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CPV in b→s penguins and T,CPT violation at BABAR and BELLE





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OUTLINE

- Introduction: B factories, datasets , and analyses method
- Direct observation of time reversal violation
- Search for time-dependent CPT violation
- Study of rare b \rightarrow s penguin decays
 - B \rightarrow KKK CPV analyses and over the Dalitz plot
 - $B^0 \rightarrow K^0_{\ S} K^0_{\ S} K^0_{\ S}$, $B^0 \rightarrow K^0_{\ S} K^+ K^-$, $B^+ \rightarrow K^0_{\ S} K^0_{\ S} K^+$, $B^+ \rightarrow K^+ K^+ K^-$
 - $B \rightarrow K^{(*)}|^{+}|^{-}$



NEW!



The Beauty factories



3.1 GeV e⁺ and 9 GeV e⁻ beams

 $\int Ldt \sim 433 \text{ fb}^{-1}@Y(4S), 550 \text{ fb}^{-1} \text{ total (Off, Y(ns))}$ ~470 *M BB*, all used in analyses shown here

3.5 GeV e⁺ and 8 GeV e⁻ beams

711 fb⁻¹@Y(4S), > 1 ab⁻¹ total (Off, Y(ns)) ~772 $M B\overline{B}$, 535 $M B\overline{B}$ used in CPT analysis

Analyses methods

Time dependent measurement



•Kinematical identification with

- $m_{ES} = \sqrt{E_{beam}^{*2} p_{B}^{*2}}$ (Beam energy substituted mass)
- $\Delta E = E_B^* E_{beam}^*$ (Energy difference)

•Event-shape variables combined in a neural network or Fisher discriminant to suppress jet-like continuum events

BB

Jet-like

'Spherical'

Direct observation of time reversal violation (1)



Direct observation of time reversal violation (2)

EPR entanglement from Y(4S)	Semileptonic decay projects:
$ i\rangle = \frac{1}{\sqrt{2}} [B^{0}(t_{1})\overline{B}^{0}(t_{2}) - \overline{B}^{0}(t_{1})B^{0}(t_{2})] = \frac{1}{\sqrt{2}} [B_{+}(t_{1})B_{-}(t_{2}) - B_{-}(t_{1})B_{+}(t_{2})] $	$B^{0} \text{ with } l^{+}, \bar{B}^{0} \text{ with } l^{-}$ $J/\Psi K_{L} \text{ projects } CP \text{ even } B_{+} = \frac{1}{\sqrt{2}} [B^{0} + \bar{B}^{0}]$ $J/\Psi K_{S} \text{ projects } CP \text{ odd } B_{-} = \frac{1}{\sqrt{2}} [B^{0} - \bar{B}^{0}]$

Final state (X, Y), one $B^0 or \overline{B}^0$, and one CP state $B_+ or B_-$, with decay time $t_x < t_y$

Physical proces Reference (X,Y)	s / Reco Final state	Physical proc T transformed	ess / Reco Final state d (X,Y)
$B^0 \rightarrow B_+$	Γ, J/Ψ K _L	$B_+ \rightarrow B^0$	J/ Ψ K $_{\rm S}$, I $^+$
$B^0 \rightarrow B$	Γ, J/Ψ K _s	$B \rightarrow B^0$	J/ Ψ K $_{L}$, I+
$\overline{B}{}^{0} \rightarrow B_{+}$	I+, J/Ψ K _L	$B_+ \rightarrow \overline{B}{}^0$	J/ΨK _s , ⊢
$\overline{B}^0 \to B$	Ι+, J/Ψ K _s	$B \rightarrow \overline{B}{}^0$	J/ΨK _L , ⊢

4 independent T comparisons (as 4 CP and 4 CPT comparisons)

T implies comparison of :

- 1. Opposite Δt sign.
- 2. Different reco states $(J/\Psi K_s vs. J/\Psi K_L)$.
- 3. Opposite tag states (B^0 vs \overline{B}^0).

Direct observation of time reversal violation (3)

Signal model

Assumes $\Delta \Gamma_d$ =0 but does NOT assume CPT

(for perfect time reconstruction: formula needs to be corrected for time resolution)

$$g_{\alpha,\beta}^{\pm}(t_{true}) \propto e^{-\Gamma|\tau|} \{1 + S_{\alpha,\beta}^{\pm} \sin(\Delta m_d \mid t_{true} \mid) + C_{\alpha,\beta}^{\pm} \cos(\Delta m_d \mid t_{true} \mid)\}$$
$$\alpha \in \{B^{\circ}, \overline{B}^{\circ}\}; \ \beta \in \{K_s, K_L\} \qquad \Delta t = t_{CP} - t_{flav}$$

The signal model has 8 different sets of S, C parameters

$$2 \Delta t \ (\Delta t > 0, \Delta t < 0) \quad \times \quad 2 f lav \ (B^0, \overline{B}^0) \quad \times \quad 2 \ CP \ (K_S, K_L)$$

To be compared to the usual CPV studies : one single set $g_{\alpha,\beta}(\Delta t) \propto e^{-\Gamma |\Delta t|} \{1 \pm [\eta_f S \sin(\Delta m_d \Delta t) + C \cos(\Delta m_d \Delta t)]\}$ Assumes $\Delta \Gamma_d = 0$ and CPT In SM and CKM formalism: S ~ sin 2 β (BABAR) = sin 2 ϕ_1 (Belle) ~ 0.7

Fit Method:

- 1. Simultaneous ML fit to B^0 , \overline{B}^0 , $c\overline{c}K_S$, and $J/\psi K_L$ for $\Delta t > 0$ and $\Delta t < 0$ events.
- 2. Obtain the 8 sets of S, C parameters, and from these, we define T, CP and CPT violating parameters ΔS , ΔC .

Direct observation of time reversal violation (4)

Illustrative : 4 independent T violating asymmetries Include experimental reconstruction effects.

Neglecting reconstruction effects : $A_T \approx \frac{1}{2} \left[\Delta S_T^{\pm} \sin(\Delta m |\Delta t|) + \Delta C_T^{\pm} \cos(\Delta m |\Delta t|) \right]$



BABAR Preliminary

Direct observation of time reversal violation (5)

BABAR Preliminary

-2 In ΔL scan with systematics included



- **T violation observed !**
- Due to CP violation in the interference between the decay with/wo B mixing ($\Delta S \neq 0$), but not directly in the decay (ΔC consistent with 0).
- No CPT violation seen

T violation seen with 14 σ significance

CP violation seen with 16.6 σ significance No CPT violation: 0.33 σ significance

BABAR Preliminary

Parameter	Result
$\Delta S_{\mathrm{T}}^{+} = S_{\ell^{-}X, J/\psi K_{L}^{0}}^{-} - S_{\ell^{+}X, c\overline{c}K_{S}^{0}}^{+}$	$-1.37 \pm 0.14 \pm 0.06$
$\Delta S_{\mathrm{T}}^{-} = S_{\ell^{-}X, J/\psi K_{L}^{0}}^{+} - S_{\ell^{+}X, c\overline{c}K_{S}^{0}}^{-}$	$1.17 \pm 0.18 \pm 0.11$
$\Delta C_{\mathrm{T}}^{+} = C_{\ell^{-}X, J/\psi K_{L}^{0}}^{-} - C_{\ell^{+}X, c\overline{c}K_{S}^{0}}^{+}$	$0.10 \pm 0.16 \pm 0.08$
$\Delta C_{\mathrm{T}}^{-} = C_{\ell^{-}X, J/\psi K_{L}^{0}}^{+} - C_{\ell^{+}X, c\overline{c}K_{S}^{0}}^{-}$	$0.04 \pm 0.16 \pm 0.08$
$\Delta S_{\rm CP}^+ = S_{\ell^- X, c\overline{c}K_S^0}^+ - S_{\ell^+ X, c\overline{c}K_S^0}^+$	$-1.30 \pm 0.10 \pm 0.07$
$\Delta S_{\rm CP}^- = S_{\ell^- X, c\overline{c}K_{\rm S}^0}^ S_{\ell^+ X, c\overline{c}K_{\rm S}^0}^-$	$1.33 \pm 0.12 \pm 0.06$
$\Delta C_{\rm CP}^+ = C_{\ell^- X, c\overline{c}K_S^0}^+ - C_{\ell^+ X, c\overline{c}K_S^0}^+$	$0.07 \pm 0.09 \pm 0.03$
$\Delta C^{\rm CP} = C^{\ell^- X, c\overline{c}K^0_S} - C^{\ell^+ X, c\overline{c}K^0_S}$	$0.08 \pm 0.10 \pm 0.04$
$\Delta S^+_{\rm CPT} = S^{\ell^+ X, J/\psi K^0_L} - S^+_{\ell^+ X, c\overline{c}K^0_S}$	$0.16 \pm 0.20 \pm 0.09$
$\Delta S^{\rm CPT} = S^+_{\ell^+ X, J/\psi K^0_L} - S^{\ell^+ X, c\overline{c}K^0_S}$	$-0.03 \pm 0.13 \pm 0.06$
$\overline{\Delta C^+_{\rm CPT}} = C^{\ell^+ X, J/\psi K^0_L} - C^+_{\ell^+ X, c\overline{c} K^0_S}$	$0.15 \pm 0.17 \pm 0.07$
$\Delta C^{\rm CPT} = C^+_{\ell^+ X, J/\psi K^0_L} - C^{\ell^+ X, c\overline{c} K^0_S}$	$0.03 \pm 0.14 \pm 0.08$



Search for time dependent CPT violation (1)



 PRD 85, 071105(R) (2012)
 535 M BB

Measurement of CPT violating z parameter and $\Delta\Gamma_d/\Gamma_d$ Im(z) and /or Re (z) $\neq 0 \Leftrightarrow$ CPT violation in $B^0 - \overline{B}^0$ mixing Mass eigenstates:

$$\begin{aligned} |B_L\rangle &= p \sqrt{1 - \mathbf{z}} |B^0\rangle + q \sqrt{1 + \mathbf{z}} |\overline{B^0}\rangle & \Delta \Gamma_{\mathsf{d}} = \Gamma_{\mathsf{H}} - \Gamma_{\mathsf{L}} \\ |B_H\rangle &= p \sqrt{1 + \mathbf{z}} |B^0\rangle - q \sqrt{1 - \mathbf{z}} |\overline{B^0}\rangle & \Gamma_{\mathsf{d}} = (\Gamma_{\mathsf{H}} + \Gamma_{\mathsf{L}})/2 \end{aligned}$$

Flavor tag one B meson using semileptonic decay and reconstruct second B meson into CP or flavor final states:

- $c\bar{c}s$ CP eigenstates : J/ Ψ K⁰_{S.L}
- Flavor state : $B^0 \rightarrow D^- \pi^+$, $D^{*-} \pi^+$, $D^{*-} \rho^+$, $D^{*-} I^+ \nu_{I}$

Previous BABAR and Belle measurements use dilepton events (2 semileptonic decays). Most precise previous measurement from BABAR with 232 M $B\overline{B}$: « Search for T, CP, and CPT violation in $B^0\overline{B}^0$ mixing with inclusive Dilepton events »

PRL 96, 251802 (2006).

This new Belle measurement improves the overall precision by factors of 1.3 to 2.0 for all parameters. $$_{
m 10}$$

Search for time dependent CPT violation (2)



Δt distribution with CP and CPT violation, applies to any neutral B decay

$$P(\Delta t, f_{rec}, f_{tag}) = \frac{\Gamma_d}{2} e^{-\Gamma_d |\Delta t|} \times \left[\frac{|a_+|^2 + |a_-|^2}{2} \cosh \frac{\Delta \Gamma_d}{2} \Delta t - Re(a_+a_-^*) \sinh \frac{\Delta \Gamma_d}{2} \Delta t + \frac{|a_+|^2 - |a_-|^2}{2} \cos \Delta m_d \Delta t - Im(a_+a_-^*) \sin \Delta m_d \Delta t \right]$$

$$a_+ = \bar{A}_{tag} A_{rec} - A_{tag} \bar{A}_{rec};$$

$$a_- = \sqrt{1 - \mathbf{z}^2} \left(\frac{p}{q} A_{tag} A_{rec} - \frac{q}{p} \bar{A}_{tag} \bar{A}_{rec} \right) + \mathbf{z} \left(\bar{A}_{tag} A_{rec} + A_{tag} \bar{A}_{rec} \right)$$

$$A_{tag} = \langle f_{tag} | H_d | B^0 \rangle; \ \bar{A}_{tag} = \langle f_{tag} | H_d | \bar{B}^0 \rangle; \ A_{rec} = \langle f_{rec} | H_d | B^0 \rangle; \ \bar{A}_{rec} = \langle f_{rec} | H_d | \bar{B}^0 \rangle;$$

Simultaneous ML fit of Δt distributions for different decay modes to extract main physics parameters : **Re (z), Im (z),** $\Delta \Gamma_d / \Gamma_d$ and τ_{B^0} , τ_{B^+} , Δm_d , $|\lambda_{CP}|$, $\arg(\lambda_{CP}\eta_{CP})$ With $\lambda_{CP} = \frac{q}{p} \times \frac{A(B^0 \to f_{CP})}{A(\bar{B}^0 \to f_{CP})}$ and Im $(\eta_{CP} \lambda_{CP})^{\sim} \sin(2\phi_1) = \sin(2\beta)$

B _{rec} decay modes (events counts)	Sensitive mainly to
$J/\psi K_{S}^{0}$ (7,713) $J/\psi K_{L}^{0}$ (10,966)	Re(z) and $\Delta \Gamma_{\rm d} / \Gamma_{\rm d}$
$D^{-}\pi^{+}$ (39,366) ; $D^{*-}\pi^{+}$ (46,292) $D^{*-}\rho^{+}$ (45,913) ; $D^{*-}l^{+}\nu_{l}$ (383,818)	lm (z)

Search for time dependent CPT violation (3)



Consistent with zero

Systematic errors

Source	$\delta(\mathcal{R}e(z))$	$\delta(\mathcal{I}m(z))$	$\delta(\Delta\Gamma_d/\Gamma_d)$
Vertex reconstruction	0.008	0.0028	0.009
Δt -resolution function	0.003	0.0004	0.002
Tag-side interference	0.028	0.0006	0.001
CSD effect	0.004	0.0008	0.003
Fit bias	0.012	0.0013	0.005
Signal fraction	0.004	0.0002	0.002
Background Δt shape	0.005	0.0001	0.002
Others	0.001	< 0.0001	0.002
Total	0.033	0.0033	0.011



BELLE

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B→KKK decays



- Modes dominated by penguin diagram : sensitive to new physics
- Related to measurement of $sin(2\beta_{eff})$
- Study of Dalitz plot structure : also look for controversial resonance f_X (1500), wide structure in K⁺K⁻ mass seen in past $B^0 \rightarrow K^0_S K^+ K^-$ and $B^+ \rightarrow K^+ K^+ K^-$ BABAR and Belle measurements. Found unnecessary according to new analyses shown here.

1) $B^0 \rightarrow K^0_s K^0_s K^0_s$

PRD 85, 054023 (2012)

« Amplitude analysis and measurement of the time-dependent CP asymmetry »

- First results on Dalitz plot structure in this final state (time integrated)
- Improved measurement of time CP-violation parameters in this decay.

2) $B^0 \rightarrow K^0_{\ S} K^+ K^-$, $B^+ \rightarrow K^0_{\ S} K^0_{\ S} K^+$, $B^+ \rightarrow K^+ K^- K^-$

arXiv:1201.5897v2 [hep-ex] Accepted by PRD

« Study of CP violation in Dalitz plot analyses»

• First ever Dalitz plot of $B^+ \rightarrow K^0_S K^0_S K^+$



PRD 85, 054023 (2012)

 $B^0 \rightarrow K^0 \, K^0 \, K^0 \, K^0$

- CP eigenstate so CPV study possible without looking at Dalitz plot
- $3 K^0{}_S \rightarrow \pi^+\pi^- \text{ or } (2 K^0{}_S \rightarrow \pi^+\pi^- \text{ and } 1 K^0{}_S \rightarrow \pi^0\pi^0)$
- Fit finds 200 ± 15 signal events among 505 candidates
- Time Dependent CP analysis : CP conservation excluded at 3.8 σ





excluded at 2.8 σ , SM predicts ~(0-4.7)%

 $B^0 \rightarrow \phi K_s$: (charmonium β =21.4 ± 0.8 deg)



Component	$\beta_{\rm eff}$ (deg)	$A_{CP}(=-C)(\%)$
$\phi(1020)K_{s}^{0}$	$21 \pm 6 \pm 2$	$-5 \pm 18 \pm 5$
$f_0(980)K_s^0$	$18 \pm 6 \pm 4$	$-28\pm24\pm9$
Other	$20.3 \pm 4.3 \pm 1.2$	$-2 \pm 9 \pm 3$

arXiv:1201.5897v2 [hep-ex] Accepted by PRD BABAR Preliminary

$A_{CP} \sim \eta_{CP} \sin(2\beta^{eff}) \times \sin(\Delta m \Delta t)$

$$sin(2\beta^{eff}) \equiv sin(2\phi_1^{eff}) \stackrel{\text{HFAG}}{\underset{\text{Moriond 2012}}{\text{PRELIMINARY}}}$$



¹⁶

G

2012



Study of $B \rightarrow K^{(*)}I^+I^-$ decays

Preliminary

Very sensitive to new physics





 $K_{s}e^{+}e^{-}, K^{+}e^{+}e^{-}, K_{s}\mu^{+}\mu^{-}, K^{+}\mu^{+}\mu^{-},$ $K^{*+}(\to K_{s}\pi^{+})e^{+}e^{-}, K^{*0}(\to K^{+}\pi^{-})e^{+}e^{-}, K^{*+}(\to K_{s}\pi^{+})\mu^{+}\mu^{-}, K^{*0}(\to K^{+}\pi^{-})\mu^{+}\mu^{-}$



Measurements versus *s* in good agreement with SM and with Belle, CDF, LHCb:

- **Partial Branching Fractions**
- **CP** asymmetries

$$\ell^+\ell^- = e^+e^-, \ \mu^+\mu^-$$
$$s \equiv (m_{\ell+\ell^-})^2$$

Lepton Flavor ratio

$$\mathcal{A}_{I}^{K^{(*)}} \equiv \frac{\mathcal{B}(B^{0} \to K^{(*)0}\ell^{+}\ell^{-}) - r_{\tau}\mathcal{B}(B^{+} \to K^{(*)+}\ell^{+}\ell^{-})}{\mathcal{B}(B^{0} \to K^{(*)0}\ell^{+}\ell^{-}) + r_{\tau}\mathcal{B}(B^{+} \to K^{(*)+}\ell^{+}\ell^{-})}$$

Isospin asymmetry: at low dilepton masses, isospin asymmetries are negative as seen by Belle, the combined BABAR/Belle $K^{(*)}\ell^+\ell^$ result indicates $\sim 3\sigma$ effect

Conclusions

• First direct observation of time reversal violation, in any system.

T violation observed with a 14 σ significance (BABAR preliminary, paper to be submitted soon) due to CP violation in the interference (B⁰ decay with/wo mixing). Different from CPLEAR - Physics Letters B 444 (1998) 43 - for which T and CP violation in $K^0 \overline{K}^0$ mixing can not be distinguished. No hint of CPT violation, that can be measured as well.

- New search for time-dependent CPT violation published by Belle. Most accurate measurement of CPT violating z parameter in $B^0\overline{B}^0$ mixing and $\Delta\Gamma_d/\Gamma_d$, all consistent with zero.
- Recent BABAR (preliminary and published) results on rare b→s penguin decays to search for new physics.
 - − Analyses of B^+ and $B^0 \rightarrow K K K$ decays over the Dalitz plot, and time dependent CP violation measurements.
 - First evidence of time-dependent CP Violation in $B^0 \rightarrow K^0_{\ S} K^0_{\ S} K^0_{\ S}$
 - Most precise measurement of $\beta^{eff}(\phi K_s) = \phi_1^{eff}(\phi K_s) = 21\pm6\pm2$ degrees
 - Indication of direct CP violation in $B^+ \rightarrow \phi K^+$ at 2.8 σ , larger than SM prediction.
 - Analysis of the electroweak penguin decays $B \rightarrow K^{(*)}|+|^{-}$. No new physics observed.





Significance of T violation only stat.

- We obtain the likelihood value of the fit to S, C for the 8 independent samples (Standard Fit).
- We repeat the fit, reassembling the parameters for T-conjugated processes, to forbid T violation.

T invariance	CP invariance	CPT invariance
$\Delta S_{\mathrm{T}}^{+} = 0$	$\Delta S_{\rm CP}^+ = 0$	$\Delta S^+_{\rm CPT} = 0$
$\Delta S_{\mathrm{T}}^{-} = 0$	$\Delta S_{CP}^- = 0$	$\Delta S_{\rm CPT}^- = 0$
$\Delta S^+_{\rm CP} = \Delta S^+_{\rm CPT}$	$\Delta S_{\rm T}^+ = \Delta S_{\rm CPT}^+$	$\Delta S_{\rm T}^+ = \Delta S_{\rm CP}^+$
$\Delta S^{\rm CP} = \Delta S^{\rm CPT}$	$\Delta S^{\rm T} = \Delta S^{\rm CPT}$	$\Delta S^{\rm T} = \Delta S^{\rm CP}$
$\Delta C_{\rm T}^+ = 0$	$\Delta C_{\rm CP}^+ = 0$	$\Delta C^+_{\rm CPT} = 0$
$\Delta C_{\rm T}^- = 0$	$\Delta C_{\rm CP}^- = 0$	$\Delta C^{\rm CPT}=0$
$\Delta C^+_{\rm CP} = \Delta C^+_{\rm CPT}$	$\Delta C_{\rm T}^+ = \Delta C_{\rm CPT}^+$	$\Delta C_{\rm T}^+ = \Delta C_{\rm CP}^+$
$\Delta C_{\rm CP}^- = \Delta C_{\rm CPT}^-$	$\Delta C_{\rm T}^- = \Delta C_{\rm CPT}^-$	$\Delta C_{\rm T}^- = \Delta C_{\rm CP}^-$

- 3. Significance of T violation $\Delta C_{CP} = \Delta C_{CF}$ evaluated from the difference of the likelihood values.
- 4. Raw asymmetries and fit projections can be now plotted in the standard way.

$$\Delta \chi^2 = -2(\ln L_{No_T_{Violation}} - \ln L)$$
$$\Delta \nu = 8$$

- 5. CP, and CPT significance is evaluated similarly.
- 6. Using Gaussian approximation, we evaluate the change of likelihood in 1σ systematic variation. $m_i^2 = -2[\ln L(q_i, o_i) - \ln L(p_0)]/s_{stat, i}^2$
- 7. We take the max $\{m_i^2\}$ and we devide our significace (s^2) by $(1 + \max\{m_i^2\})$

Signal model description



FPCP 12

$(\Delta S^{\pm}, \Delta C^{\pm})$ parameters



Systematic uncertainties

Systematic source	ΔS_{T}^{+}	ΔS _T -
misID flavour	0.019	0.019
Δt resolution function	0.02	0.05
Outlier's scale factor	0.012	-0.013
m _{ES} parameters	0.012	0.0018
ΔE parameters	0.017	0.017
K _L systematics	0.03	0.03
Differences between B _{CP} and B _{flav}	0.02	0.02
Background effects	0.03	0.04
Uncertainty on fit bias from MC	0.010	0.08
Detector and vertexing effects.	0.011	0.04
$\Delta\Gamma \neq 0$ effects	0.004	0.003
External physics parameters	0.005	0.006
Normalization effects	0.012	0.009
Total Systematics	0.06	0.11

FPCP 12

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CPLEAR - **Physics Letters B 444 (1998) 43** - used $K^0 \to \overline{K}^0$ versus $\overline{K}^0 \to K^0$, so CP and T are experimentally identical since CP $[K^0 \to \overline{K}^0] = \overline{K}^0 \to K^0 = T [K^0 \to \overline{K}^0]$

Same as measurements of |q/p| with dileptons (flavor oscillations), i.e. |q/p| is a CP and T violating parameter (in the $B^0 \overline{B}{}^0$ mixing), so we cannot conclude from it if the effect is T or CP, or both.



We cannot distinguish CP and T. Not a DIRECT observation of TRV Interference TRV searches



CPV time dependent (TD) studies:

- There are no exchanges $t \leftrightarrow -t$ and $|in \rangle \leftrightarrow |out \rangle$.
- Assumes CPT invariance and $\Delta \Gamma = 0$.

P, Villanueva-Pérez IFIC-Valencia

Dalitz plot analysis

$$\mathcal{A} \equiv \mathcal{A}(B \to KKK; m_{12}, m_{23}) = \sum_{j} a_j F_j(m_{12}, m_{23})$$
$$a_j = c_j (1 + b_j) e^{i(\phi_j + \delta_j)}$$
F; are resonant or

F_j are resonant or nonresonant lineshapes: relativistic Breit-Wigner, spin-factors, etc.

From isobar coefficients can derive: partial branching fractions, A_{CP} (= -2b/(1+b²)), β_{eff} (= β + δ), etc.

 $\bar{a_j} = c_j(1-b_j)e^{i(\phi_j-\delta_j)}$



$B^0 \rightarrow K^0_s K^0_s K^0_s$ Amplitude Analysis

- Significance estimated from change in log-likelihood when the magnitude of each resonance's amplitude is set to 0
- The fit fractions (FF) indicate a high degree of destructive interference

$$FF_j \equiv \frac{\int \int \left(|\mathcal{A}_j|^2 + |\overline{\mathcal{A}}_j|^2 \right) ds_{12} ds_{23}}{\int \int \left(|\mathcal{A}|^2 + |\overline{\mathcal{A}}|^2 \right) ds_{12} ds_{23}}$$

 Inclusive branching fraction consistent with PDG

Mode	\mathcal{B} [×10 ⁻⁶]
Inclusive $B^0 \rightarrow K^0_S K^0_S K^0_S$	$6.19 \pm 0.48 \pm 0.15 \pm 0.12$
$f_0(980)K^0_S, f_0(980) \to K^0_S K^0_S$	$2.7^{+1.3}_{-1.2}\pm0.4\pm1.2$
$f_0(1710)K_S^0, f_0(1710) \to K_S^0K_S^0$	$0.50{}^{+0.46}_{-0.24}\pm 0.04\pm 0.10$
$f_2(2010)K^0_S, f_2(2010) \to K^0_S K^0_S$	$0.54^{+0.21}_{-0.20}\pm0.03\pm0.52$
NR, $K_{S}^{0}K_{S}^{0}K_{S}^{0}$	$13.3^{+2.2}_{-2.3} \pm 0.6 \pm 2.1$
$\chi_{c0}K^0_S, \ \chi_{c0} ightarrow K^0_SK^0_S$	$0.46^{+0.25}_{-0.17} \pm 0.02 \pm 0.21$

Mode	Parameter	
$f_0(980)K_S^0$	FF	$0.44^{+0.20}_{-0.19}$
	Phase [rad]	0.09 ± 0.16
	Significance $[\sigma]$	3.0
$f_0(1710)K_S^0$	FF	$0.07 {}^{+0.07}_{-0.03}$
	Phase [rad]	1.11 ± 0.23
	Significance $[\sigma]$	3.3
$f_2(2010)K_S^0$	FF	$0.09^{+0.03}_{-0.03}$
	Phase [rad]	2.50 ± 0.20
	Significance $[\sigma]$	3.3
NR	FF 🤇	$2.16^{+0.36}_{-0.37}$
	Phase [rad]	0.0
	Significance $[\sigma]$	8.0
$\chi_{c0}K_S^0$	FF	$0.07 {}^{+0.04}_{-0.02}$
	Phase [rad]	0.63 ± 0.47 -
	Significance $[\sigma]$	3.9
	Total FF 💦 🤇	$2.84^{+0.71}_{-0.66}$

Fit Results For Amplitude Analysis

• $B^0 \rightarrow K^+ K^- K^+$ Results:

Decay mode	$\mathcal{B}(B^+ \rightarrow K^+ K^- K^+) \times FF_j (10^-$	$^{-6}) = A_{CP} (\%)$	$\Delta \phi_j$ (deg)
$\phi(1020)K^{+}$	$4.48 \pm 0.22^{+0.33}_{-0.24}$	$(12.8 \pm 4.4 \pm 1.3)$	$>23 \pm 13^{+4}_{-5}$
$f_0(980)K^+$	$9.4 \pm 1.6 \pm 2.8$	$-8 \pm 8 \pm 4$	$9 \pm 7 \pm 6$
$f_0(1500)K^+$	$0.74 \pm 0.18 \pm 0.52$		
$f'_{2}(1525)K^{+}$	$0.69 \pm 0.16 \pm 0.13$	$14 \pm 10 \pm 4$	$-2 \pm 6 \pm 3$
$f_0(1710)K^+$	$1.12 \pm 0.25 \pm 0.50$		
$\chi_{c0}K^+$	$1.12 \pm 0.15 \pm 0.06$		$-4 \pm 13 \pm 2$
NR	$22.8 \pm 2.7 \pm 7.6$	$6.0 \pm 4.4 \pm 1.9$	0 (fixed)
NR (S-wave)	$52^{+23}_{-14} \pm 27$		
NR (P-wave)	$24^{+22}_{-12} \pm 27$		

 $B^+ \rightarrow K^0_s K^0_s K^+$ Results:

Decay mode	$\left \mathcal{B}(B^+ \to K^0_S K^0_S K^+) \times FF_j \ (10^{-6}) \right $
$f_0(980)K^+$	$14.7 \pm 2.8 \pm 1.8$
$f_0(1500)K^+$	$0.42 \pm 0.22 \pm 0.58$
$f_2'(1525)K^+$	$0.61 \pm 0.21^{+0.12}_{-0.09}$
$f_0(1710)K^+$	$0.48^{+0.40}_{-0.24} \pm 0.11$
$\chi_{c0}K^+$	$0.53 \pm 0.10 \pm 0.04$
NR (S-wave)	$19.8 \pm 3.7 \pm 2.5$

• $B^0 \to K^+ K^- K_s^0$ Results:

Decay mode	$ \mathcal{B}(B^0 \to K^+ K^- K^0) \times FF_j (10^{-6}) $
$\phi(1020)K^{0}$	$3.48 \pm 0.28^{+0.21}_{-0.14}$
$f_0(980)K^0$	$7.0^{+2.6}_{-1.8} \pm 2.4$
$f_0(1500)K^0$	$0.57^{+0.25}_{-0.19} \pm 0.12$
$f_2'(1525)K^0$	$0.13^{+0.12}_{-0.08} \pm 0.16$
$f_0(1710)K^0$	$4.4 \pm 0.7 \pm 0.5$
$\chi_{c0}K^0$	$0.90 \pm 0.18 \pm 0.06$
NR	$33 \pm 5 \pm 9$
NR (S-wave)	$30 \pm 5 \pm 8$
NR (P-wave)	$3.1\pm0.7\pm0.4$