local model

We propose a model

- which preserves Lorentz invariance
- and breaks the CPT symmetry through a (nonlocal) interaction.
- Free field theory is a local one.
- Nonlocal field theories appear, in general, as effective field theories of a larger theory.

Consider a field theory with the nonlocal interaction Hamiltonian of the type

$$\mathcal{H}_{int}(x) = g \int d^4y \phi^*(x)\phi(x)\phi^*(x)\theta(x_0 - y_0)\theta((x-y)^2)\phi(y) + h.c., \quad (1)$$

where $\phi(x)$ is a Lorentz-scalar field and θ is the Heaviside step function, with values 0 or 1, for its negative and positive argument, respectively. The combination $\theta(x_0 - y_0)\theta((x-y)^2)$ in (1) ensures the Lorentz invariance, i.e. invariance under the proper orthochronous Lorentz transformations, since the order of the times x_0 and y_0 remains unchanged for time-like intervals, while for space-like distances the interaction vanishes.

Also, the same combination makes the nonlocal interaction causal at the tree level, which dictates that there is no interaction when the fields are separated by space-like distances and thus there is a maximum speed of $c = 1$ for the propagation of information.
On the other hand, it is clear that C and P invariance are trivially satisfied in (1), while T invariance is broken due to the presence of $\theta(x_0 - y_0)$ in the integrand.

One can always insert into the Hamiltonian (1), without changing its symmetry properties, a weight function or form-factor $F((x-y)^2)$, for instance of a Gaussian type:

$$F = \exp\left(-\frac{(x-y)^2}{l^2}\right), \tag{2}$$

with l being a nonlocality length in the considered theory. Such a weight function would smear out the interaction and would guarantee the desired behaviour of the integrand in (1); in the limit of fundamental length $l \to 0$ in (2), the Hamiltonian (1) would correspond to a local, CPT- and Lorentz-invariant theory.

A weight function such as (2) would make the acausality of the model (see the next section) restricted only to very small distances, of the order of l. The latter could be looked upon as being a characteristic parameter relating the effective field theory to its parent one, for instance the radius of a compactified dimension when the parent theory is a higher-dimensional one. Furthermore, with such a weight function, the interaction vanishes at infinite $(x-y)^2$ separations and thus one can envisage the existence of in- and out-fields.

There exists a whole class of such CPT-violating, Lorentz invariant field theories involving different, scalar, spinor or higher-spin interacting fields. Typical simplest examples are:

$$\mathcal{H}_{int}(x) = g_1 \int d^4y \phi_1^*(x)\phi_1(x)\theta(x_0 - y_0)\theta((x-y)^2)\phi_2(y) + h.c., \quad (3)$$

$$\mathcal{H}_{int}(x) = g_2 \int d^4y \, \bar{\psi}(x)\psi(x)\theta(x_0 - y_0)\theta((x-y)^2)\phi(y) + h.c., \quad (4)$$

$$\mathcal{H}_{int}(x) = g_3 \int d^4y \, \phi(x)\theta(x_0 - y_0)\theta((x-y)^2)\phi^2(y) + h.c. \quad (5)$$

Quantum theory of nonlocal interactions

The S-matrix in the interaction picture is obtained as solution of the Lorentz-covariant Tomonaga- Schwinger equation :

$$i\frac{\delta}{\delta\sigma(x)}\Psi[\sigma] = \mathcal{H}_{int}(x)\Psi[\sigma]\,, \tag{6}$$

with σ a space-like hypersurface, and the boundary condition:

$$\Psi[\sigma_0] = \Psi\,. \tag{7}$$

where $\mathcal{H}_{int}$ is for instance the Hamiltonian (5) with the fields in the interaction picture. Then Eq. (6) with the boundary condition (7) represent a well-posed Cauchy problem.

The existence of a unique solution for the Tomonaga-Schwinger equation is ensured if the integrability condition

$$\frac{\delta^2\Psi[\sigma]}{\delta\sigma(x)\delta\sigma(x')} - \frac{\delta^2\Psi[\sigma]}{\delta\sigma(x')\delta\sigma(x)} = 0, \tag{8}$$

with x and x' on the surface σ, is satisfied. The integrability condition (8), inserted into (6), requires that the commutator of the interaction Hamiltonian densities vanishes at space-like separation:

$$[\mathcal{H}_{int}(x), \mathcal{H}_{int}(y)] = 0\,, \quad \text{for } (x-y)^2 < 0\,. \tag{9}$$

Since in the interaction picture the field operators satisfy free-field equations, they automatically satisfy Lorentz invariant commutation rules. The Lorentz invariant commutation relations are such that (9) is fulfilled only when x and y are space-like separated, $(x-y)^2 < 0$, i.e. when σ is a space-like surface. As a result, the integrability condition (9) is equivalent to the microcausality condition for local relativistic QFT. When the surfaces σ are hyperplanes of constant time, the Tomonaga-Schwinger equations reduce to the single-time Schrödinger equation.

Inserting the expression (5) into (9), we have:

$$[\mathcal{H}_{int}(x), \mathcal{H}_{int}(y)] =$$

$$= \int d^4a d^4b \theta((x-a)^2)\theta(x^0-a^0)\theta((y-b)^2)\theta(y^0-b^0)\times$$

$$\times \left[\phi(x)\phi^2(a) + h.c., \phi(y)\phi^2(b) + h.c.\right]. \qquad (10)$$

The commutator on the r.h.s. will open up into a sum of products of field at the points x, y, a, b, multiplied by commutators of free fields like $[\phi(x), \phi(y)]$, $[\phi(x), \phi(b)]$, $[\phi(a), \phi(y)]$, $[\phi(a), \phi(b)]$.

In order for the commutator (10) to vanish, all the coefficients of the products of fields in the expansion have to vanish, since the fields at different space-time points are independent. Clearly, the terms with the coefficient $\Delta(x-y) = [\phi(x), \phi(y)]$ vanish for $(x-y)^2 < 0$. However, the commutator (10) does not vanish for $(x-y)^2 < 0$. In order to show this, is is enough to show that one independent product of fields has nonzero coefficient. Let us consider the products which contain the fields $\phi(x), \phi(y), \phi(a), \phi(b)$.

A straightforward calculation shows that the terms containing these fields are:

$$\int d^4a\, d^4b\; \theta((x-a)^2)\theta(x^0-a^0)\theta((y-b)^2)\theta(y^0-b^0)\times$$

$$2\Delta(a-b)\{\phi(a),\phi(b)\}\phi(x)\phi(y)+h.c. \qquad (11)$$

A closer study of the expression (11) shows that it does not vanish at space-like distances between x and y and thus the causality condition (9) is not satisfied.

This, in turn, implies that the field operators in the Heisenberg picture, $\Phi_H(x)$ and $\Phi_H(y)$, do not satisfy the locality condition

$$[\Phi_H(x), \Phi_H(y)] = 0, \qquad \text{for } (x-y)^2 < 0, \tag{12}$$

when the quantum corrections are taken into account. This is in accord with the requirement of locality condition (12) for the validity of CPT theorem both in the Hamiltonian proof (Luders,Pauli)and as well in the axiomatic one (Jost, Bogoliubov), taking into account that there is no example of a QFT, which satisfies the weak local commutativity condition (WLC) but not the local commutativity (LC).

Lorentz-invarinace violating but CPT-invariant quantum field theories: Reciprocal theorem

During the last decade, we have learned that the violation of Lorentz invariance does not necessarily lead to the violation of the CPT theorem. The example comes from the quantum field theory on noncommutative space-time (NC QFT) with the canonical, Heisenberg-like, commutation relations for coordinate operators:

$$[x^\mu, x^\nu] = i\theta^{\mu\nu}, \tag{13}$$

with $\theta^{\mu\nu}$ an antisymmetric constant matrix.

In this case, by the nature of the above noncommutativity parameter $\theta^{\mu\nu}$ being a constant but not a tensor, Lorentz invariance is broken, but not the CPT symmetry. Translational invariance is valid. In addition to the Lorentz invariance violation, such NC QFTs are nonlocal in the noncommuting coordinates. However, the Lorentz symmetry violation is of a very particular form, and invariance under the stability group of the matrix $\theta^{\mu\nu}$ is preserved under the so-called residual symmetry $SO(2) \times SO(1,1)$. This reduced symmetry is enough to prove the CPT theorem only for the scalar fields (for which the C operation is a simple Hermitian conjugation) on the noncommutative space-time (13).

A full proof of the CPT theorem in Lorentz-violating noncommutative quantum field theory, however, could be achieved only by using the twisted Poincaré symmetry which these theories possess. The twisted Poincaré invariance is a deformation of the Poincaré symmetry, considered as a Hopf algebra, a concept coming from the theory of quantum groups, as compared with the Lie algebra. The irreducible representations of twisted Poincaré are identical to those of the usual Poincaré algebra, i.e. labeled by the mass and spin of the particles.

Therefore, the meaning of the charge conjugation has survived intact in the noncommutative quantum field theories. While parity and time reversal symmetries can be defined with any concept of space and time, the notion of charge conjugation has meaning only in the framework of Lorentz symmetry. Antiparticles are a consequence of special relativity. Particle and antiparticle are in the same irreducible representation of the Poincaré group. The CPT theorem is thus strongly connected to the Poincaré group representations, and not so much to the Lorentz symmetry, as the validity of the CPT theorem in the noncommutative space-time shows.

Conclusion

- We have presented a very simple class of interacting nonlocal quantum field theories, which violate CPT invariance and preserve Lorentz invariance.
- This result invalidates a general claim made previously by Greenberg, that "CPT violation implies violation of Lorentz invariance".
- Violation of Lorentz invariance does not necessarily lead to CPT violation.