Neutrino in extreme external conditions

"Particle Physics and Cosmology"
XXIV Rencontres de Blois
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Outline

- ν electromagnetic properties

  (short review)

  - results of recent experimental searches for upper bound on μ_ν

    (GEMMA Coll. JINR-ITEP)

  - our corresponding theoretical studies of ν-e scattering

- ν quantum states in matter

  (new approach)

  Spin Light of ν in matter $SL\nu$

  ν energy quantization in matter

ν is the only known particle Beyond Standard Model ν is quite

invisible

particle

ν exhibits unexpected properties (puzzles)

W. Pauli, 1930

- neutral "neutron" $\Longrightarrow$ ν — E.Fermi, 1933
- probably $\mu_\nu \neq 0$!?
- Pauli himself wrote to Baade:

…recent claim for new experimental bound on μ_ν (with atomic ionization effect) continue chain of puzzles…

"Today I did something a physicist should never do. I predicted something which will never be observed experimentally...".

H.Bethe, R.Peierls, «The 'neutrino'»
Nature 133 (1934) 532,

- «There is no practically possible way of observing the neutrino»

- …up to now absolute value

$m_\nu \neq 0$?

… however … $\Longrightarrow$

Crucial role of neutrino

ν is a "tiny" particle :

- very light $m_{\nu_f} \ll m_f, \quad f = e, \mu, \tau$
- electrically neutral $q_\nu = 0$ $\boxed{q_\nu < 4 \times 10^{-17} e}$
- with very small magnetic moment μ_ν **?**
- weak interactions are indeed weak $\bar{\nu} + p \to e^+ + n$ $\sigma \sim 10^{-43}\ cm^2$ $\boxed{L \sim 10^{15} km}$

$\sigma_{\nu_e N} \sim 10^{-39}\ cm^2$	ν-N scattering
$\sigma_{\bar{\nu}_e p} \sim 10^{-40}\ cm^2$	inverse β-decay
$\sigma_{\nu_e e} \sim 10^{-43}\ cm^2$	ν-e scattering

$E_\nu \sim 3 MeV$ … free path in water…

at the final stages of development of particular elementary particle physics framework

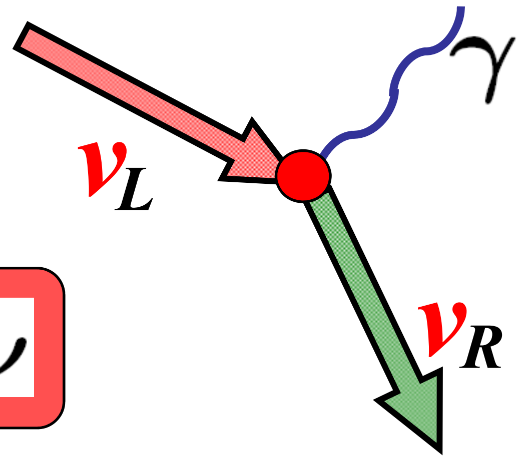
ν manifests itself most vividly under the influence of extreme external conditions:

- dense background matter

and

- strong external (electromagnetic ...) fields

2 problems:



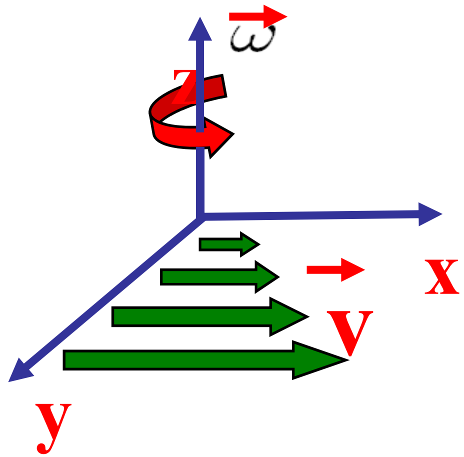
1. Spin Light of ν

in matter

$SL\nu$

2. ν energy quantization

in rotating matter



ν quantum states in matter

New approach to particles in matter

Method of exact solutions

Modified Dirac equations for ν (and e)

(containing the correspondent effective matter potentials)

+

exact solutions (particles wave functions)

↓

a basis for investigation of different phenomena which can proceed when neutrinos (and electrons) move in dense media (astrophysical and cosmological environments).

«method of exact solutions»

Interaction of particles in external electromagnetic fields

(Furry representation in quantum electrodynamics)

$$\mathbf{B}_\perp$$

$$e \longrightarrow e + \gamma$$

synchrotron radiation

Potential of electromagnetic field

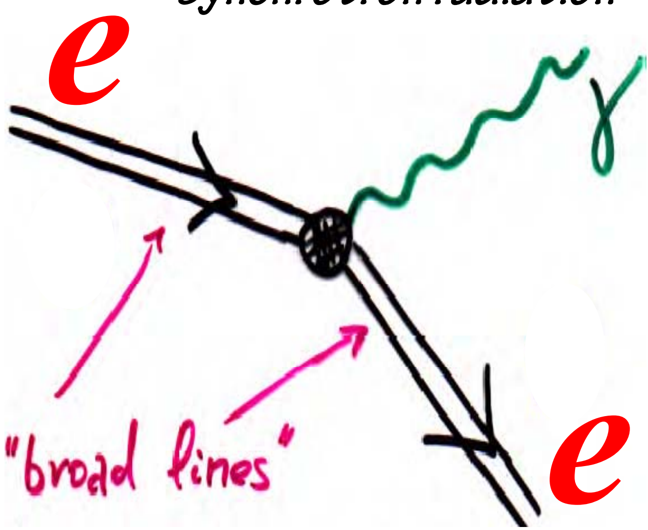
$$A_\mu(x) = A^q_\mu(x) + A^{ext}_\mu(x),$$

quantized part of potential

evolution operator

$$U_F(t_1, t_2) = Texp\Big[- i \int_{t_1}^{t_2} j^\mu(x) A^q_\mu(x) dx \Big],$$

charged particles current $j_\mu(x) = \frac{e}{2}[\overline{\Psi}_F \gamma_\mu, \Psi_F]$,



Dirac equation in external classical (non-quantized) field $A^{ext}_\mu(x)$

$$\Big\{ \gamma^\mu \Big(i\partial_\mu - eA^{ext}_\mu(x) \Big) - m_e \Big\} \Psi_F(x) = 0$$

...beyond perturbation series expansion,

strong fields and non linear effects...

. . . evaluation of the method

- *- within a project of research on* ν *in dense matter and external fields*
  - stimulated by need to obtain a consistent theory of "spin light of neutrino" in matter

A.S.,
"Neutrinos and electrons in background matter: a new approach",
Ann.Fond. de Broglie 31 (2006) 289;

"Method of wave equations exact solutions in studies of neutrinos and electron interactions in dense matter",
J.Phys.A: Math.Theor. 41 (2008) 164047

. . . «method of exact solutions »

ν and e in matter treated within

«method of exact solutions»

(of quantum wave equations)

A.Studenikin, A.Ternov, "Neutrino quantum states in matter", Phys.Lett.B 608 (2005) 107;

"Generalized Dirac-Pauli equation and neutrino quantum states in matter" hep-ph/0410296,

A.Grigoriev, A.Studenikin, A.Ternov, Phys.Lett.B 608 622 (2005)19

- *ν energy quantization in rotating matter ...*

A.Studenikin,"Method of wave equations exact solutions in studies of neutrino and electron interactions in dense matter",

- J.Phys.A:Math.Theor. 41 (2008) 16402

"Neutrinos and electrons in background matter: a new approach",

- Ann. Fond. de Broglie 31 (2006) 289,
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I.Balantsev, Yu.Popov, A.Studenikin, "On a problem of relativistic particles motion in a strong magnetic field and dense matter",

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I.Balantsev, Yu.Popov, A.Studenikin, **J.Phys.A: Math.Theor. 44** (2011) 255301

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A.Studenikin, **Phys.Atom.Nucl. 70** (2007) 1275; *ibid* **67** (2004)1014

A.Grigoriev, A.Savochkin, A.Studenikin, **Russ.Phys. J. 50** (2007) 845

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Phys.Atom.Nucl. 67 (2004) 719
JETP 99 (2004) 254; **JHEP 09** (2002) 016

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Phys.Lett.B 564 (2003) 27
Phys.Lett.B 515 (2001) 94

A.Grigoriev, A.Lobanov, A.Studenikin, **Phys.Lett.B 535** (2002) 187

A.Egorov, A.Lobanov, A.Studenikin, **Phys.Lett.B 491** (2000) 137

Outline (in addition to 3 mentioned above main problems)

- electromagnetic properties of ν
  - ν magnetic moment (th. & exp.)
- direct influence of e.m. fields on ν
  - spin (spin-flavour) oscillations in $B_\perp$
  - spin oscillations in arbitrary (e.m.) external fields
- indirect influence of e.m. fields on
  - beta-decay of neutron in $B_\perp$ and
  - spin-flavour ν oscillations in magnetized

e.m.f. of laser

$\mu \to e + \nu + \nu$

…Why

electromagnetic
properties of ν
provide a kind of
window to

NEW Physics ?

Beyond Standard Model

⇒ ① Carlo Giunti, Alexander Studenikin :
"Neutrino electromagnetic properties"
Phys.Atom.Nucl. 73, 2089-2125 (2009)
arXiv:0812.3646 v5, Apr 12, 2010

② A.Studenikin :
"Neutrino magnetic moment: a window to new physics"
Nucl.Phys.B (Proc.Supl.) 188, 220 (2009)

③ C. Giunti, A. Studenikin :
"Electromagnetic properties of neutrinos"
J.Phys.: Conf.Series. 203 (2010) 012100
arXiv:1006.3646 June 8, 2010

④ C.Giunti, A.Studenikin : "Theory and phenomenology of neutrino electromagnetic properties"
Rev.Mod.Phys. (in preparation)

... in spite of

- *results of terrestrial laboratory experiments on ν EM properties*

as well as

- *data from astrophysics and cosmology*

are in agreement with "ZERO" ν EM properties

... However, in course of recent development of knowledge on ν mixing and oscillations,

Outline (III)

Direct **and** **Indirect**

influence

of electromagnetic fields

on ν

through non-trivial neutrino electromagnetic properties (magnetic moment):

- ★ **neutrino spin**
- ★ **spin-flavour oscillations…**
- ★ **different** $\nu\gamma$ **processes**

due to e.m. field influence on charged particles coupled to neutrinos

- ★ **neutron beta-decay in** B
- ★ **change of** ν **oscillation pattern due to matter polarization under influence of external e.m. fields …**

Main results of our previous studies

1994-1997 ✓ • $\nu_L \leftrightarrow \nu_R$ in $B_\perp$, $(B_{cr} = B_{cr}(\Delta m^2, \theta, \rho))$ (spin oscillations)

1998-2000 ✓ $\nu_L \leftrightarrow \nu_R$ in arbitrary e.m. fields,

2000-2002 ✓ $\nu_L \leftrightarrow \nu_R$ in moving matter,

1995-2002 $\nu_e \leftrightarrow \nu_\mu$ in moving matter,

2003-2005 ✓ "spin light of neutrino" in matter and e.m. fields and gravitational fields

2004-2006… ✓ quantum theory of neutrino motion in background matter

NB ! These studies are performed within the Standard Model of interaction

$$m_\nu \neq 0$$

In the Standard Model : $m_\nu = 0$,

there is no ν_R $\Rightarrow$

ν magnetic moment $\mu_\nu = 0$.

Thus, $\mu_\nu \neq 0$ ← beyond the SM.

!

*...a tool for studying physics **B**eyond **S**tandard **M**odel...*

... a tool for studying physics Beyond Standard Model...

$m_\nu \neq 0$

Theory (Standard Model with ν_R)

$$\mu_{\nu_e} = \frac{3eG_F}{8\sqrt{2}\pi^2} m_{\nu_e} \sim 3\cdot 10^{-19} \mu_B \left(\frac{m_{\nu_e}}{1 eV}\right), \quad \mu_B = \frac{e}{2m_e}$$

Lee Shrock, 1977; Fujikawa Shrock, 1980

$$a_e = \frac{\alpha_{QED}}{2\pi} \sim 10^{-3}$$

... much greater values are desired

for astrophysical or cosmology

visualization of μ_ν

Astrophysical bounds

$$\mu_\nu \lesssim 3 \cdot 10^{-12} \mu_B$$

(Red Giant lumin.) etc.

G. Raffelt, D. Dearborn, J. Silk, 1989.

Theory (Standard Model with ν_R)

$$\mu_e = \frac{3eG_F}{8\sqrt{2}\pi^2} m_{\nu_e} \sim 3 \cdot 10^{-19} \mu_B \left(\frac{m_{\nu_e}}{1\,\text{eV}}\right), \quad \mu_B = \frac{e}{2m_e}$$

Lee Shrock, 1977; Fujikawa Shrok, 1980

...the present status...

to have visible $m_{\nu} \neq 0$

is not an easy task for

theoreticians

and experimentalists

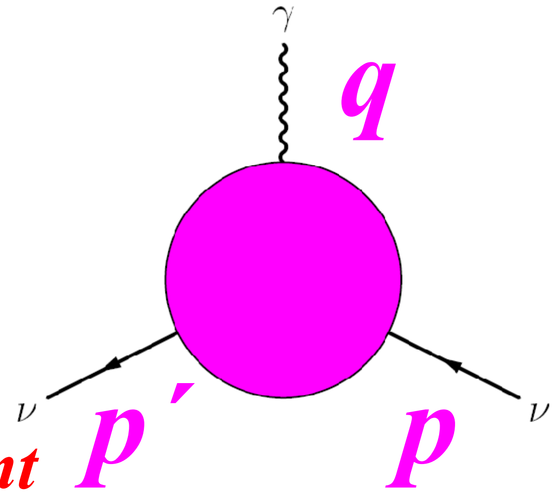
… puzzling

ν electromagnetic properties

something that is tiny or probably even does not exist at all…

… a bit of ν electromagnetic properties theory …

ν electromagnetic vertex function



$$<\psi(p')|J_\mu^{EM}|\psi(p)> = \bar{u}(p')\Lambda_\mu(q,l)u(p)$$

Matrix element of electromagnetic current is a Lorentz vector

$\Lambda_\mu(q,l)$ ***should be constructed using***

matrices $\hat{1}, \quad \gamma_5, \quad \gamma_\mu, \quad \gamma_5\gamma_\mu, \quad \sigma_{\mu\nu},$

tensors $g_{\mu\nu}, \quad \epsilon_{\mu\nu\sigma\gamma}$

vectors q_μ ***and*** l_μ

$$q_\mu = p'_\mu - p_\mu, \; l_\mu = p'_\mu + p_\mu$$

Lorentz covariance (1) and electromagnetic gauge invariance (2) $\Longrightarrow$

Matrix element of electromagnetic current between neutrino states

$$\langle \nu(p')|J_\mu^{EM}|\nu(p)\rangle = \bar{u}(p')\Lambda_\mu(q)u(p)$$

where vertex function generally contains 4 *form factors*

$$\Lambda_\mu(q) = f_Q(q^2)\gamma_\mu + f_M(q^2) i\sigma_{\mu\nu}q^\nu - f_E(q^2)\sigma_{\mu\nu}q^\nu\gamma_5 + f_A(q^2)(q^2\gamma_\mu - q_\mu \not{q})\gamma_5$$

1. electric

dipole

2. magnetic

3. electric

4. anapole

- **Hermiticity and discrete symmetries of EM current J_μ^{EM} put constraints on form factors**

Dirac ν

1) *CP invariance + hermiticity* ⟹ $f_E = 0$,

2) *at zero momentum transfer* ***only*** *electric charge* $f_Q(0)$ *and magnetic moment* $f_M(0)$ *contribute to* $H_{int} \sim J_\mu^{EM} A^\mu$,

3) *hermiticity* **itself** ⟹ *three form factors are real:* $Im f_Q = Im f_M = Im f_A = 0$

Majoran ν

1) *from CPT invariance (regardless CP or CP).*

$$f_Q = f_M = f_E = 0$$

…as early as 1939, W.Pauli…

EM properties ⟹ a way to distinguish Dirac and Majorana ν

In general case matrix element of $J_\mu^{\rm EM}$ can be considered between different initial $\psi_i(p)$ and final $\psi_j(p')$ states of different masses

$p^2 = m_i^2, \ p'^2 = m_j^2$:

$$< \psi_j(p')|J_\mu^{EM}|\psi_i(p) >= \bar{u}_j(p')\Lambda_\mu(q)u_i(p)$$

… beyond SM…

and

$$\Lambda_\mu(q) = \Big(f_Q(q^2)_{ij} + f_A(q^2)_{ij}\gamma_5\Big)(q^2\gamma_\mu - q_\mu \not{q}) + f_M(q^2)_{ij} i\sigma_{\mu\nu}q^\nu + f_E(q^2)_{ij}\sigma_{\mu\nu}q^\nu\gamma_5$$

form factors are matrices in ν mass eigenstates space.

(off-diagonal case $i \neq j$)

Dirac ν

1) *hermiticity itself does not apply restrictions on form factors,*

2) *CP invariance + hermiticity*

$f_Q(q^2), \ f_M(q^2), \ f_E(q^2), \ f_A(q^2)$

are relatively real (no relative phases).

Majorana ν

1) *CP invariance + hermiticity*

$\mu_{ij}^M = 2\mu_{ij}^D \ \ and \ \ \epsilon_{ij}^M = 0$ *or*

$\mu_{ij}^M = 0 \ \ and \ \ \epsilon_{ij}^M = 2\epsilon_{ij}^D$

… quite different EM properties …

… importance of μ_ν studies…

If diagonal $\mu_\nu \neq 0$

were confirmed

! then ν^{Dirac}

! … for $\nu^{Majorana}$
non-diagonal = transitional
$\mu_\nu \neq 0$

… progress
in experimental
studies of μ_ν $\Longrightarrow$

ν magnetic moment in experiments

Samuel Ting
(wrote on the wall at Department of Theoretical Physics of Moscow State University) :

"Physics is an experimental science"

Studies of ν-e scattering - most sensitive method of experimental investigation of μ_ν

Cross-section:

- $$\frac{d\sigma}{dT}(\nu + e \rightarrow \nu + e) = \left(\frac{d\sigma}{dT}\right)_{SM} + \left(\frac{d\sigma}{dT}\right)_{\mu_\nu}$$

where the Standard Model contribution

- $$\left(\frac{d\sigma}{dT}\right)_{SM} = \frac{G_F^2 m_e}{2\pi}\left[(g_V + g_A)^2 + (g_V - g_A)^2\left(1 - \frac{T}{E_\nu}\right)^2 + (g_A^2 - g_V^2)\frac{m_e T}{E_\nu^2}\right],$$

T is the electron recoil energy and

- $$\left(\frac{d\sigma}{dT}\right)_{\mu_\nu} = \frac{\pi\alpha_{em}^2}{m_e^2}\left[\frac{1 - T/E_\nu}{T}\right]\mu_\nu^2$$

$$\mu_\nu^2 = \sum_{j=\nu_e,\ \nu_\mu,\ \nu_\tau} |\ \mu_{ij} - \epsilon_{ij}\ |^2$$

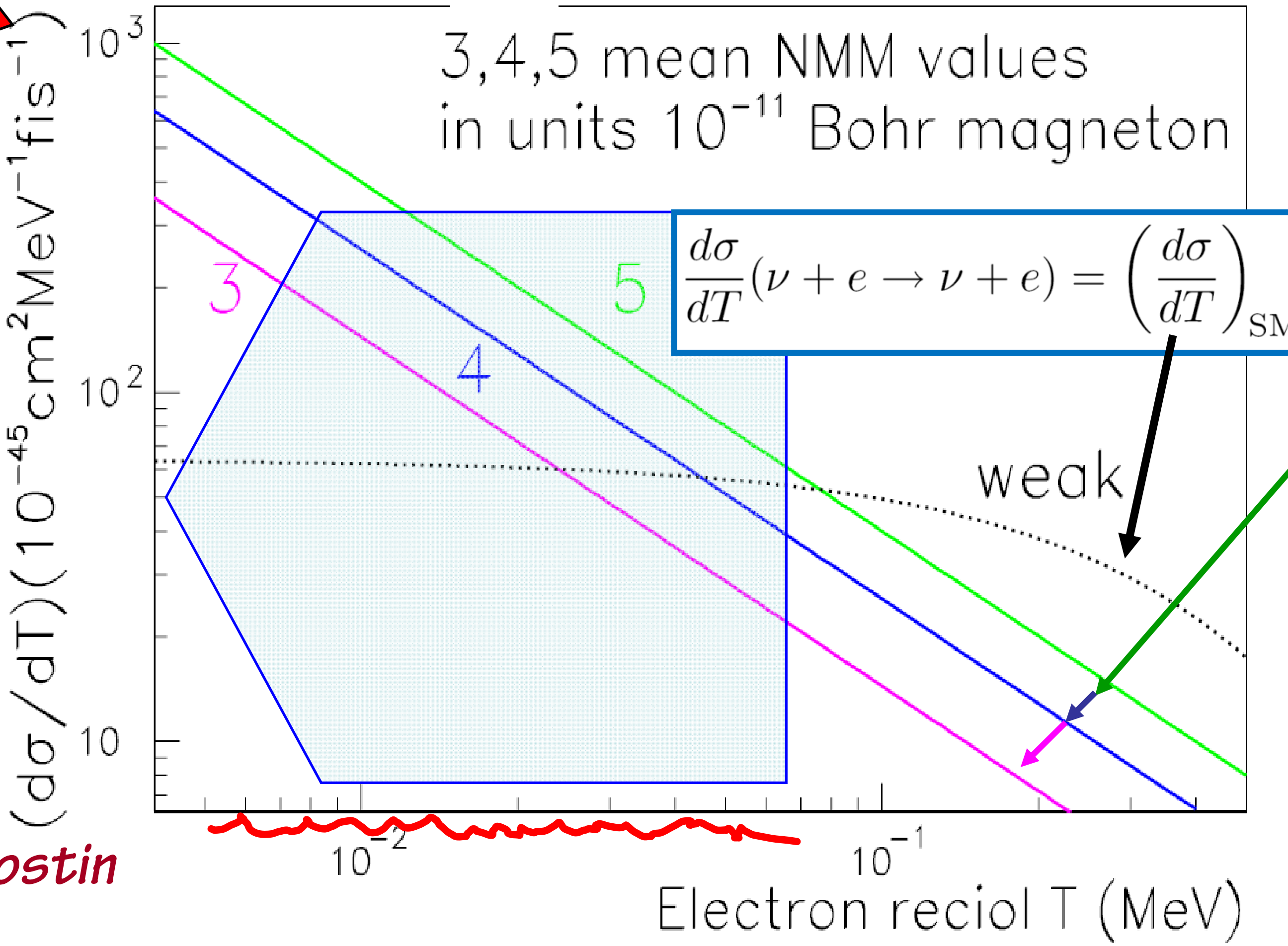
$$g_V = \begin{cases} 2\sin^2\theta_W + \frac{1}{2} & \text{for } \nu_e, \\ 2\sin^2\theta_W - \frac{1}{2} & \text{for } \nu_\mu, \nu_\tau, \end{cases} \qquad g_A = \begin{cases} \frac{1}{2} & \text{for } \nu_e, \\ -\frac{1}{2} & \text{for } \nu_\mu, \nu_\tau \end{cases}$$

for anti-neutrinos $g_A \rightarrow -g_A$

to incorporate charge radius: $g_V \rightarrow g_V + \frac{2}{3}M_W^2\langle r^2\rangle \sin^2\theta_W$

Magnetic moment contribution is dominated at low electron recoil energies when $\left(\frac{d\sigma}{dT}\right)_{\mu_\nu} > \left(\frac{d\sigma}{dT}\right)_{SM}$ *and* $\frac{T}{m_e} < \frac{\pi^2 \alpha_{em}}{G_F^2 m_e^4}\mu_\nu^2$

… the lower the smallest measurable electron recoil energy is, the smaller values of μ_ν^2 *can be probed in scattering experiments …*



$$\frac{d\sigma}{dT}(\nu + e \to \nu + e) = \left(\frac{d\sigma}{dT}\right)_{SM} + \left(\frac{d\sigma}{dT}\right)_{\mu_\nu}$$

from A.Starostin

MUNU experiment at Bugey reactor (2005)

$$\mu_\nu \leq 9 \times 10^{-11} \mu_B$$

TEXONO collaboration at Kuo-Sheng power plant (2006)

$$\mu_\nu \leq 7 \times 10^{-11} \mu_B$$

GEMMA (2007)

$$\mu_\nu \leq 5.8 \times 10^{-11} \mu_B$$

GEMMA I 2005 - 2007

BOREXINO (2008)

$$\mu_\nu \leq 5.4 \times 10^{-11} \mu_B$$

...was considered as the world best constraint...

$$\mu_\nu \leq 8.5 \times 10^{-11} \mu_B \quad (\nu_\tau,\ \nu_\mu)$$

Montanino, Picariello, Pulido, PRD 2008

based on first release of BOREXINO data

GEMMA (2005-2008)

Germanium Experiment on measurement of Magnetic Moment of Antineutrino

JINR (Dubna) + ITEP (Moscow) at Kalinin Nuclear Power Plant

$$\mu_\nu < 3.2 \times 10^{-11} \mu_B$$

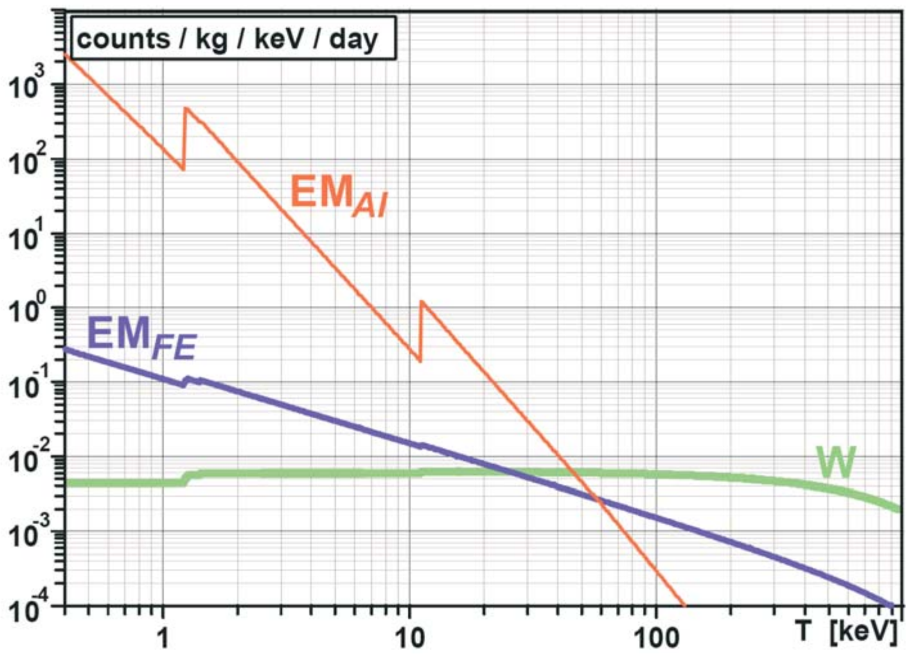
…till 13 January 2010 and again since autumn 2010
best limit on ν magnetic moment

A.Beda et al, Phys.Atom.Nucl.Lett. 7 (2010) 406

result known since 2009:
A.Beda, E.Demidova, A.Starostin et al,
arXiv:09.06.1926 , June 10, 2009,
A.Beda, V.Brudanin, E.Demidova et al,
in: "Particle Physics on the Eve of LHC",
ed. A.Studenikin, World Scientific (Singapore),
p.112, 2009 (13th Lomonosov Conference) www.icas.ru

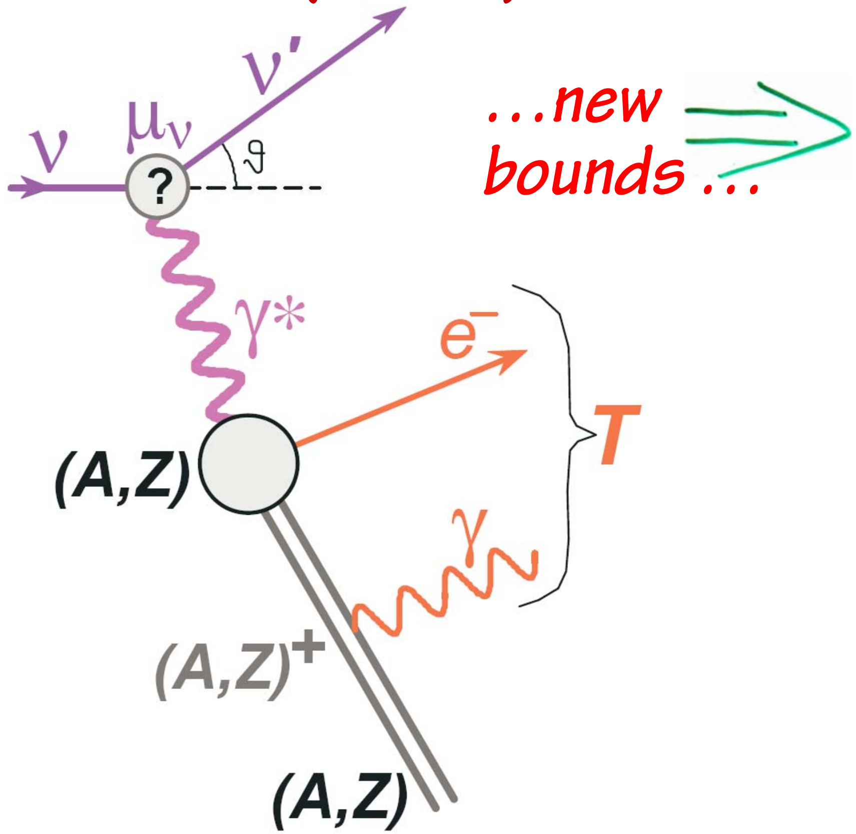
... quite recent claim that ν-e cross section should be increased by Atomic Ionization effect:

$$\nu + (A,Z) \longrightarrow \nu\,' + (A,Z)^+ + e^-$$
$$\downarrow \text{recombination}$$
$$(A,Z) \quad + \gamma$$



?

H.Wong et al. (TEXONO Coll.), arXiv: 1001.2074, 13 Jan 2010, reported at Neutrino 2010 Conference (Athens, June 2010), PRL 105 (2010) 061801



...new bounds ...

…much better limits on ν effective magnetic moment …

$$\mu_\nu < 1.3 \times 10^{-11} \mu_B$$?

… atomic ionization effect accounted for …

H.Wong et al., (TEXONO Coll.), arXiv: 1001.2074, 13 Jan 2010, PRL 105 (2010) 061801

Neutrino 2010 Conference, Athens

… however …

$$\mu_\nu < 5.0 \times 10^{-12} \mu_B$$?

… atomic ionization effect accounted for …

A.Beda et al. (GEMMA Coll.), arXiv: 1005.2736, 16 May 2010

$$\mu_\nu < 3.2 \times 10^{-11} \mu_B$$

… ν-e scattering on free electrons …
(without atomic ionization)

- K.Kouzakov, A.Studenikin,
  "Magnetic neutrino scattering on atomic electrons revisited" •
  Phys.Lett. B 105 (2011) 061801, arXiv: 1011.5847

- "Electromagnetic neutrino-atom collisions: The role of electron binding"
  Nucl.Phys.B (Proc.Suppl.) 217 (2011) 353
  arXiv: 1108.2872, 14 Aug 2011

- K.Kouzakov, A.Studenikin, M.Voloshin,
  "Neutrino-impact ionization of atoms in search for neutrino magnetic moment", **Phys.Rev.D** 83 (2011) 113001
  arXiv: 1101.4878, 25 Jan 2011

- "On neutrino-atom scattering in searches for neutrino magnetic moments" **Nucl.Phys.B** (Proc.Supp.) 2011 (Proc. of Neutrino 2010 Conf.)
  arXiv: 1102.0643, 3 Feb 2011

- "Testing neutrino magnetic moment in ionization of atoms by neutrino impact", **JETP Lett.** 93 (2011) 699
  arXiv: 1105.5543, 27 May 2011

- M.Voloshin,
  "Neutrino scattering on atomic electrons in search for neutrino magnetic moment"
  Phys.Rev.Lett. 105 (2010) 201801, arXiv: 1008.2171

No important effect of Atomic Ionization
on cross section in μ_ν experiments
once all possible final electronic states
accounted for

- M.Voloshin, 23 Aug 2010;
K.Kouzakov, A.Studenikin, 26 Nov 2010;
H.Wong et al, arXiv: 1001.2074 V3, 28 Nov 2010

GEMMA (2005-2008)

Germanium Experiment on measurement of Magnetic Moment of Antineutrino

JINR (Dubna) + ITEP (Moscow) at Kalinin Nuclear Power Plant

$$\mu_\nu < 3.2 \times 10^{-11} \mu_B$$

A.Beda et al, Phys.Atom.Nucl.Lett. 7 (2010) 406

$$\mu_\nu < 2.9 \times 10^{-11} \mu_B$$

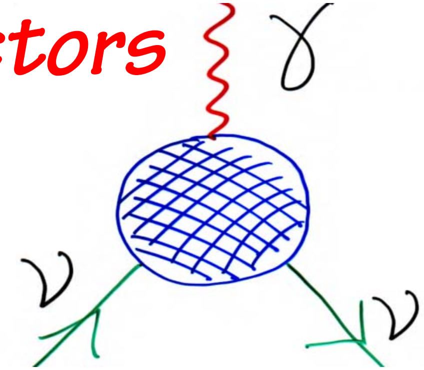
GEMMA , June 2012 ,
to be published in:

special Issue on "Neutrino Physics"

editors:
Jose Bernabeu,
Gianluigi Fogli,
Arthur McDonald,
Koichiro Nishikawa

A.Starostin,
private communication

ν e.m. vertex function $\Rightarrow$ 4 form factors



charge   dipole magnetic and electric

- $$\Lambda_\mu(q) = f_Q(q^2)\gamma_\mu + f_M(q^2) i\sigma_{\mu\nu}q^\nu + f_E(q^2)\sigma_{\mu\nu}q^\nu\gamma_5$$
  $$f_A(q^2)(q^2\gamma_\mu - q_\mu \not{q})\gamma_5$$ anapole

- **EM properties $\Rightarrow$ a way to distinguish Dirac and Majorana ν**

- Standard Model with ν_R ($m_\nu \neq 0$): $\mu_e = \frac{3eG_F}{8\sqrt{2}\pi^2} m_{\nu_e} \sim 3 \cdot 10^{-19}\mu_B \left(\frac{m_{\nu_e}}{1eV}\right)$

- In extensions of SM $\rightarrow$ enhancement of ν magnetic moment, even electrically millicharged ν

- Limits from reactor ν-e scattering experiments (2009, 2010):

  $$\mu_\nu < 3.2 \times 10^{-11}\mu_B$$ A.Beda et al. (GEMMA Coll.)

- Limits from astrophysics, star cooling (1990):

  $$\mu_\nu < 3 \times 10^{-12}\mu_B$$ G.Raffelt

… A remark on electric charge of ν …

ν neutrality $Q=0$ is attributed to **gauge invariance + anomally cancellation constraints** imposed in SM of electroweak interactions

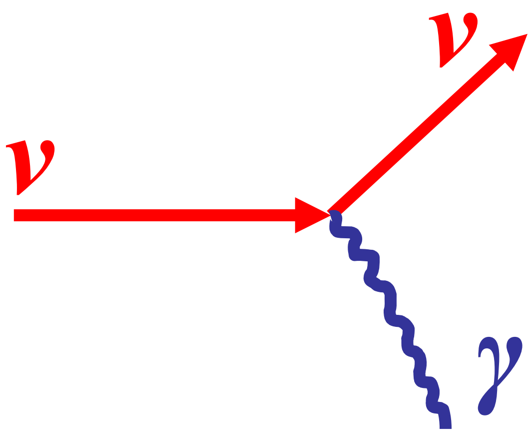
Foot, Joshi, Lew, Volkas, 1990; Foot, Lew, Volkas, 1993; Babu, Mohapatra, 1989, 1990

- *…General proof:*
- *In SM :* $Q = I_3 + \frac{Y}{2}$ ($SU(2)_L \times U(1)_Y$)
- *In SM (without ν_R) triangle anomalies cancellation constraints* → *certain relations among particle hypercharges Y, that is enough to fix all Y so that they, and consequently Q, are quantized* → $Q=0$
- $Q=0$ *is proven also by direct calculation in SM within different gauges and methods*

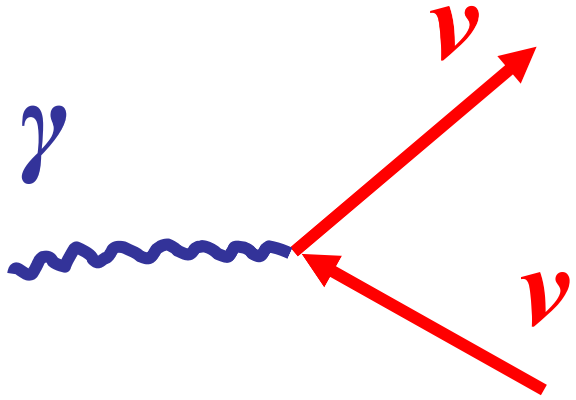
Bardeen, Gastmans, Lautrup, 1972; Cabral-Rosetti, Bernabeu, Vidal, Zepeda, 2000; Beg, Marciano, Ruderman, 1978; Marciano, Sirlin, 1980; Sakakibara, 1981;
- *M.Dvornikov, A.S., 2004 (for extended SM in one-loop calculations)*

- *… However, strict requirements for Q quantization may disappear in extensions of standard $SU(2)_L \times U(1)_Y$ EW model if ν_R with $Y \neq 0$ are included : in the absence of Y quantization electric charges Q gets dequantized* → **millicharged ν**

Neutrino–photon couplings

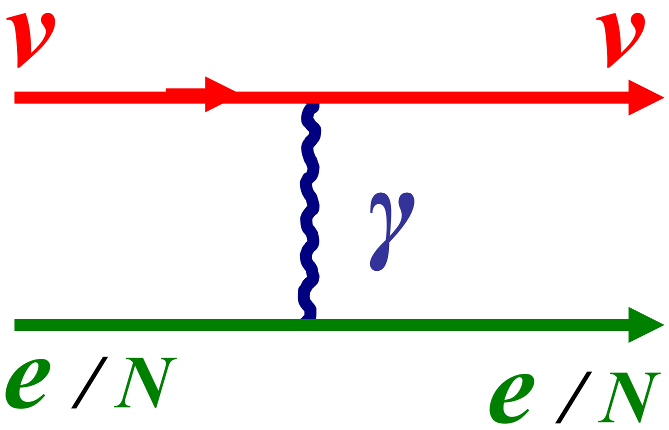


ν decay, Cherenkov radiation

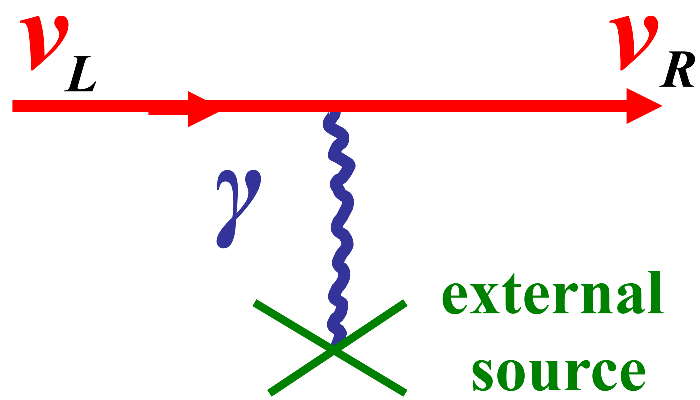


γ decay in plasma

!!!



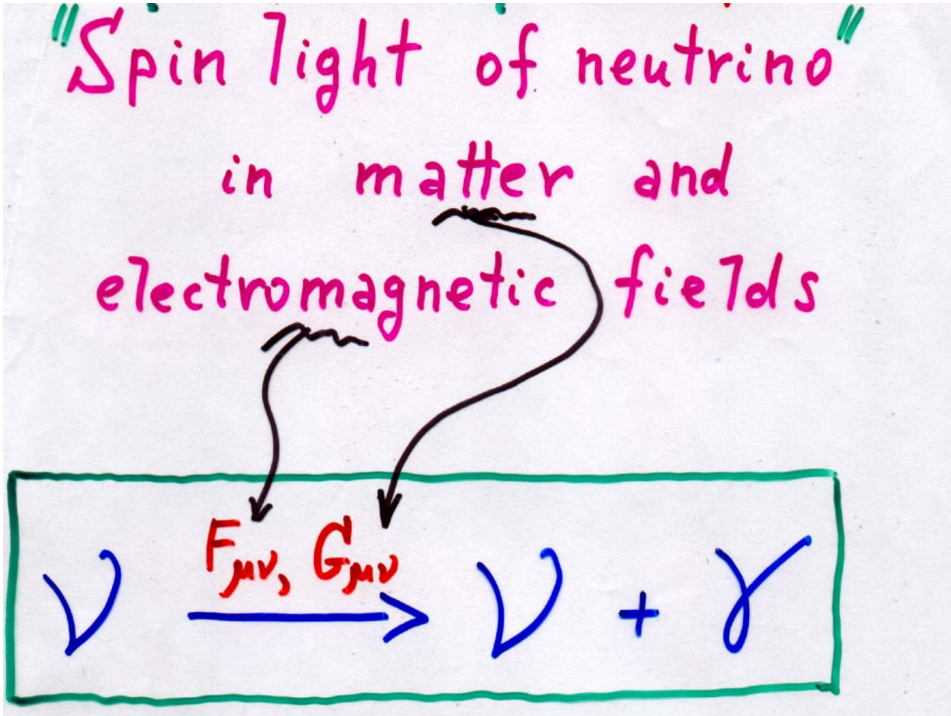
Scattering



Spin precession

- *New mechanism of electromagnetic radiation*

SLν



A.Lobanov, A.Studenikin, Phys.Lett. B 564 (2003) 27, Phys.Lett. B 601 (2004) 171

A.Studenikin, A.Ternov, Phys.Lett. B 608 (2005) 107

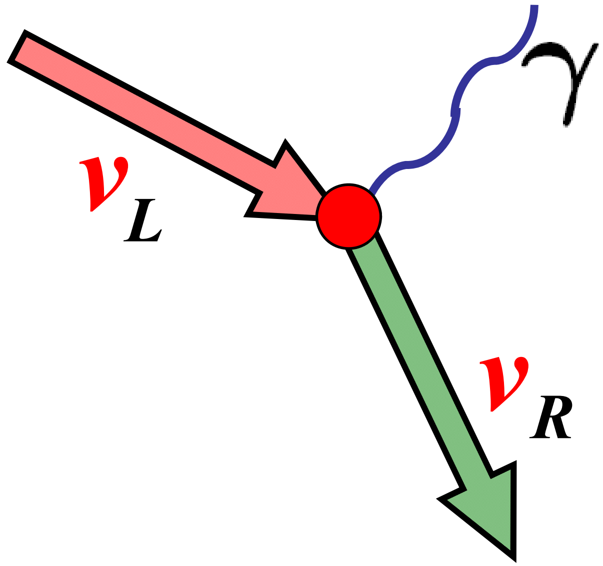
A.Grigoriev, A.S., A.Ternov, Phys.Lett. B 622 (2005) 199

A.Studenikin, J.Phys.A: Math.Gen. 39 (2006) 6769, Ann.Fond. De Broglie 31 (2006) 286, J.Phys.A: Math.Theor. 41 (2008) 16402,

A.Grigoriev, A.Lokhov, A.Studenikin, A.Ternov, Nuovo Cim. 35 C (2012) 57,

arXiv:1112.5263

$SL\nu$



Spin light of neutrino

- ***new mechanism** of the electromagnetic process stimulated by the presence of background environment, in which neutrino with **nonzero magnetic moment** emits light*

A.Lobanov, A.Studenikin, Phys.Lett. B 564 (2003) 27,
Phys.Lett. B 601 (2004) 171
A.S., A.Ternov, Phys.Lett. B 608 (2005) 107
A.Grigoriev, A.S., A.Ternov, Phys.Lett. B 622 (2005) 199
A.S., J.Phys.A: Math.Gen. 39 (2006) 6769
A.S., J.Phys.A: Math.Theor. 41 (2008) 16402

"Spin light of neutrino" in matter and electromagnetic fields

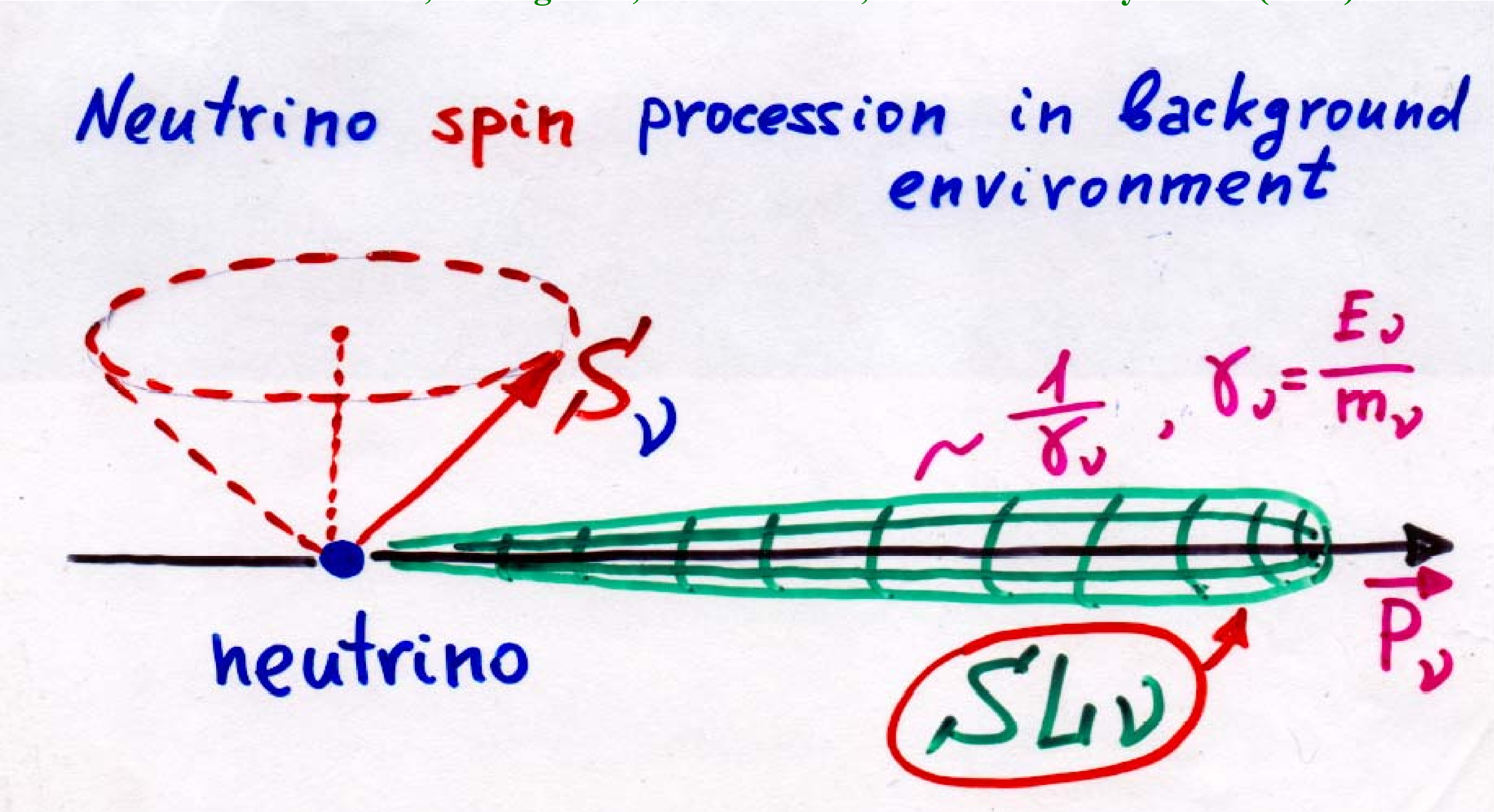
$SL\nu$

$$\nu \xrightarrow{F_{\mu\nu},\; G_{\mu\nu}} \nu + \gamma$$

Quasi-classical theory of spin light of neutrino in matter and gravitational field

$SL\nu$

A.Lobanov, A.Studenikin, Phys.Lett. B 564 (2003) 27,
Phys.Lett. B 601 (2004) 171;
M.Dvornikov, A.Grigoriev, A.Studenikin, Int.J.Mod.Phys. D 14 (2005) 309



New mechanism of electromagnetic radiation

? **Why Spin Light** of neutrino $SL\nu$ / of electron SLe **in matter.**

Analogies with:

* classical electrodynamics

an object with charge $Q=0$ and magnetic moment $\vec{m} = \frac{1}{2}\sum_i e_i[\vec{r}_i \times \vec{v}_i] \neq 0$

$$I^{cl.el.} = \frac{2}{3}\ddot{\vec{m}}^2$$

← magnetic dipole radiation power

Modified Dirac equation for neutrino in matter

Addition to the vacuum neutrino Lagrangian

$$\Delta L_{eff} = \Delta L_{eff}^{CC} + \Delta L_{eff}^{NC} = -f^{\mu}\Big(\bar{\nu}\gamma_{\mu}\frac{1+\gamma^{5}}{2}\nu\Big)$$

matter current

where $$f^{\mu} = \frac{G_F}{\sqrt{2}}\Big((1+4\sin^2\theta_W)j^{\mu} - \lambda^{\mu}\Big)$$

matter polarization

$$\Big\{ i\gamma_{\mu}\partial^{\mu} - \frac{1}{2}\gamma_{\mu}(1+\gamma_5)f^{\mu} - m \Big\}\Psi(x) = 0$$

A.Studenikin, A.Ternov, hep-ph/0410297;
Phys.Lett.B **608 (2005) 107**

It is supposed that there is a macroscopic amount of electrons in the scale of a neutrino de Broglie wave length. Therefore, **the interaction of a neutrino with the matter (electrons) is coherent.**

L.Chang, R.Zia,’88; J.Panteleone,’91; K.Kiers, N.Weiss, M.Tytgat,’97-’98; P.Manheim,’88; D.Nötzold, G.Raffelt,’88; J.Nieves,’89; V.Oraevsky, V.Semikoz, Ya.Smorodinsky,89; W.Naxton, W-M.Zhang’91; M.Kachelriess,’98; A.Kusenko, M.Postma,’02.

This is the most general equation of motion of a neutrino in which the effective potential accounts for both the **charged** and **neutral-current** interactions with the background matter and also for the possible effects of the matter **motion** and **polarization**.

Quantum theory of spin light of neutrino (I)

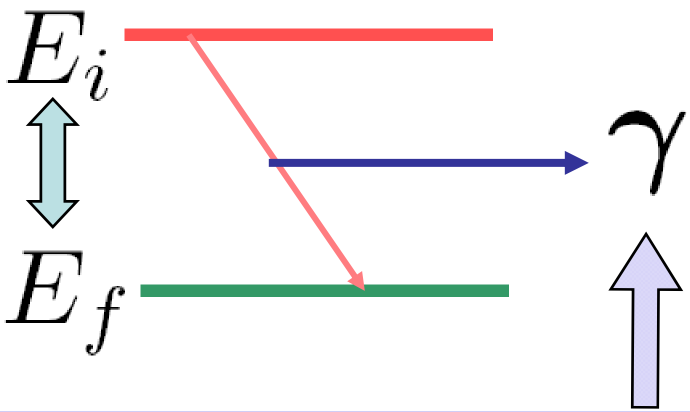
Quantum treatment of *spin light of neutrino* in matter shown that this process originates from the **two subdivided phenomena:**

$SL\nu$

the **shift** of the neutrino **energy levels** in the presence of the background matter, which is different for the two opposite **neutrino helicity states**,

$$E = \sqrt{\mathbf{p}^2\left(1 - s\alpha\frac{m}{p}\right)^2 + m^2 + \alpha m}$$

$$s = \pm 1$$



the radiation of the photon in the process of the neutrino transition from the **"excited" helicity state** to the **low-lying helicity state** in matter

A.Studenikin, A.Ternov, Phys.Lett.B 608 (2005) 107;
A.Grigoriev, A.Studenikin, A.Ternov, Phys.Lett.B 622 (2005) 199;
Grav. & Cosm. 14 (2005) 132;

neutrino-spin self-polarization effect in the matter

A.Lobanov, A.Studenikin, Phys.Lett.B 564 (2003) 27;
Phys.Lett.B 601 (2004) 171

It is possible to have $\tau = \frac{1}{\Gamma_{SL\nu}} << $ age of the Universe ?

For ultra-relativistic ν
with momentum $p \sim 10^{20} eV$
and magnetic moment $\mu \sim 10^{-10} \mu_B$
in very dense matter $n \sim 10^{40} cm^{-3}$
from

$$\Gamma_{SL\nu} = 4\mu^2 \alpha^2 m_\nu^2 p$$

$p \gg m_{plasmon}$

recently also discussed by A.Kuznetsov, N.Mikheev, 2006

A.Lobanov, A.S., PLB 2003; PLB 2004
A.Grigoriev, A.S., PLB 2005
A.Grigoriev, A.S., A.Ternov, PLB 2005

$$\alpha m_\nu = \frac{1}{2\sqrt{2}} G_F n (1 + sin^2\theta_W)$$

it follows that

$$\tau = \frac{1}{\Gamma_{SL\nu}} = 1.5 \times 10^{-8} s$$

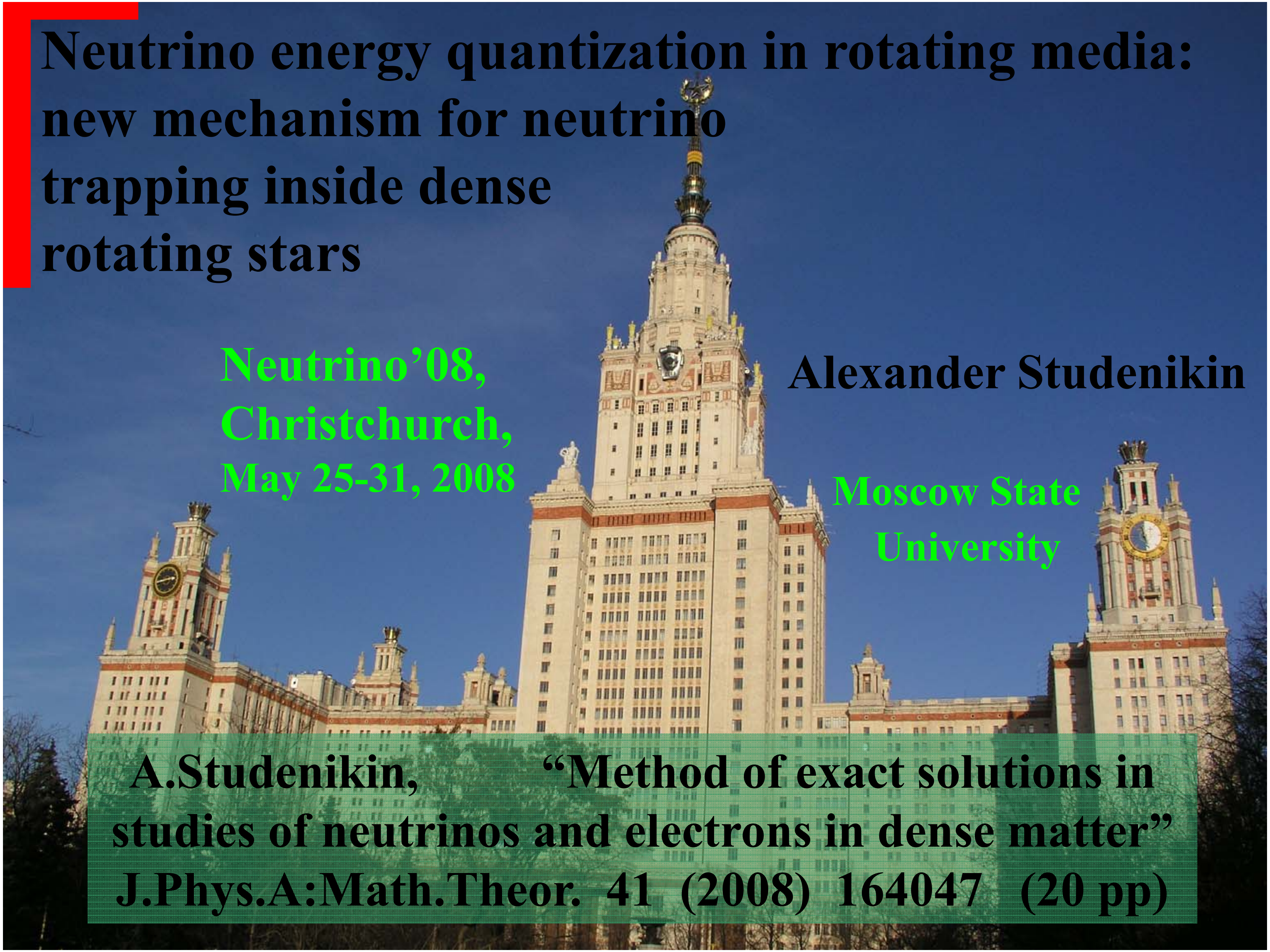
Neutrino energy quantization in rotating media: new mechanism for neutrino trapping inside dense rotating stars

Neutrino'08,
Christchurch,
May 25-31, 2008

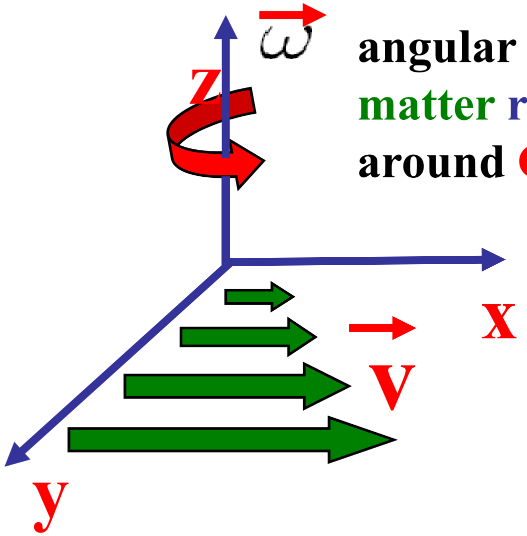
Alexander Studenikin

Moscow State
University

A.Studenikin, "Method of exact solutions in studies of neutrinos and electrons in dense matter"
J.Phys.A:Math.Theor. 41 (2008) 164047 (20 pp)



Neutrino energy quantization in matter



angular speed of
matter rotation
around OZ

A.Grigoriev, A.Savochkin,A.Studenikin (2007)
A.Studenikin (2008)
I.Balantsev, Yu.Popov, A.Studenikin (2011)

Consider ν moving in rotating medium composed of neutrons (generalization s.f.):

ν wave function

$$\left\{ i\gamma_\mu \partial^\mu - \frac{1}{2}\gamma_\mu (1+\gamma_5) f^\mu - m \right\} \Psi(x) = 0$$

where matter potential $f^\mu = -G(n, n\mathbf{v}), \quad \mathbf{v} = (\omega y, 0, 0), \ \rho = Gn\omega \qquad G = \frac{G_F}{\sqrt{2}}$

neutron number density **speed of matter** **angular speed of rotation**

ν energy spectrum $\tilde{p}_0 = \sqrt{m^2 + p_3^2 + 4N\rho + Gn} \qquad N = 0, 1, 2, \ldots$

circular orbits ⇒ **trapping inside dense stars**

Energy spectrum of active left-handed neutrino

$$p_0 = \sqrt{p_3^2 + 2\rho N} - Gn, \quad N = 0, 1, 2, \ldots$$

$$\rho = Gn\omega$$

Antineutrino → **"negative sign" energy eigenvalues**

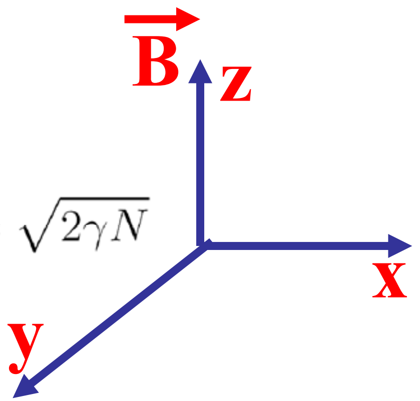
$$\tilde{p}_0 = \sqrt{p_3^2 + 2\rho N} + Gn, \quad N = 0, 1, 2, \ldots$$

→ ν energy quantization

Transversal motion of active relativistic ν
is quantized in rotating medium
like electron motion is quantized
in magnetic field (Landau energy levels):

$$p_0^{(e)} = \sqrt{m_e^2 + p_3^2 + 2\gamma N}, \quad \gamma = eB, \quad N = 0, 1, 2, \ldots$$

$$p_\perp^{(e)} = \sqrt{2\gamma N}$$



… consistent model of a rotating matter with account for ν mass

I.Balantsev, Yu.Popov, A.Studenikin,
Nuov.Cim.B 32 (2009) 53,
arXiv: 0906.2391

$$\left\{ i\gamma_\mu \partial^\mu - \frac{1}{2}\gamma_\mu (1+\gamma_5) f^\mu - m \right\} \Psi(x) = 0$$

$$f^\mu = -G(n, n\mathbf{v}), \quad \mathbf{v} = (-\omega y, \omega x, 0)$$

Energy spectra

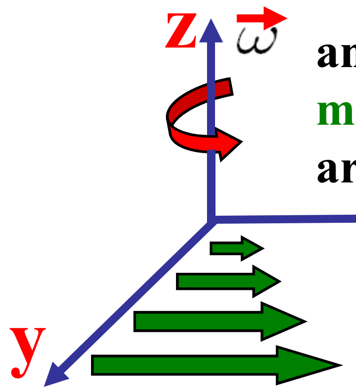
$$p_0 = \sqrt{m^2 + p_3^2 + 4N\rho} - Gn$$ *for* ν

$$\tilde{p}_0 = \sqrt{m^2 + p_3^2 + 4N\rho} + Gn$$ *for* $\tilde{\nu}$

$N = 0, 1, 2, \ldots$ $\qquad \rho = Gn\omega$

One example: consider antineutrino $\tilde{\nu}$ in rotating neutron matter, then energy of transversal motion

$$\boxed{\tilde{p}_{\perp} = \sqrt{2\rho N}} \qquad \rho = Gn\omega$$



angular speed of matter rotation around OZ

Quantum number N also determines **radius** of antineutrino quasi-classical orbit in moving matter:

$$\boxed{R = \sqrt{\frac{2N}{Gn\omega}}} \Rightarrow$$

binding orbits inside a Neutron Star !?

NS:

$$R_{NS} = 10\ km$$
$$n = 10^{37} cm^{-3}$$
$$\omega = 2\pi \times 10^{3}\ s^{-1}$$

radius of trajectory

for this set $R = \sqrt{\frac{2N}{Gn\omega}} < R_{NS} = 10\ km$

if $\boxed{N \leq N_{max} = 10^{10}}$, $\tilde{\nu}$ **with** $N \leq 10^{10}$ can be bound inside the star

thus, $\tilde{\nu}$ **with energy** $\tilde{p}_0 \sim 1\ eV$ **can be bound inside NS**

$N \gg 1$ and $p_3 = 0$

ν quantum states in rotating matter

A.Studenikin, J.Phys.A: Math.Theor. 41 (2008) 164047

quasi-classical circular orbits due to central force

$$\mathbf{F}_m^{(\nu)} = q_m^{(\nu)} \boldsymbol{\beta} \times \mathbf{B}_m$$

$$\mathbf{B}_m = \boldsymbol{\nabla} \times \mathbf{A}_m, \quad \mathbf{A}_m = n\mathbf{v}$$

"magnetic field" vector potential

"charge"

$$q_m^{(\nu)} = -G$$

matter-induced "Lorentz force",

$$\mathbf{F}_m^{(\nu)} \perp \boldsymbol{\beta}$$

Generalization to non-constant matter density:

$$\mathbf{F}_m^{(\nu)} = q_m^{(\nu)} \mathbf{E}_m + q_m^{(\nu)} \boldsymbol{\beta} \times \mathbf{B}_m,$$

L.Silva, R.Bingham, J.Dawson, J.Mendoca, P.Shukla, Phys.Plasma 7 (2000) 2166

"magnetic field" $\mathbf{B}_m = n\boldsymbol{\nabla} \times \mathbf{v} - \mathbf{v} \times \boldsymbol{\nabla} n$

"electric field" $\mathbf{E}_m = -\boldsymbol{\nabla} n - \mathbf{v}\dfrac{\partial n}{\partial t} - n\dfrac{\partial \mathbf{v}}{\partial t}$

Conclusions

μ_ν is “presently known” to be in the range

$$10^{-20}\mu_B \leq \mu_\nu \leq 10^{-\mathbf{11}}\mu_B$$

μ_ν provides a tool for exploration possible physics beyond the Standard Model

- Due to smallness of neutrino-mass-induced magnetic moments,

$$\mu_{ii} \approx 3.2 \times 10^{-19}\left(\frac{m_i}{1\ eV}\right)\mu_B$$

any indication for non-trivial electromagnetic properties of ν,
that could be obtained within reasonable time in the future,
would give evidence for interactions
beyond extended Standard Model

IOP PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND THEORETICAL

J. Phys. A: Math. Theor. **44** (2011) 255301 (12pp) doi:10.1088/1751-8113/44/25/255301

- # On the problem of relativistic particles motion in a strong magnetic field and dense matter

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[2] Skobeltsyn Institute of Nuclear Physics, Moscow State University, Moscow, Russia
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Abstract
We consider a problem of electron motion in different media and magnetic fields. It is shown that in the case of an immovable medium and constant homogenous magnetic field the electron energies are quantized. We also discuss the general problem of eigenvectors and eigenvalues of a given class of Hamiltonians. We examine obtained exact solutions for the particular case of the electron motion in a rotating neutron star which account for matter and magnetic field effects. We argue that all of these considerations can be useful for astrophysical applications, in particular for the description of electrons' and neutrinos motion in different environments.

PACS numbers: 03.65.Ge, 03.65.Pm

- *A.Balantsev, A.Studenikin, I.Tokarev*

- *A.Studenikin, I.Tokarev*

in preparation...

e quantum states in rotating matter
quasi-classical circular orbits due to central force

A.Studenikin,
J.Phys.A:Math.Theor. 41 (2008) 164047

Matter-induced "Lorentz force" on electron

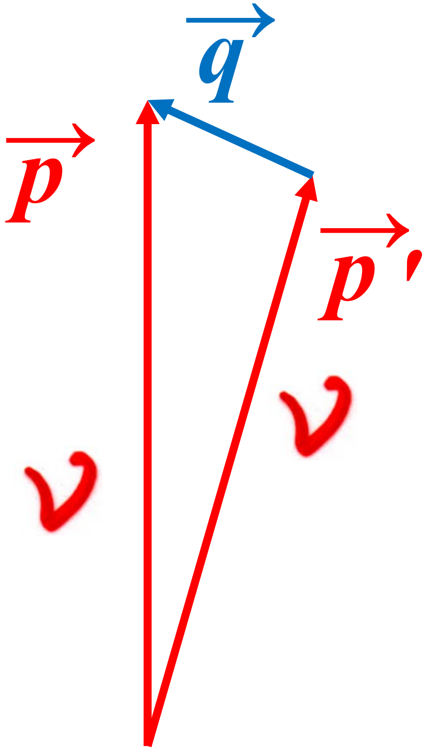
$$\mathbf{F}_m^{(e)} = q_m^{(e)} \mathbf{E}_m + q_m^{(e)} \boldsymbol{\beta} \times \mathbf{B}_m$$

We predict that there could be an electromagnetic radiation emitted by an electron moving in radial direction inside a neutrino flow $(m = \nu)$ **emitted from a central part of a star (dipole radiation):**

$$I = \frac{2}{3} q_\nu^{(e)} \left[\frac{\mathbf{a}^2}{(1-\beta^2)^2} + \frac{(\mathbf{a}\boldsymbol{\beta})^2}{(1-\beta^2)^3} \right]$$

acceleration of electron
due to mater-induced "Lorentz force"

• Neutrino-impact ionization of atoms in in serarch for μ_ν



scattering on atoms (Ge) at low energy transfer

$T \sim$ few keV and lower so that $\frac{T}{E_\nu} \ll 1$ for most of reactor ν

Ge atom recoil energy $< \frac{2E_\nu^2}{M_{Ge}} \ll T$, $M_{Ge} \to \infty$

ν interaction with nucleus is neglected

ν scatterina on atomic e is important:

$$\Lambda^i_{(\mu)} = \frac{\mu_\nu}{2m_e} \sigma^{ik} q_k$$

Four momentum transfer

$$q = p - p'$$

$$q_\mu = (T, \vec{q}), \quad q^2 = \vec{q}^{\,2}$$

energy and spatial momentum transfer from neutrinos to atomic electrons

Kouzakov, Studenikin, 2010; Kouzakov, Studenikin, Voloshin, 2011

Double differential ν-e cross section

$$\frac{d^2\sigma_{(\mu)}}{dT\,dq^2} = 4\pi\,\alpha\,\frac{\mu_\nu^2}{q^2}\left[\left(1-\frac{T^2}{q^2}\right)S(T,q^2)+\left(1-\frac{q^2}{4E_\nu^2}\right)R(T,q^2)\right]$$

$$\frac{d^2\sigma_{(\mu)}}{dTdq^2} = \left(\frac{d^2\sigma_{(\mu)}}{dTdq^2}\right)_{\parallel} + \left(\frac{d^2\sigma_{(\mu)}}{dTdq^2}\right)_{\perp}$$

Kouzakov, Studenikin, 2010; Kouzakov, Studenikin, Voloshin, 2011

where dynamical structure factor (Van Hove, 1954)

$$S(T,q^2) = \sum_n \delta(T - E_n + E_0)\,|\langle n|\rho(\vec{q})|0\rangle|^2$$

Fourier transforms of electron density and current

$(\vec{j}_\perp \cdot \vec{q}) = 0$ and $R(T,q^2) = \sum_n \delta(T - E_n + E_0)\,|\langle n|j_\perp(\vec{q})|0\rangle|^2$

summ is over all states $|n\rangle$ of electron system, $|0\rangle$ initial state

For single-differential inclusive cross section measured in experiment

$$\frac{d\sigma_{(\mu)}}{dT} = 4\pi\,\alpha\,\mu_\nu^2 \int_{T^2}^{4E_\nu^2} S(T,q^2)\,\frac{dq^2}{q^2}$$

$$R(T,q^2) = \frac{T^2}{q^2} S(T,q^2)$$

transversal contribution practically for most q^2 is negligible

SM electroweak contribution to cross section

$$\frac{d\sigma_{EW}}{dT} = \frac{G_F^2}{4\pi}\left(1 + 4\,\sin^2\theta_W + 8\,\sin^4\theta_W\right) \int_{T^2}^{4E_\nu^2} S(T,q^2)\,dq^2$$

nonrelativistic limit $\int_{T^2}^{4E_\nu^2} \Rightarrow \int_0^\infty$

For free electron $S_{(FE)}(T,q^2) = \delta(T - q^2/2m)$

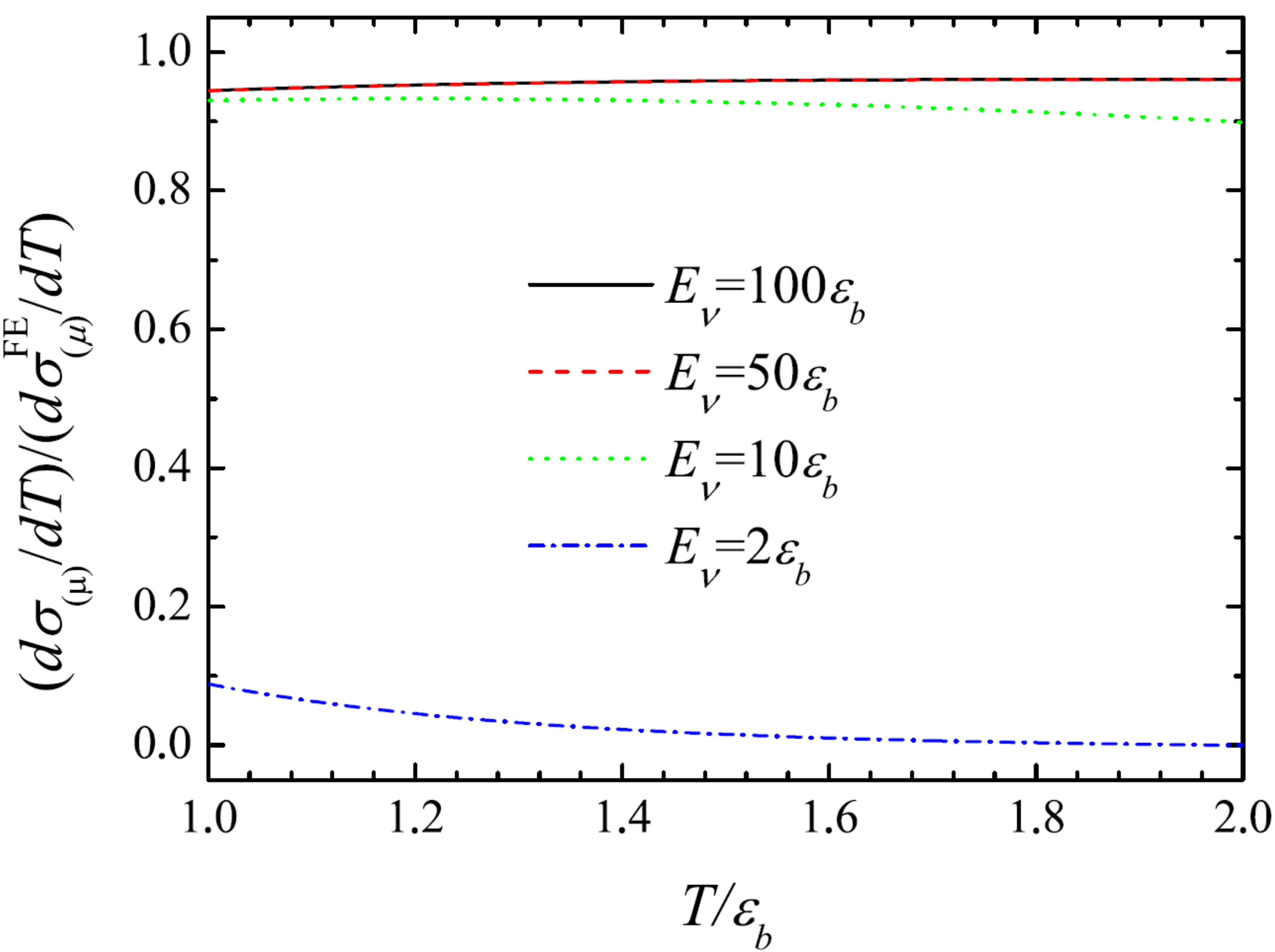
$$\int_0^\infty S_{(FE)}(T,q^2)\,\frac{dq^2}{q^2} = \frac{1}{T}\,, \qquad \int_0^\infty S_{(FE)}(T,q^2)\,dq^2 = 2\,m$$

$$\frac{d\sigma_{(\mu)}}{dT} = 4\pi\,\alpha\,\mu_\nu^2\left(\frac{1}{T} - \frac{1}{E_\nu}\right) = \pi\,\frac{\alpha^2}{m^2}\left(\frac{\mu_\nu}{\mu_B}\right)^2\left(\frac{1}{T} - \frac{1}{E_\nu}\right)$$

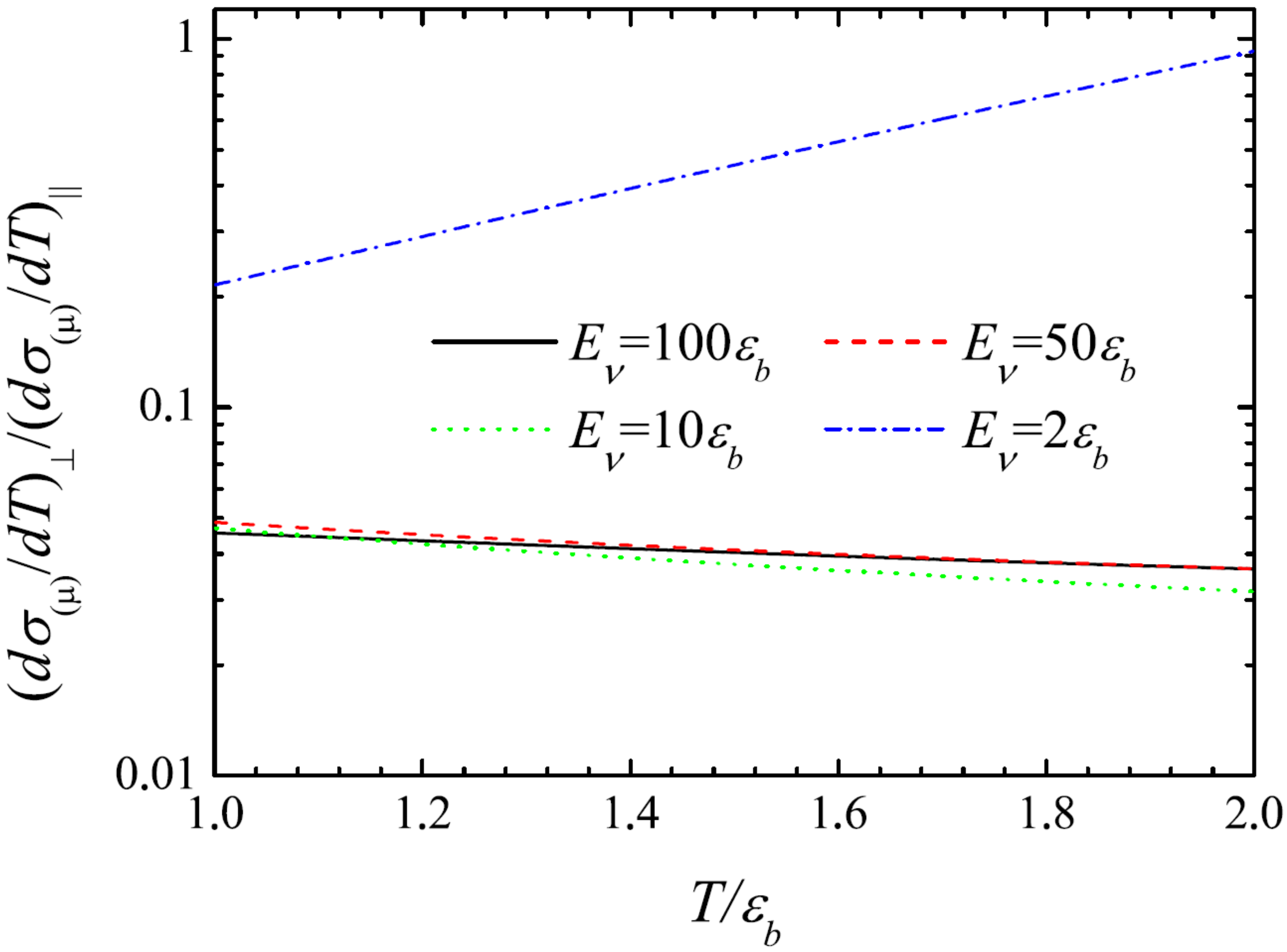
free electron approximation is valid

- ... the same for electron bound in atom ...

(ν-e scattering on free electrons)



1.0
0.8
0.6
0.4
0.2
0.0
1.0
1.2
1.4
1.6
1.8
2.0
$(d\sigma_{(\mu)}/dT)/(d\sigma^{\mathrm{FE}}_{(\mu)}/dT)$
T/ε_b
$E_\nu=100\varepsilon_b$
$E_\nu=50\varepsilon_b$
$E_\nu=10\varepsilon_b$
$E_\nu=2\varepsilon_b$



$(d\sigma_{(\mu)}/dT)_\perp/(d\sigma_{(\mu)}/dT)_\parallel$
1
0.1
0.01
1.0
1.2
1.4
1.6
1.8
2.0
T/ε_b
$E_\nu=100\varepsilon_b$
$E_\nu=50\varepsilon_b$
$E_\nu=10\varepsilon_b$
$E_\nu=2\varepsilon_b$

16th Lomonosov Conference on Elementary Particle Physics, www.icas.ru

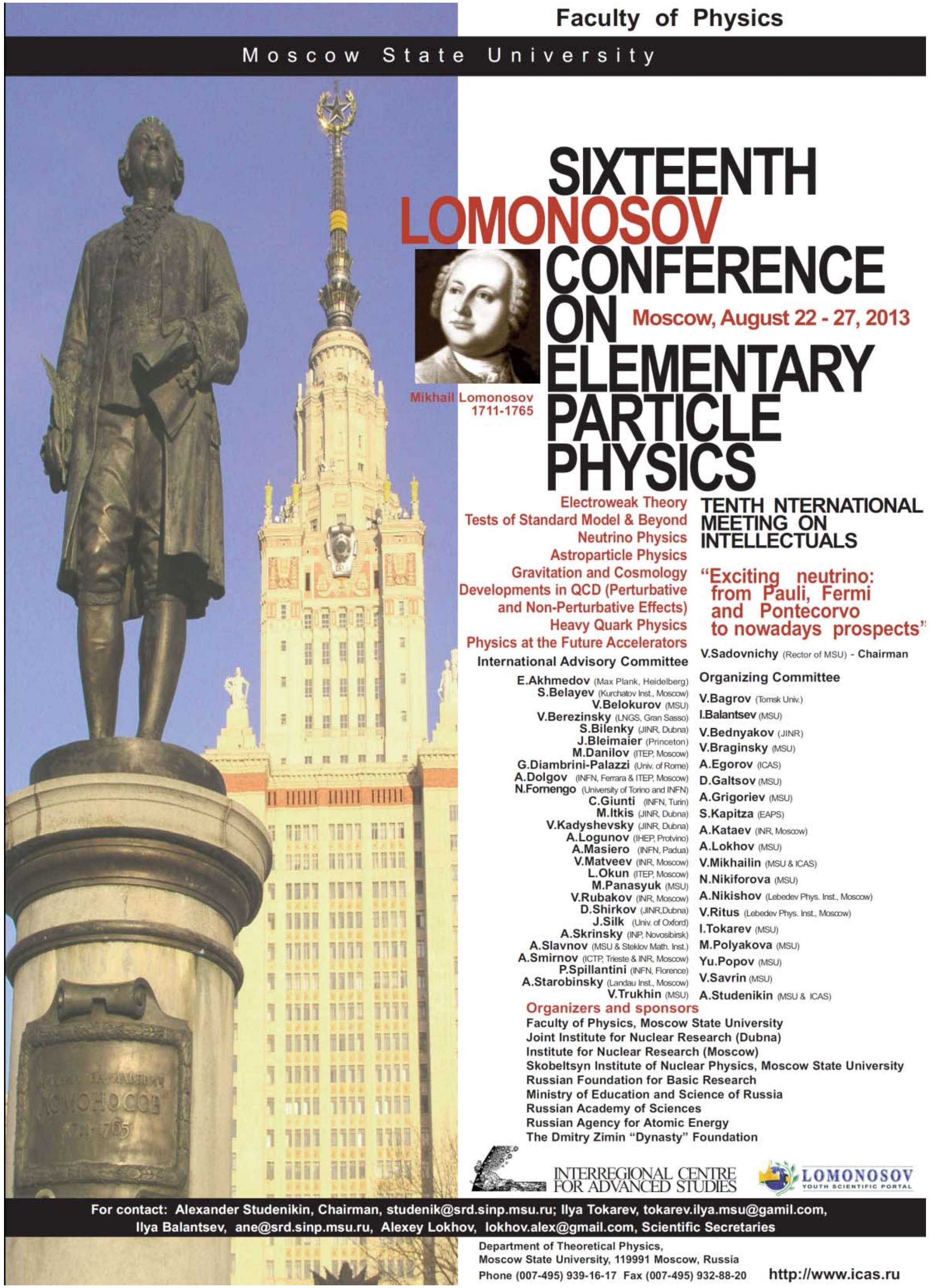
Moscow State University,

August 22-27, 2013

Бруно Понтекорво

1913-1993

centennial anniversary



… we very much hope that

ν electromagnetic properties

will not follow the presentiment of Pauli

… *situation with*

ν electromagnetic properties

is better that it was for ν in the time of W. Pauli, 1930

… once they will be observed experimentally

… are important in astrophysics

… there is a need for further theoretical and experimental studies

Astrophysics bounds on μ_ν

$$\mu_\nu(\text{astro}) < 10^{-10}\text{–}10^{-12}\ \mu_B$$

Mostly derived from consequences of helicity-state change in astrophysical medium:

- available degrees of freedom in BBN,
- stellar cooling via plasmon decay,
- cooling of SN1987a.

Red Giant lumin. !
$\mu_\nu \lesssim 3 \cdot 10^{-12} \mu_B$
G. Raffelt, D. Dearborn, J. Silk, 1989.

Bounds depend on

- modeling of astrophysical systems,
- on assumptions on the neutrino properties.

Generic assumption:

- absence of other nonstandard interactions except for μ_ν .

A global treatment would be desirable, incorporating oscillation and matter effects as well as the complications due to interference and competitions among various channels.

16th Lomonosov Conference on Elementary Particle Physics
MSU. Moscow. August 22-28, 2013

Бруно Понтекорво
1913-1993

August 22, 2013
is the centenary of
Bruno Pontecorvo birth

... our 3 proposals:

- 16LomCon will be dedicated to the memory of Bruno Pontecorvo
- Scientific programme of 16LomCon to be devoted to neutrino physics astroparticle physics and related subjects

3

… a bit of ν electromagnetic properties theory

3.1 ν vertex function

The most general study of the **massive neutrino** vertex function (including electric and magnetic form factors) in arbitrary R_ξ gauge in the context of the SM + SU(2)-singlet ν_R accounting for masses of particles in polarization loops

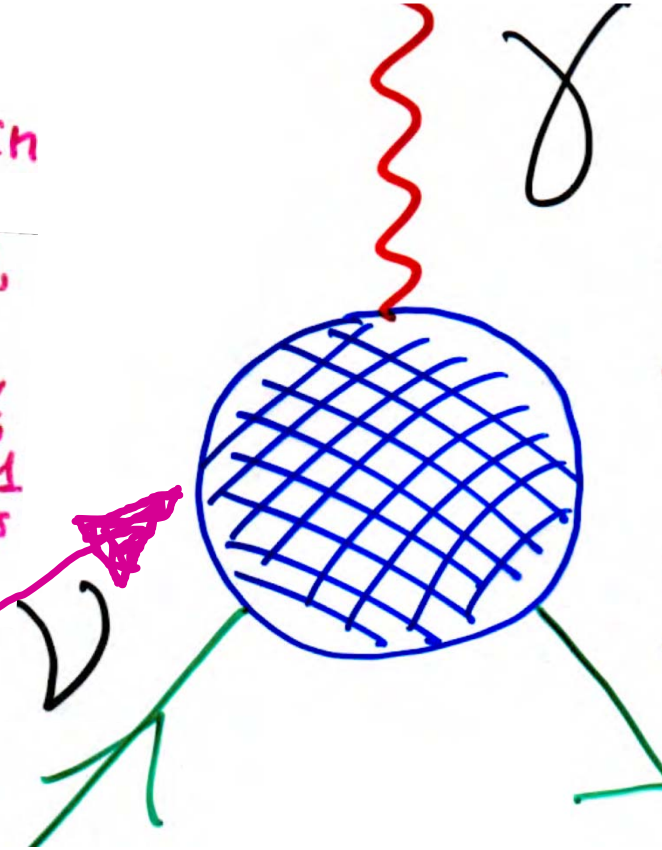
M. Dvornikov, A. Studenikin

⊛ Phys. Rev. D 63, 073001, 2004,

"Electric charge and magnetic moment of massive neutrino";

JETP 126 (2004), N8, 1

⊛ "Electromagnetic form factors of a massiv neutrino."



R_ξ-gauge and $(q^2 \neq 0)$

charge — magnetic moment

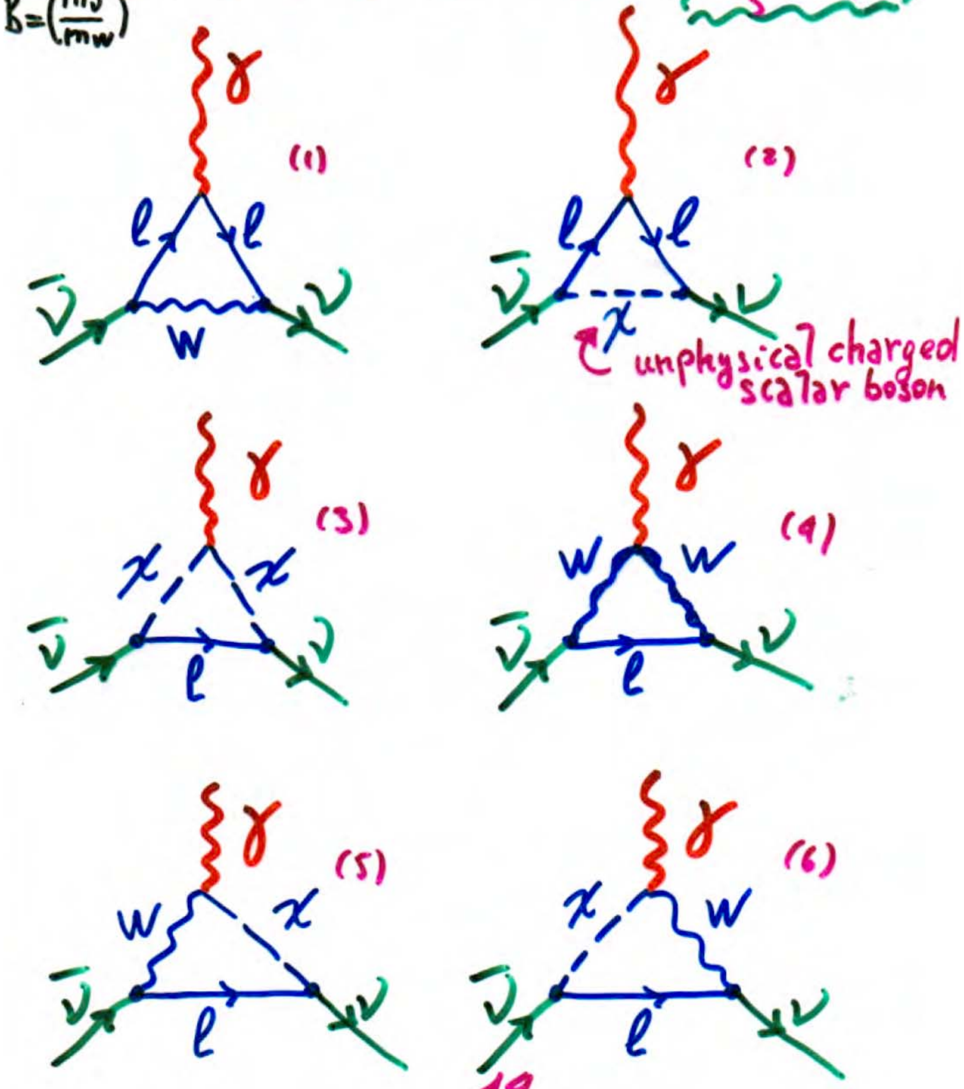
$$\Delta_\mu(q) = f_Q(q^2)\gamma_\mu + f_M(q^2)\, i\sigma_{\mu\nu} q^\nu -$$

$$- f_E(q^2)\, i\sigma_{\mu\nu} q^\nu \gamma_5 - f_A(q^2)(q^2\gamma_\mu - q_\mu \not{q})\gamma_5$$

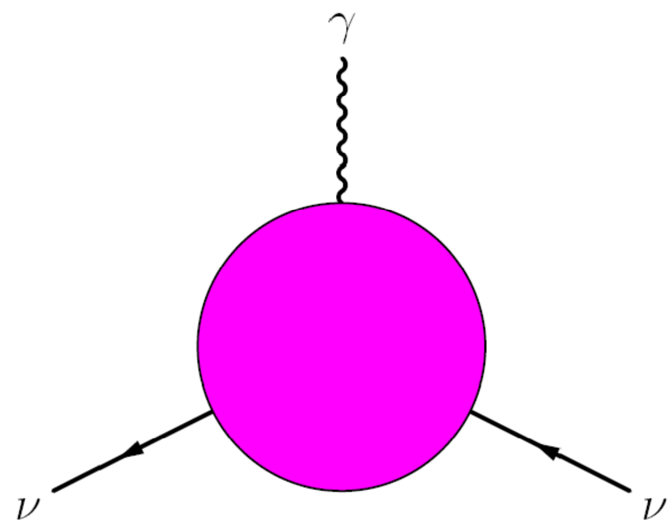
electric moment — anapole moment

$a=\left(\frac{m_\ell}{m_W}\right)^2$, $b=\left(\frac{m_\nu}{m_W}\right)^2$

Proper vertices — R_ξ-gauge



$$\Lambda_\mu(q)=\sum_{i=1}^{14}\Lambda_\mu^i(q)$$



$$\Lambda_\mu(q)$$

Contributions of proper vertices diagrams

(dimensional-regularization scheme)

- $$\Lambda_\mu^{(1)}=i\frac{eg^2}{2}\int\frac{d^Nk}{(2\pi)^N}\left[g^{\kappa\lambda}-(1-\alpha)\frac{k^\kappa k^\lambda}{k^2-\alpha M_W^2}\right]\times\frac{\gamma_\kappa^L(\not p\,'-\not k+m_\ell)\gamma_\mu(\not p-\not k+m_\ell)\gamma_\lambda^L}{[(p'-k)^2-m_\ell^2][(p-k)^2-m_\ell^2][k^2-M_W^2]},$$

- $$\Lambda_\mu^{(2)}=i\frac{eg^2}{2M_W^2}\int\frac{d^Nk}{(2\pi)^N}\frac{(m_\nu P_L-m_\ell P_R)(\not p\,'-\not k+m_\ell)\gamma_\mu(\not p-\not k+m_\ell)(m_\ell P_L-m_\nu P_R)}{[(p'-k)^2-m_\ell^2][(p-k)^2-m_\ell^2][k^2-\alpha M_W^2]},$$

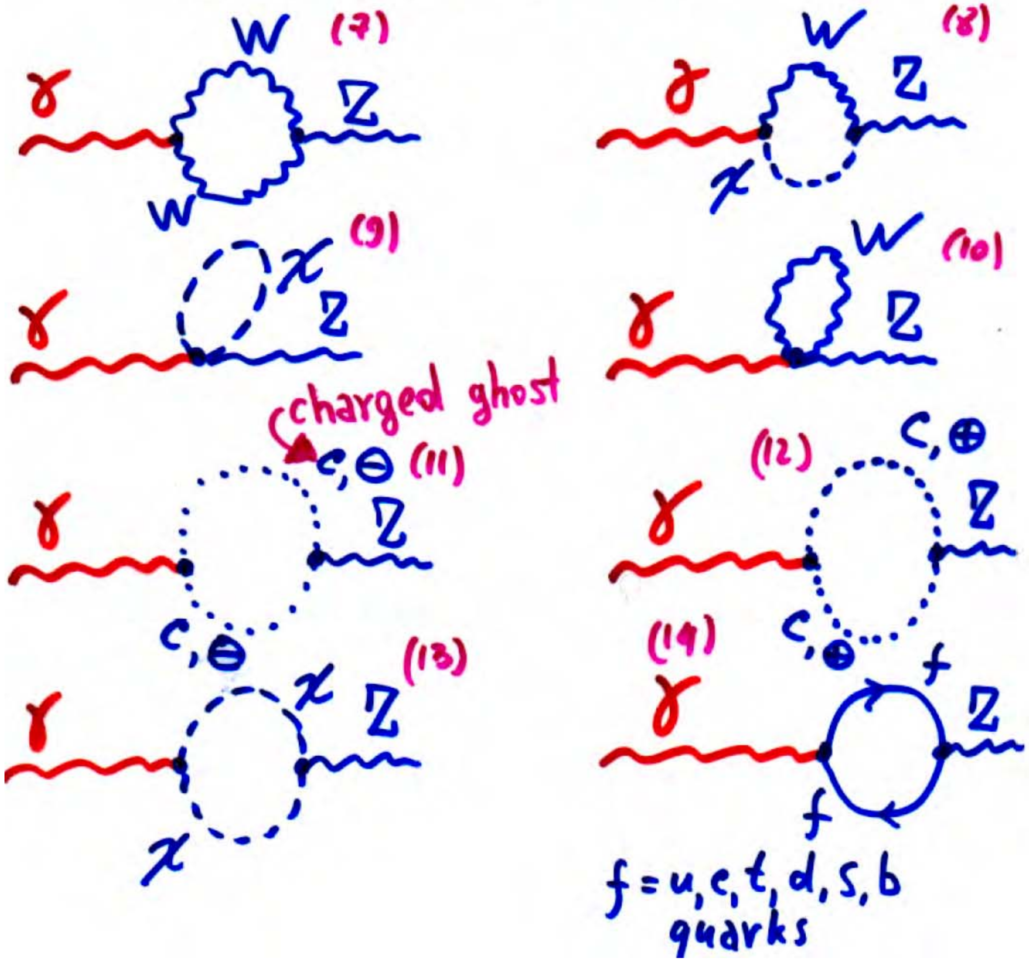
- $$\Lambda_\mu^{(3)}=i\frac{eg^2}{2M_W^2}\int\frac{d^Nk}{(2\pi)^N}(2k-p-p')_\mu\frac{(m_\nu P_L-m_\ell P_R)(\not k+m_\ell)(m_\ell P_L-m_\nu P_R)}{[(p'-k)^2-\alpha M_W^2][(p-k)^2-\alpha M_W^2][k^2-m_\ell^2]},$$

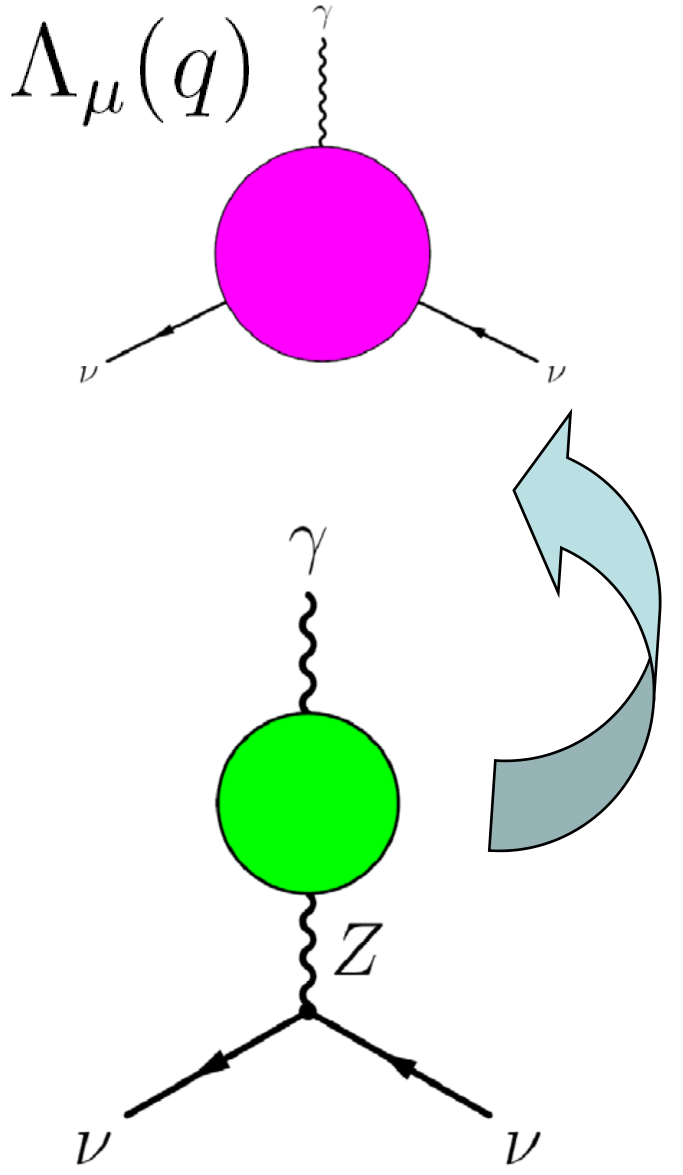
- $$\Lambda_\mu^{(4)}=i\frac{eg^2}{2}\int\frac{d^Nk}{(2\pi)^N}\gamma_\kappa^L(\not k+m_\ell)\gamma_\lambda^L\left[\delta_\beta^\kappa-(1-\alpha)\frac{(p'-k)^\kappa(p'-k)_\beta}{(p'-k)^2-\alpha M_W^2}\right]\left[\delta_\gamma^\lambda-(1-\alpha)\frac{(p-k)^\lambda(p-k)_\gamma}{(p-k)^2-\alpha M_W^2}\right]$$
$$\times\frac{\delta_\mu^\beta(2p'-p-k)^\gamma+g^{\beta\gamma}(2k-p-p')_\mu+\delta_\mu^\gamma(2p-p'-k)^\beta}{[(p'-k)^2-M_W^2][(p-k)^2-M_W^2][k^2-m_\ell^2]},$$

- $$\Lambda_\mu^{(5)+(6)}=i\frac{eg^2}{2}\int\frac{d^Nk}{(2\pi)^N}$$
$$\times\left\{\frac{\gamma_\beta^L(\not k-m_\ell)(m_\ell P_L-m_\nu P_R)}{[(p'-k)^2-M_W^2][(p-k)^2-\alpha M_W^2][k^2m_\ell^2]}\left[\delta_\mu^\beta-(1-\alpha)\frac{(p'-k)^\beta(p'-k)_\mu}{(p'-k)^2-\alpha M_W^2}\right]\right.$$
$$\left.-\frac{(m_\nu P_L-m_\ell P_R)(\not k-m_\ell)\gamma_\beta^L}{[(p'-k)^2-\alpha M_W^2][(p-k)^2-M_W^2][k^2-m_\ell^2]}\left[\delta_\mu^\beta-(1-\alpha)\frac{(p-k)^\beta(p-k)_\mu}{(p-k)^2-\alpha M_W^2}\right]\right\}$$

$$\Lambda_\mu^j(q) = \frac{g}{2\cos\theta_W}\,\Pi^{(j)}_{\mu\nu}(q)\,\frac{1}{q^2-M_Z^2} \times \left\{ g^{\nu\alpha} - (1-\alpha_Z)\frac{q^\nu q^\alpha}{q^2-\alpha_Z M_Z^2} \right\}\gamma^4_\alpha, \quad j=7,\dots,14$$

γ-Z self-energy diagrams





$\gamma - Z$ self-energy diagrams

Matrix element of electromagnetic current between massive and zero-mass neutrino states differ radically

- **For massless** ν

$$f_A(q^2)(q^2\gamma_\mu - q_\mu \not{q})\gamma_5$$

$$\overline{u}(p')\Lambda_\mu(q)u(p) = f_D(q^2)\overline{u}(p')\gamma_\mu(1+\gamma_5)u(p)$$

$f_Q(q^2) = f_D(q^2)$ ***electric*** **form factor** ***anapole*** $f_A(q^2) = f_D(q^2)/q^2$

- **For massive** ν

$$\Lambda_\mu(q) = f_Q(q^2)\gamma_\mu + f_M(q^2)i\sigma_{\mu\nu}q^\nu - f_E(q^2)\sigma_{\mu\nu}q^\nu\gamma_5 + f_A(q^2)(q^2\gamma_\mu - q_\mu \not{q})\gamma_5$$

one cannot disregard

- **Calculations of massive** ν **vertex function (calculation the complete set of Feynman diagrams)**

additional term

$$\Lambda_\mu(q) \sim f_5(q^2)\gamma_\mu\gamma_5$$

Dvornikov, Studenikin, PRD 2004

- **Direct calculation of these contributions**

$$f_5(q^2) = f_5^{(\gamma - Z)}(q^2) + f_5^{(\text{prop. vert.})}(q^2) = 0$$

Direct calculations of complete set of one-loop contributions to ν vertex function in minimally extended Standard Model

for a massive Dirac neutrino:

M.Dvorniko
A.Studenikin
PRD, 200

… in case CP conservati

- $\Lambda_\mu(q) \Rightarrow f_Q(q^2),\ f_M(q^2),\ f_E(q^2),\ f_A(q^2)$
- **Electric char** $f_Q(0)$ **= 0 and is gauge-independent**
- **Magnetic mome** $f_M(0)$ **is finite and gauge-indepen**
- **Gauge and *q*x*q* dependence** ⇒

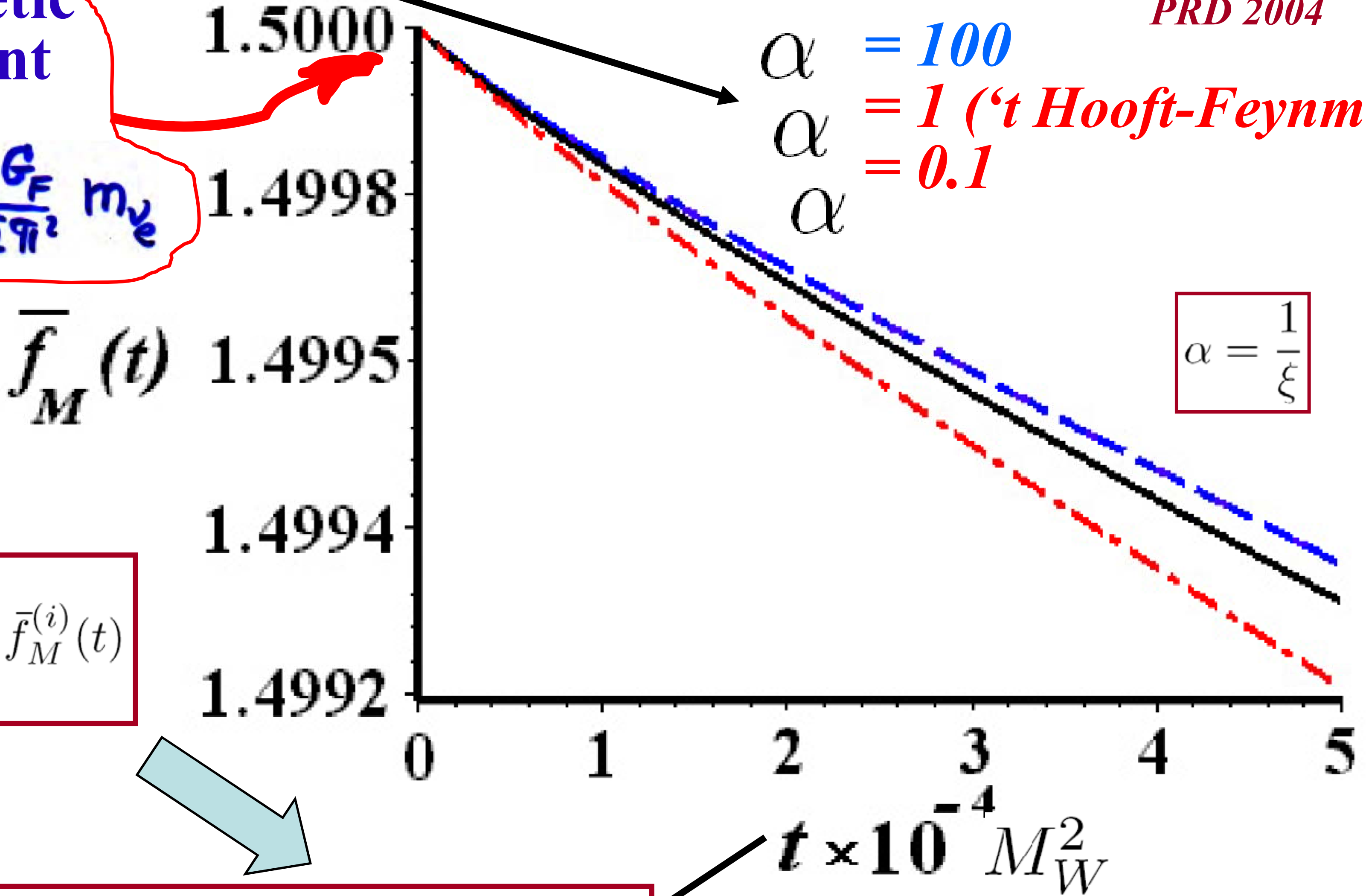
ν

Gauge and *q*x*q* dependence …

Dvornikov, Studenikin, PRD 2004

ν magnetic moment

$$\mu_e = \frac{3eG_F}{8\sqrt{2}\pi^2} m_{\nu_e}$$



$$\bar{f}_M(t) = \sum_{i=1}^{6} \bar{f}_M^{(i)}(t)$$

$$f_M(q^2) = \frac{eG_F}{4\pi^2\sqrt{2}} m_\nu \sum_{i=1}^{6} \bar{f}_M^{(i)}(q^2)$$

ν dipole magnetic form f

Magnetic moment dependence

$$\mu_\nu = \mu_\nu(m_\nu) \quad ?$$

on neutrino mass

Calculation of ν magnetic moment (massive ν, arbitrary R_ξ - gauge)

3.2

Dvornikov, Studenikin, PRD 2004

$$\Lambda_\mu(q) = f_Q(q^2)\gamma_\mu + f_M(q^2) i\sigma_{\mu\nu}q^\nu - f_E(q^2)\sigma_{\mu\nu}q^\nu\gamma_5 + f_A(q^2)(q^2\gamma_\mu - q_\mu \not{q})\gamma_5$$

magnetic moment

$$\mu(a,b,\alpha) = f_M(q^2=0)$$

two mass parameters

$$a = \left(\frac{m_\ell}{M_W}\right)^2 \qquad b = \left(\frac{m_\nu}{M_W}\right)^2$$

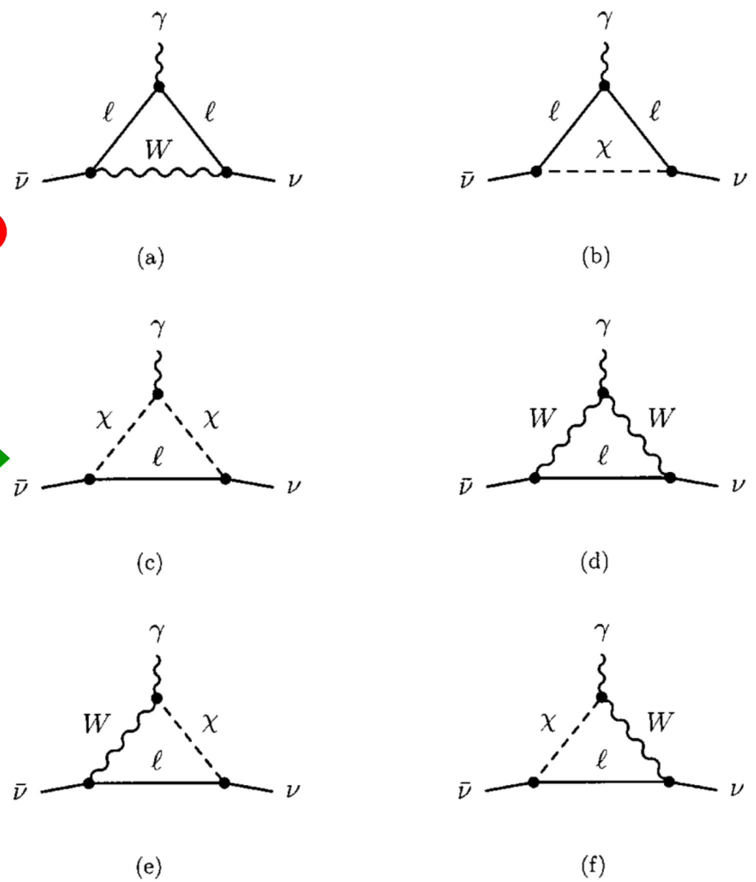
$$\mu(a,b,\alpha) = \sum_{i=1}^{6} \mu^{(i)}(a,b,\alpha)$$

and gauge-fixing parameter

$$\alpha = \frac{1}{\xi}$$

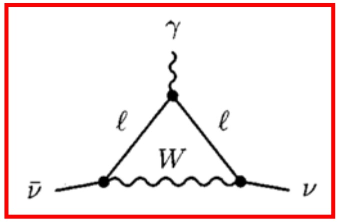
$\xi = 0$ - unitary gauge, $\xi = 1$ - 't Hooft-Feynman gauge

Proper vertices

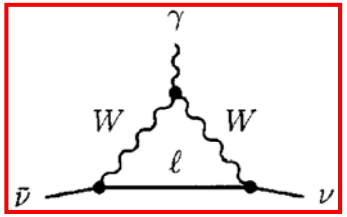


… after loop integrals calculations (e.g., for diagrams (a) and (d) contributing in unitary gauge)

$$\mu^{(1)}(a,b,\alpha)=\frac{eG_F}{4\pi^2\sqrt{2}}m_\nu\left\{\int_0^1 dz\, z(1-z^2)\frac{1}{D}-\frac{1}{2}\int_0^1 dz(1-z)^3(a-bz)\left[\frac{1}{D_\alpha}-\frac{1}{D}\right]\right.$$
$$\left.-\frac{1}{2}\int_0^1 dz(1-z)(1-3z)[\ln D_\alpha-\ln D]\right\},$$



$$\mu^{(4)}(a,b,\alpha)=\frac{eG_F}{4\pi^2\sqrt{2}}m_\nu\left\{\frac{1}{2}\int_0^1 dz\, z^2(1+2z)\frac{1}{D}\right.$$
$$+\frac{b}{2}\int_0^1 dz\int_0^z dy(1-z)^2[z(1-z)-2y]\left[\frac{1}{D_\alpha+y(1-\alpha)}-\frac{1}{D}\right]$$
$$\left.+\frac{1}{2}\int_0^1 dz\int_0^z dz(-2+9z-4z^2-6y)\{\ln[D_\alpha+y(1-\alpha)]-\ln D\}\right\},$$



where $D_\alpha=a+(\alpha-a)z-bz(1-z)$ **and** $D=D_{\alpha=1}$

Dvornikov, Studenikin,
PRD 2004, JETP 2004

… within exact calculations it is possible to expand over mass parameter $b=\left(\frac{m_\nu}{M_W}\right)^2$

$$\mu(a,b,\alpha)=\frac{eG_F}{4\pi^2\sqrt{2}}m_\nu\sum_{i=1}^{6}\{\overline{\mu}_0^{(i)}(a,\alpha)+b\,\overline{\mu}_1^{(i)}(a,\alpha)+\mathcal{O}(b^2)\}$$

Cabral-Rosetti, Bernabéu, Vidal, Zepeda, EPJ 2000

$$\mu_0(a,\alpha)=\frac{eG_F}{4\pi^2\sqrt{2}}m_\nu\frac{3}{4(1-a)^3}(2-7a+6a^2-2a^2\ln a-a^3)+\mathcal{O}(a^2)$$

$a=\left(\frac{m_\ell}{M_W}\right)^2$

$$\overline{\mu}_1(a,\alpha)=\sum_{i=1}^{6}\overline{\mu}_1^{(i)}(a,\alpha)=\frac{1}{12(1-a)^5}(5-26a+6a\ln a-36a^2-60a^2\ln a+58a^3-18a^3\ln a-a^4)$$

… μ_ν **gauge independent and finite value…**

ν magnetic moment

(for arbitrary neutrino mass, heavy neutrino…)

- LEP data →

only 3 light νs coupled to Z°,

for any additional neutrino

$$m_{\nu} \geq 45 \ Gev$$

$m_\nu \ll m_e \ll M_W$ **light** ν

$\mu_e = \frac{3eG_F}{8\sqrt{2}\pi^2} m_{\nu_e}$

$$\mu_\nu = \frac{eG_F}{4\pi^2\sqrt{2}} m_\nu \frac{3}{4(1-a)^3}\left(2 - 7a + 6a^2 - 2a^2 \ln a - a^3\right), \quad a = \left(\frac{m_e}{M_W}\right)^2$$

Gabral-Rosetti, Bernabeu, Vidal, Zepeda, Eur.Phys.J C 12 (2000) 633

Dvornikov, Studenikin, Phys.Rev.D 69 (2004) 073001; JETP 99 (2004) 254

$m_e \ll m_\nu \ll M_W$ **intermediate** ν

$$\mu_\nu = \frac{3eG_F}{8\pi^2\sqrt{2}} m_\nu \left\{1 + \frac{5}{18} b\right\}, \quad b = \left(\frac{m_\nu}{M_W}\right)^2$$

$m_e \ll M_W \ll m_\nu$ **heavy** ν

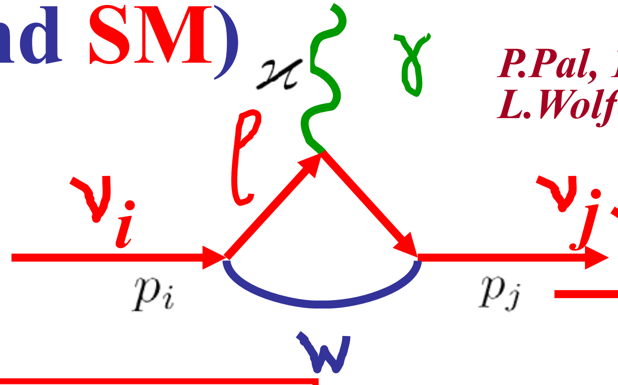
$$\mu_\nu = \frac{eG_F}{8\pi^2\sqrt{2}} m_\nu$$

$\sim 10^{-19} \mu_B \left(\frac{m_{\nu_e}}{1\,eV}\right)$

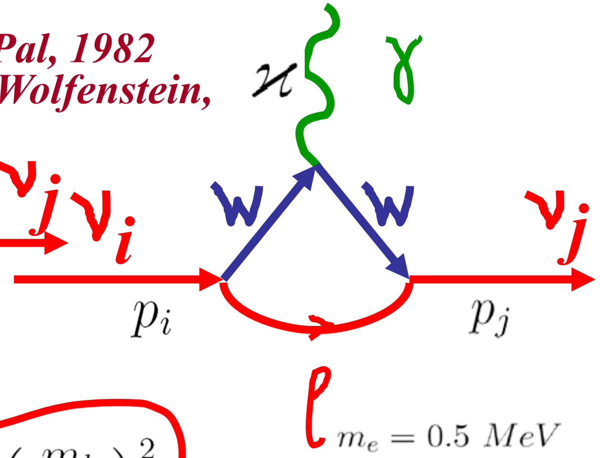
... μ_ν *in case of mixing*

3.5 Neutrino (beyond SM) dipole moments

(+ transition moments)



P.Pal, 1982
L.Wolfenstein,



- **Dirac neutrino**

$$\left.\begin{matrix}\mu_{ij}\\ \epsilon_{ij}\end{matrix}\right\} = \frac{eG_F m_i}{8\sqrt{2}\pi^2}\left(1 \pm \frac{m_j}{m_i}\right)\sum_{l=e,\,\mu,\,\tau} f(r_l)U_{lj}U^*_{li}$$

$$r_l = \left(\frac{m_l}{m_W}\right)^2$$

$m_e = 0.5\ MeV$
$m_\mu = 105.7\ MeV$
$m_\tau = 1.78\ GeV$
$m_W = 80.2\ GeV$

- $m_i, m_j \ll m_l, m_W$

$$\Rightarrow\quad f(r_l) \approx \frac{3}{2}\left(1 - \frac{1}{2}r_l\right),\quad r_l \ll 1$$

transition moments vanish because unitarity of U implies that its rows or columns represent orthogonal vectors

- **Majorana neutrino only for** $i \neq j$

$$\mu^M_{ij} = 2\mu^D_{ij} \quad and \quad \epsilon^M_{ij} = 0$$

or

$$\mu^M_{ij} = 0 \quad and \quad \epsilon^M_{ij} = 2\epsilon^D_{ij}$$

… depending on relative *CP* phase of ν_i and ν_j

- **transition moments are suppressed, Glashow-Iliopoulos-Maiani cancellation,**
- **for diagonal moments there is no GIM cancellation**

The first nonzero contribution from neutrino transition moments

$$f_{r_l} \to -\frac{3}{2} + \frac{3}{4}\left(\frac{m_l}{m_W}\right)^2 \ll 1$$

GIM cancellation

$$\left.\begin{matrix}\mu_{ij}\\ \epsilon_{ij}\end{matrix}\right\} = \frac{3eG_F m_i}{32\sqrt{2}\pi^2}\left(1 \pm \frac{m_j}{m_i}\right)\left(\frac{m_\tau}{m_W}\right)^2 \sum_{l=e,\mu,\tau}\left(\frac{m_l}{m_\tau}\right)^2 U_{lj}U^*_{li}$$

$$\mu_B = \frac{e}{2m_e}$$

- $$\left.\begin{matrix}\mu_{ij}\\ \epsilon_{ij}\end{matrix}\right\} = 4\times 10^{-23}\mu_B\left(\frac{m_i \pm m_j}{1\ eV}\right)\sum_{l=e,\mu,\tau}\left(\frac{m_l}{m_\tau}\right)^2 U_{lj}U^*_{li}$$

… neutrino radiative decay is very slow

- **Dirac ν diagonal (i=j) magnetic moment**

$\epsilon^D_{ii} = 0$ for *CP*-invariant interactions

$\mu^M_{ii} = \epsilon^M_{ii} = 0$

$$\mu_{ii} = \frac{3eG_F m_i}{8\sqrt{2}\pi^2}\left(1 - \frac{1}{2}\sum_{l=e,\mu,\tau} r_l |U_{li}|^2\right) \approx 3.2\times 10^{-19}\left(\frac{m_i}{1\ eV}\right)\mu_B$$

$$r_l = \left(\frac{m_l}{m_W}\right)^2$$

no GIM cancellation

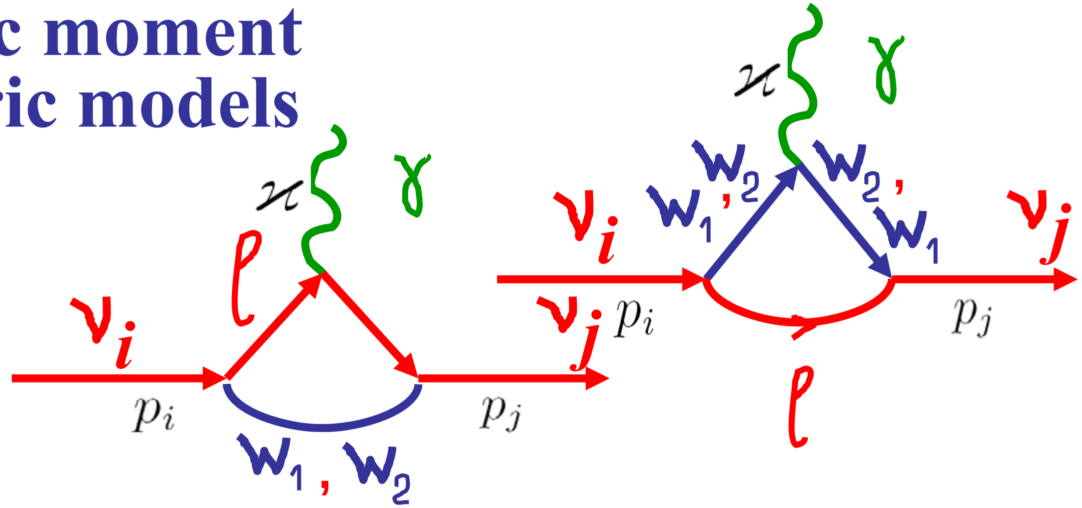
Lee, Shrock, Fujikawa, 19

- μ^D_{ii} - to leading order - independent on U_{li} and $m_{l=e,\mu,\tau}$
- $\mu_e^2 = \sum_{i=1,2,3} |U_{ie}|^2 \mu^2_{ii}$ …possibility to measure fundamental μ^D_{ii}

$\mu^D_{ii} = 0$ for massless ν (in the absence of right-handed charged currents)

3.6 Neutrino magnetic moment in left-right symmetric models

$SU_L(2) \times SU_R(2) \times U(1)$



Gauge bosons mass states

$$W_1 = W_L \cos\xi - W_R \sin\xi$$
$$W_2 = W_L \sin\xi + W_R \cos\xi$$

with mixing angle ξ **of gauge bosons** $W_{L,R}$ **with pure** $(V \pm A)$ **couplings**

Kim, 1976; Marciano, Sanda, 1977; Beg, Marciano, Ruderman, 1978

$$\mu_{\nu_l} = \frac{eG_F}{2\sqrt{2}\pi^2}\left[m_l\left(1 - \frac{m_{W_1}^2}{m_{W_2}^2}\right)\sin 2\xi + \frac{3}{4}m_{\nu_l}\left(1 + \frac{m_{W_1}^2}{m_{W_2}^2}\right)\right]$$

… charged lepton mass … *… neutrino mass …*

...the present status...

to have visible $m_\nu \neq 0$

is not an easy task for

experimentalists

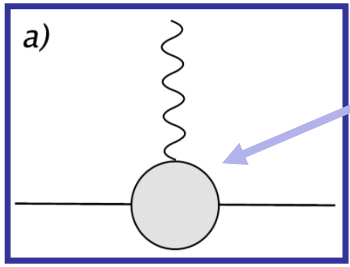
and theoreticians

3.3 Naïve relationship between the size of m_ν and μ_ν

… problem to get la μ_ν and still acceptable m_ν

If μ_ν is generated by physics beyond the SM at energy scale Λ,

P.Vogel e.a., 2006

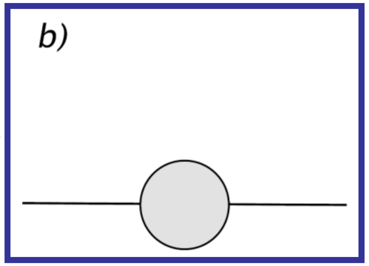


then

$$\mu_\nu \sim \frac{eG}{\Lambda},$$

…combination of constants and loop factors…

contribution to m_ν given by



, then $m_\nu \sim G\Lambda$

Voloshin, 1988; Barr, Freire, Zee, 1990

$$m_\nu \sim \frac{\Lambda^2}{2m_e}\frac{\mu_\nu}{\mu_B} \sim \frac{\mu_\nu}{10^{-18}\mu_B}[\Lambda(\mathrm{TeV})]^2 \ \mathrm{eV}$$

from quadratic divergence appearing in renormalization of dimension four neutrino mass

Large magnetic moment

$\mu_\nu = \mu_\nu(m_\nu, m_{B^+}, m_{e^-})$

- In the L-R symmetric models ($SU(2)_L \times SU(2)_R \times U(1)$)

Kim, 1976
Beg, Marciano, Ruderman, 1978

- *Voloshin, 1988*

"On compatibility of m_ν all with large μ_ν of neutrino", Sov.J.Nucl.Phys. 48 (1988) 512

… there may be $SU(2)_\nu$ symmetry that forbids m_ν but not μ_ν

- *Bar, Freire, Zee, 1990*
- supersymmetry
- extra dimensions
- model-independent constraint μ_ν

considerable enhancement of μ_ν to experimentally relevant range

$\mu_\nu^D \leq 10^{-15}\mu_B$

$\mu_\nu^M \leq 10^{-14}\mu_B$

for BSM ($\Lambda \sim 1$ TeV) without fine tuning and under the assumption that $\delta m_\nu \leq 1$ eV

Bell, Cirigliano, Ramsey-Musolf, Vogel, Wise, 2005

3.11

ν charge radius and anapole moment

$$\Lambda_\mu(q) = f_Q(q^2)\gamma_\mu + f_M(q^2) i\sigma_{\mu\nu}q^\nu - f_E(q^2)\sigma_{\mu\nu}q^\nu\gamma_5 + f_A(q^2)(q^2\gamma_\mu - q_\mu \not{q})\gamma_5$$

1. electric; 2. magnetic; 3. electric (dipole); 4. anapole

Although it is usually assumed that ν are electrically neutral (charge quantization implies $Q \sim \frac{1}{3}e$),

ν can dissociates into charged particles so that : $f_Q(q^2) \neq 0 \; for \; q^2 \neq 0$

$$f_Q(q^2) = f_Q(0) + q^2 \frac{df_Q}{dq^2}(0) + \cdots,$$

where the massive ν charge radius

$$\langle r_\nu^2 \rangle = -6 \frac{df_Q}{dq^2}(0)$$

For massless ν anapole moment

$$a_\nu = f_A(q^2) = \frac{1}{6}\langle r_\nu^2 \rangle$$

Interpretation of charge radius as an observable is rather delicate issue: $\langle r_\nu^2 \rangle$ represents a correction to tree-level electroweak scattering amplitude between ν and charged particles, which receives radiative corrections from several diagrams (including γ exchange) to be considered simultaneously calculated CR is infinite and gauge dependent quantity. For massless ν, a_ν and $\langle r_\nu^2 \rangle$ can be defined (finite and gauge independent) from scattering cross section.

? ? ? For massive ν ? ? ?

Bernabeu, Papavassiliou, Vidal, 2004