Theoretical Aspects of Flavour in the Leptonic Sector

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Where Do We Stand?

- Exciting Time in ν Physics: recent hints/evidences of large θ_{13} from T2K, MINOS, Double Chooz, Daya Bay and RENO
- Latest 3 neutrino global analysis (including recent results from reactor experiments and T2K):

Capozzi, Fogli, Lisi, Marrone, Montanino, Palazzo (2013, updated March 2014)

Parameter	Best fit	1σ range	2σ range	3σ range
$\delta m^2/10^{-5}$ eV^2 (NH or IH)	7.54	7.32 – 7.80	7.15 – 8.00	6.99 – 8.18
$\sin^2\theta_{12}/10^{-1}$ (NH or IH)	3.08	2.91 – 3.25	2.75 – 3.42	2.59 – 3.59
$\Delta m^2/10^{-3}$ eV^2 (NH)	2.43	2.37 – 2.49	2.30 – 2.55	2.23 – 2.61
$\Delta m^2/10^{-3}$ eV^2 (IH)	2.38	2.32 – 2.44	2.25 – 2.50	2.19 – 2.56
$\sin^2\theta_{13}/10^{-2}$ (NH)	2.34	2.15 – 2.54	1.95 – 2.74	1.76 – 2.95
$\sin^2\theta_{13}/10^{-2}$ (IH)	2.40	2.18 – 2.59	1.98 – 2.79	1.78 – 2.98
$\sin^2\theta_{23}/10^{-1}$ (NH)	4.37	4.14 – 4.70	3.93 – 5.52	3.74 – 6.26
$\sin^2\theta_{23}/10^{-1}$ (IH)	4.55	4.24 – 5.94	4.00 – 6.20	3.80 – 6.41
δ/π (NH)	1.39	1.12 – 1.77	0.00 – 0.16 $\oplus$ 0.86 – 2.00	—
δ/π (IH)	1.31	0.98 – 1.60	0.00 – 0.02 $\oplus$ 0.70 – 2.00	—

➡ Evidence of $\theta_{13} \neq 0$

➡ hints of $\theta_{23} \neq \pi/4$

➡ expectation of Dirac CP phase δ

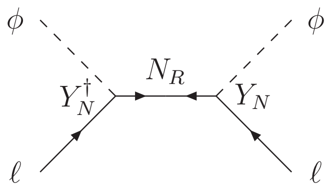
➡ no clear preference for hierarchy

➡ Majorana vs Dirac

Theoretical Challenges

(i) Absolute mass scale: Why $m_\nu << m_{u,d,e}$?

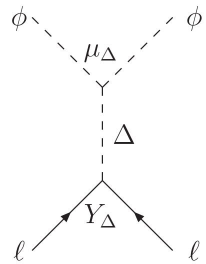
- seesaw mechanism: most appealing scenario ⇒ Majorana
- UV completions of Weinberg operators HHLL
  - Type-I seesaw: exchange of singlet fermions



Minkowski, 1977; Yanagida, 1979;
Glashow, 1979;
Gell-mann, Ramond, Slansky,1979;
Mohapatra, Senjanovic, 1979;

N_R: $SU(3)_C \times SU(2)_W \times U(1)_Y$ ~(1,1,0)

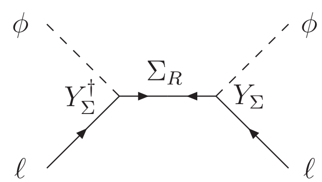
  - Type-II seesaw: exchange of weak triplet scalar



Lazarides, 1980; Mohapatra, Senjanovic, 1980

Δ: $SU(3)_C \times SU(2)_W \times U(1)_Y$ ~(1,3,2)

  - Type-III seesaw: exchange of weak triplet fermion



Foot, Lew, He, Joshi, 1989; Ma, 1998

Σ_R: $SU(3)_C \times SU(2)_W \times U(1)_Y$ ~(1,3,0)

Theoretical Challenges

(i) Absolute mass scale: Why $m_\nu << m_{u,d,e}$?

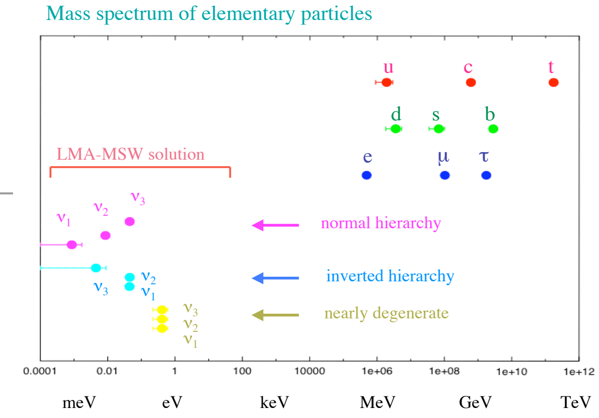
- seesaw mechanism: most appealing scenario ⇒ Majorana
  - GUT scale (type-I) vs TeV scale (type-II, III, inverse seesaw)
- TeV scale new physics (SUSY, extra dimension, U(1)´) ⇒ Dirac or Majorana

(ii) Flavor Structure: Why neutrino mixing large while quark mixing small?

- neutrino anarchy: no parametrically small number — Hall, Murayama, Weiner (2000); de Gouvea, Murayama (2003)
  - near degenerate spectrum, large mixing
  - predictions strongly depend on choice of statistical measure
  - still alive and kicking — de Gouvea, Murayama (2012)
- family symmetry: there's a structure, expansion parameter (~~symmetry~~ effect)
  - mixing result from dynamics of underlying symmetry
  - for leptons only (normal or inverted)
  - for quarks and leptons: quark-lepton connection ↔ GUT (normal)
- Alternative?

- In this talk: assume 3 generations, no LSND/MiniBoone/Reactor Anomaly
- **These scenarios have drastically different predictions**
- **precision measurements allow for distinguishing models**

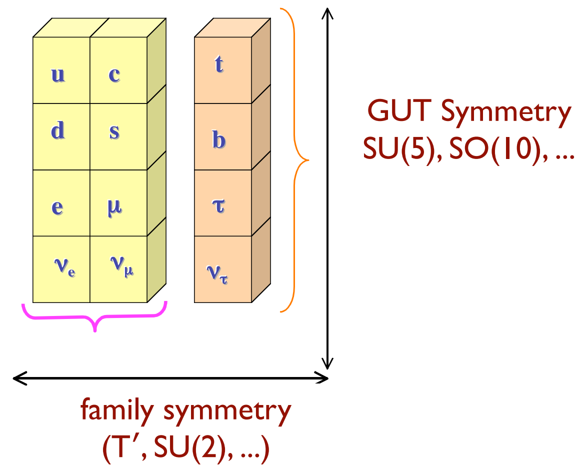
Origin of Mass Hierarchy and Mixing



- In the SM: 22 physical quantities which seem unrelated
- Question arises whether these quantities can be related
- No fundamental reason can be found in the framework of SM
- less ambitious aim ⇒ reduce the # of parameters by imposing symmetries
  - SUSY Grand Unified Gauge Symmetry
    - GUT relates quarks and leptons: quarks & leptons in same GUT multiplets
      - one set of Yukawa coupling for a given GUT multiplet ⇒ intra-family relations
    - seesaw mechanism naturally implemented
    - proton decay, leptogenesis, LFV charged lepton decay
  - Family Symmetry
    - relate Yukawa couplings of different families
      - inter-family relations ⇒ further reduce the number of parameters

⇒ Experimentally testable *correlations* among physical observables

Origin of Flavor Mixing and Mass Hierarchy



- Several models have been constructed based on
  - GUT Symmetry [SU(5), SO(10)] ⊕ Family Symmetry G_F
- Family Symmetries G_F based on continuous groups:
  - U(1)
  - SU(2)
  - SU(3)
- Recently, models based on discrete family symmetry groups have been constructed
  - A_4 (tetrahedron)
  - T´ (double tetrahedron)
  - S_3 (equilateral triangle)
  - S_4 (octahedron, cube)
  - A_5 (icosahedron, dodecahedron)
  - Δ_{27}
  - Q_4

Motivation: Tri-bimaximal (TBM) neutrino mixing

Tri-bimaximal Neutrino Mixing

- Tri-bimaximal Mixing Pattern

Harrison, Perkins, Scott (1999)

$$U_{TBM} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix}$$

$$\sin^2\theta_{\mathrm{atm,\,TBM}} = 1/2 \qquad \sin^2\theta_{\odot,\mathrm{TBM}} = 1/3 \qquad \sin\theta_{13,\mathrm{TBM}} = 0.$$

- General approach:
  - PMNS = LO prediction (TBM, BM, ...) + corrections
  - corrections:
    - higher order terms in super potential (family symmetry)
    - contributions from charged lepton sector (GUT symmetry)

Non-Abelian Finite Family Symmetry A4

- TBM mixing matrix: can be realized with finite group family symmetry based on A_4 Ma, Rajasekaran (2001); Babu, Ma, Valle (2003); ...

- A_4: even permutations of 4 objects

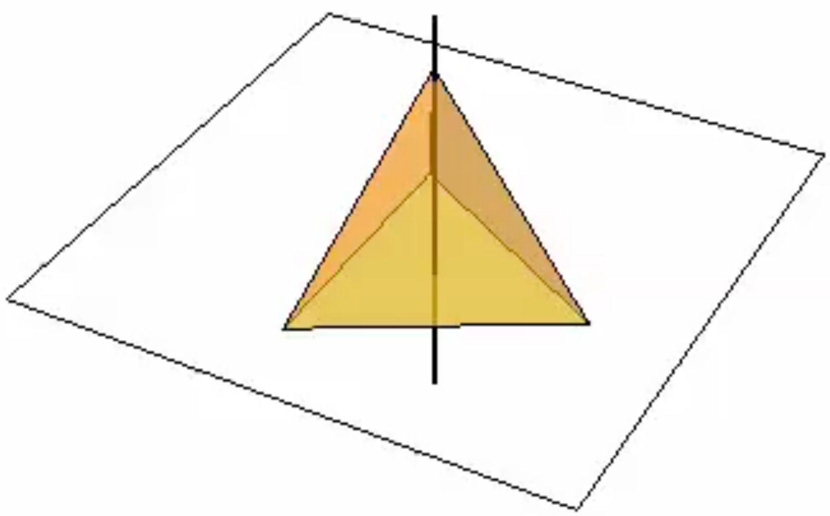
  S: (1234) → (4321)

  T: (1234) → (2314)

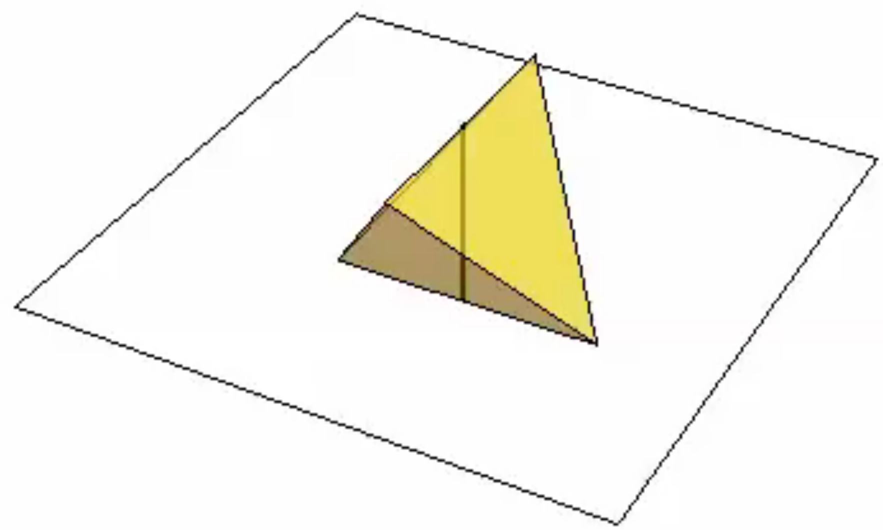
- Group of order 12
- Invariant group of tetrahedron

Invariant Group of Tetrahedron

T: (1234) → (2314)



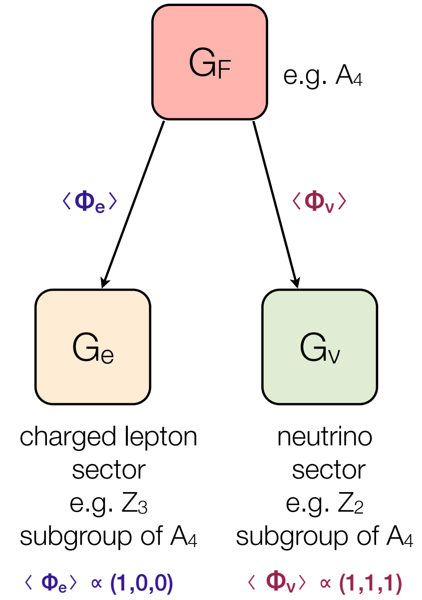
S: (1234) →(4321)



Non-Abelian Finite Family Symmetry A4

- TBM mixing matrix: can be realized with finite group family symmetry based on A_4

Ma, Rajasekaran (2001); Babu, Ma, Valle (2003); .



- A_4: even permutations of 4 objects

  S: (1234) → (4321)

  T: (1234) → (2314)

- Group of order 12
- Invariant group of tetrahedron
- TBM arises due to the misalignment of symmetry breaking patterns
- Problem: A_4 does not seem to give rise to quark mixing

Example: T′ Family Symmetry

- SU(5) compatibility ⇒ Double Tetrahedral Group T´ M.-C.C, K.T. Mahanthappa (2007, 2009)
- Symmetries ⇒ 9 parameters in Yukawa sector ⇒ 22 physical observables
- neutrino mixing angles from group theory (CG coefficients)
- TBM: misalignment of symmetry breaking patterns
  - neutrino sector: T’ → G_{TST2} , charged lepton sector: T’ → G_T
- GUT symmetry ⇒ contributions to mixing parameters from charged lepton sector

⇒ deviation from TBM related to Cabibbo angle θ_c ,consequence of Georgi-Jarlskog relations

$$\theta_{13} \simeq \theta_c/3\sqrt{2}$$ ← CG's of SU(5) & T´

$$\tan^2\theta_\odot \simeq \tan^2\theta_{\odot,TBM} + \frac{1}{2}\theta_c\cos\delta$$

prediction in 2009 for Dirac CP phase: δ = 227 degrees

quark CP phase: γ = 45.6 degrees

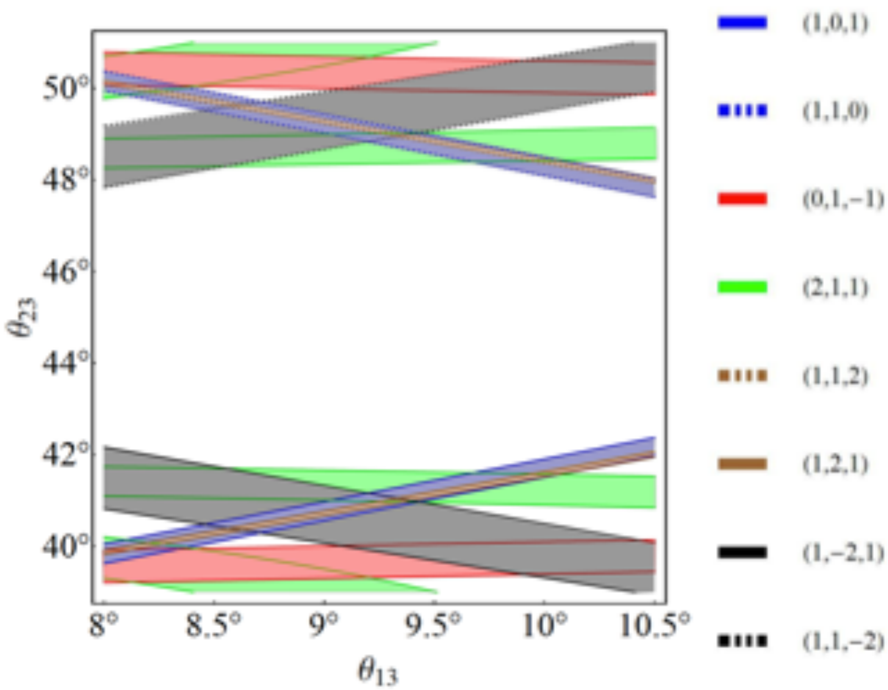
- large θ_{13} possible with one additional singlet flavon

M.-C. C., J. Huang, K.T. Mahanthappa, A. Wijiangco (2013)

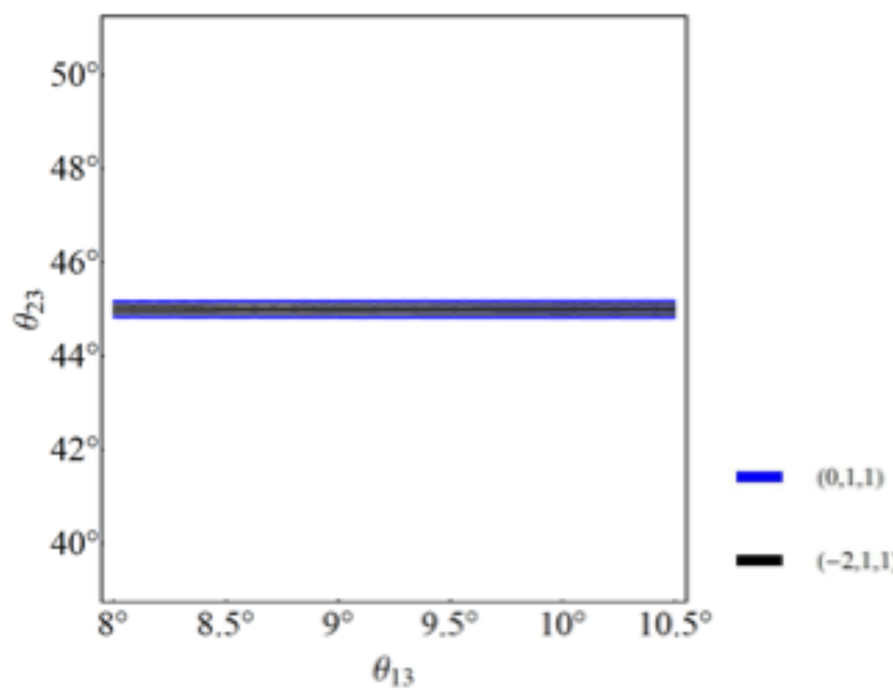
"Large" Deviations from TBM in A_4

M.-C.C, J. Huang, J. O'Bryan, A. Wijangco, F. Yu, (2012)

- other A4 breaking patterns:



normal

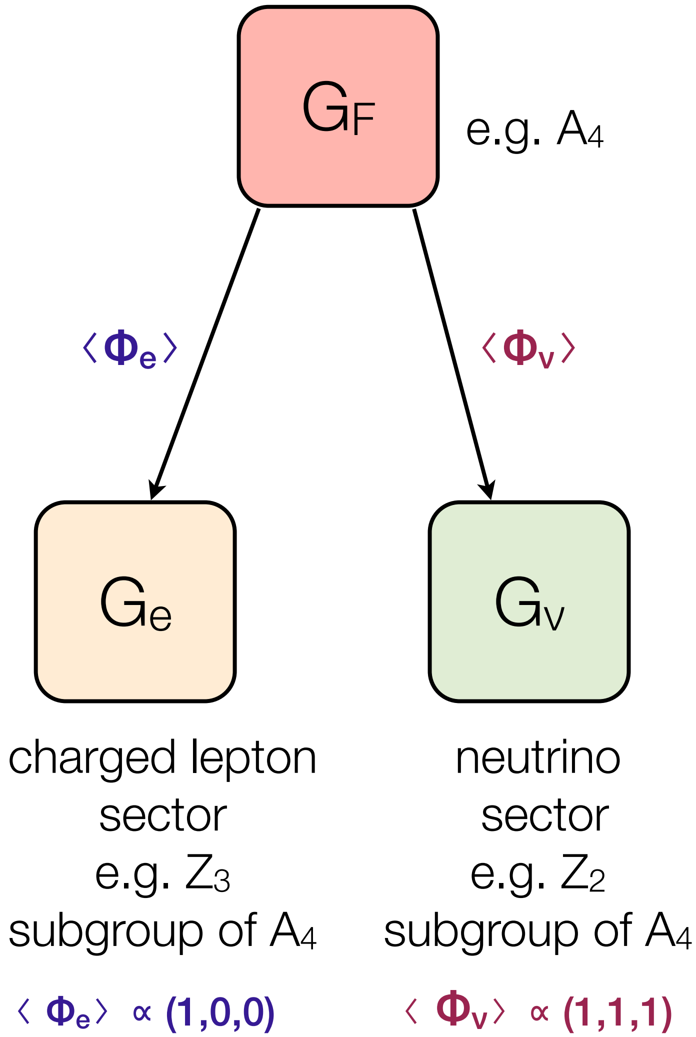


inverted

deviations correlated

non-maximal θ_{23} ⇨ normal hierarchy

Flavor Model Structure: A_4 Example



- interplay between the symmetry breaking patterns in two sectors lead to lepton mixing (BM, TBM, ...)
- symmetry breaking achieved through flavon VEVs
- each sector preserves different residual symmetry
- full Lagrangian does not have these residual symmetries
- general approach: include high order terms in holomorphic superpotential
- possible to construct models where higher order holomorphic superpotential terms vanish to ALL orders
- quantum correction?

  ⇒ uncertainty in predictions due to

  Kähler corrections

Leurer, Nir, Seiberg (1993); Dudas, Pokorski, Savoy (1995); Dreiner, Thomeier (2003);

Kähler Corrections

M.-C.C., M. Fallbacher, M. Ratz, C. Staudt (2012)

- Superpotential: holomorphic

$$\mathscr{W}_{\text{leading}} \;=\; \frac{1}{\Lambda}(\Phi_e)_{gf}\, L^g\, R^f\, H_d + \frac{1}{\Lambda\,\Lambda_\nu}(\Phi_\nu)_{gf}\, L^g\, H_u\, L^f\, H_u$$

$$\longrightarrow \mathscr{W}_{\text{eff}} \;=\; (Y_e)_{gf}\, L^g\, R^f\, H_d + \frac{1}{4}\kappa_{gf}\, L^g\, H_u\, L^f\, H_u$$

order parameter
<flavon vev> / Λ ~ θ_c

- Kähler potential: non-holomorphic

$$K \;=\; K_{\text{canonical}} + \Delta K$$

- Canonical Kähler potential

$$K_{\text{canonical}} \;\supset\; \left(L^f\right)^\dagger\, \delta_{fg}\, L^g + \left(R^f\right)^\dagger\, \delta_{fg}\, R^g$$

- Correction

$$\Delta K \;=\; \left(L^f\right)^\dagger\, (\Delta K_L)_{fg}\, L^g + \left(R^f\right)^\dagger\, (\Delta K_R)_{fg}\, R^g$$

- can be induced by flavon VEVs
- important for order parameter ~ θ_c
- can lead to non-trivial mixing

Kähler Corrections

M.-C.C., M. Fallbacher, M. Ratz, C. Staudt (2012)

- Consider infinitesimal change, x :

$$K \;=\; K_{\text{canonical}} + \Delta K \;=\; L^{\dagger}\,(1-2x\,P)\,L$$

- rotate to canonically normalized L':

$$L \;\to\; L' \;=\; (1-x\,P)\,L$$

⇒ corrections to neutrino mass matrix

$$m_\nu(x) \;\simeq\; m_\nu + x\,P^T\,m_\nu + x\,m_\nu\,P$$

⇒ differential equation

$$\frac{\mathrm{d}m_\nu}{\mathrm{d}x} \;=\; P^T\,m_\nu + m_\nu\,P$$

- same structure as the RG evolutions for neutrino mass operator
- size of Kähler corrections can be substantially larger (no loop suppression)

A_4 Example

M.-C.C., M. Fallbacher, M. Ratz, C. Staudt (2012)

- Kähler corrections due to flavon field:
  - quadratic in flavon

$$\Delta K^{\mathbf{quadratic}}_{\phi^{(\prime)}} \supset \frac{1}{\Lambda^2} \sum_{X}^{6} \kappa^{X}_{\phi^{(\prime)},\mathbf{quadratic}} \left(L\phi^{(\prime)}\right)^{\dagger}_{X} \left(L\phi^{(\prime)}\right)_{X} + \text{h.c.}$$

$$(L\Phi_\nu)^\dagger (L\Phi_\nu) \quad \text{and} \quad (L\Phi_e)^\dagger (L\Phi_e)$$

  - such terms cannot be forbidden by any (conventional) symmetry
  - Kähler corrections once flavon fields attain VEVs
  - additional parameters $\kappa^{X}_{\phi^{(\prime)}}$ reduce predictivity of the scheme

Back to A_4 Example

M.-C.C., M. Fallbacher, M. Ratz, C. Staudt (2012)

- Contributions from Flavon VEVs (1,0,0) and (1,1,1)
  - five independent "basis" matrices

$$P_{\mathrm{I}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad P_{\mathrm{II}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad P_{\mathrm{III}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$P_{\mathrm{IV}} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}, \quad P_{\mathrm{V}} = \begin{pmatrix} 0 & \mathrm{i} & -\mathrm{i} \\ -\mathrm{i} & 0 & \mathrm{i} \\ \mathrm{i} & -\mathrm{i} & 0 \end{pmatrix}$$

  - RG correction: essentially along P_{III} = diag(0,0,1) direction due to y_τ dominance
  - Kähler corrections can be along different directions than RG

Enhanced θ13

M.-C.C., M. Fallbacher, M. Ratz, C. Staudt (2012)

- consider change due to correction along P_V direction
- Kähler metric:

$$\mathcal{K}_L = 1 - 2xP \quad \text{with} \quad P_{\mathrm{V}} = \begin{pmatrix} 0 & \mathrm{i} & -\mathrm{i} \\ -\mathrm{i} & 0 & \mathrm{i} \\ \mathrm{i} & -\mathrm{i} & 0 \end{pmatrix}$$

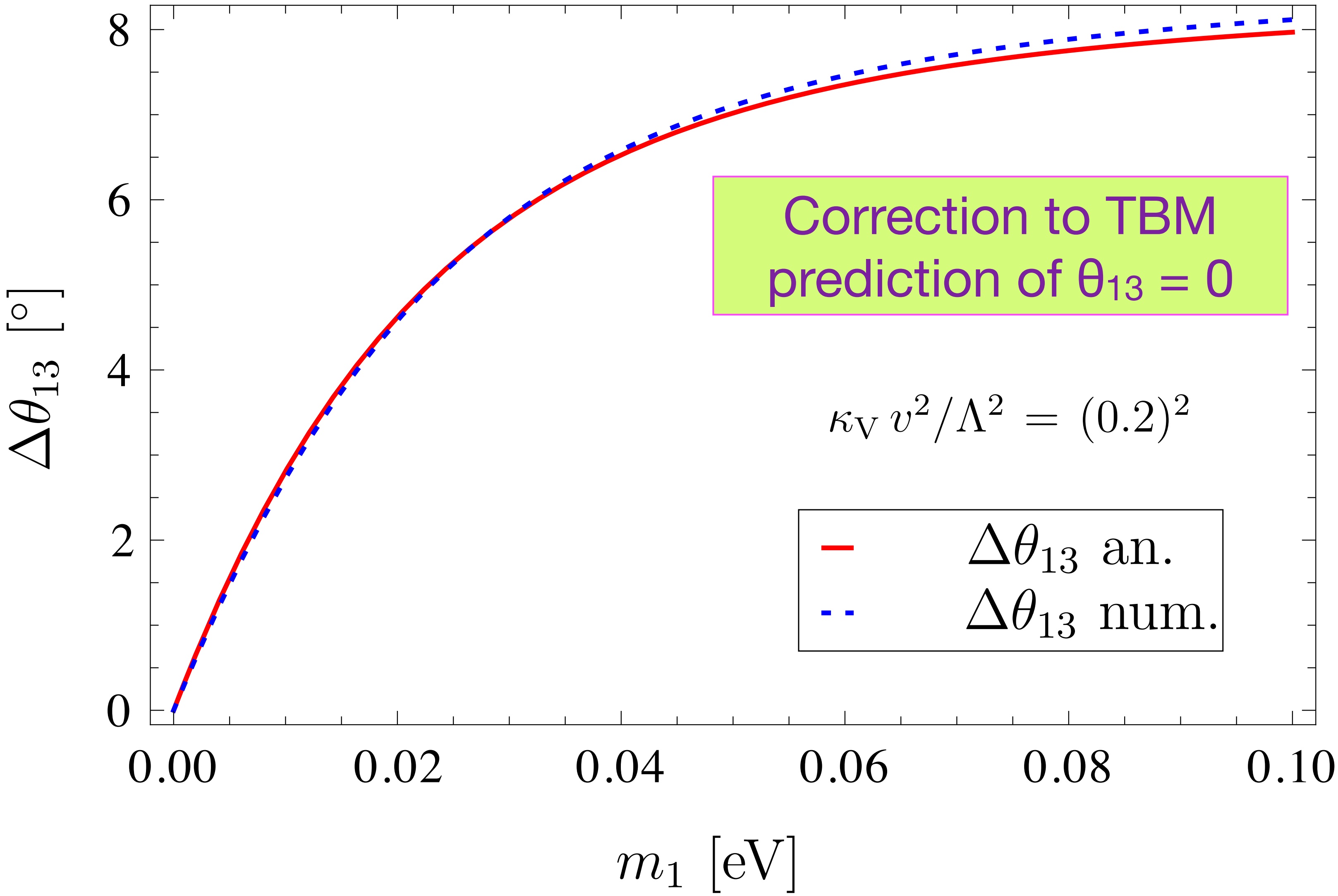
- Contributions of flavon VEV: $\langle \Phi \rangle = (1, 1, 1)\ v$
- Corrections to the leading order TBM prediction ($m_e \ll m_\mu \ll m_\tau$)

$$\Delta\theta_{13} \simeq \kappa_{\mathrm{V}} \cdot \frac{v^2}{\Lambda^2} \cdot 3\sqrt{6}\, \frac{m_1}{m_1 + m_3}$$

- Complex matrix $P_V \Rightarrow$ CP violation induced
- for the example considered: $\delta \approx \pi/2$

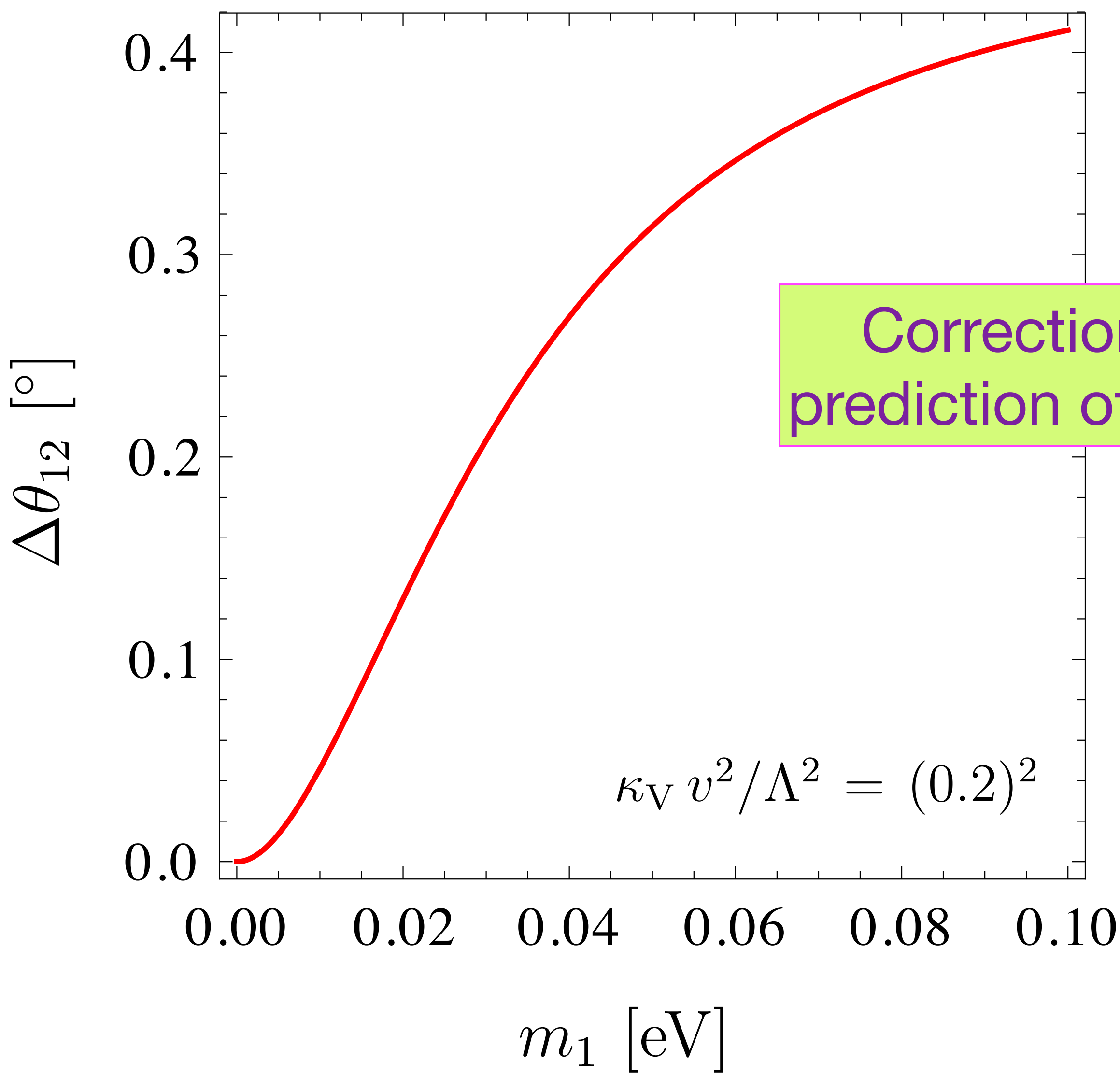
An Example: Enhanced θ_{13}

M.-C.C., M. Fallbacher, M. Ratz, C. Staudt (2012)



Corresponding Change in θ_{12}

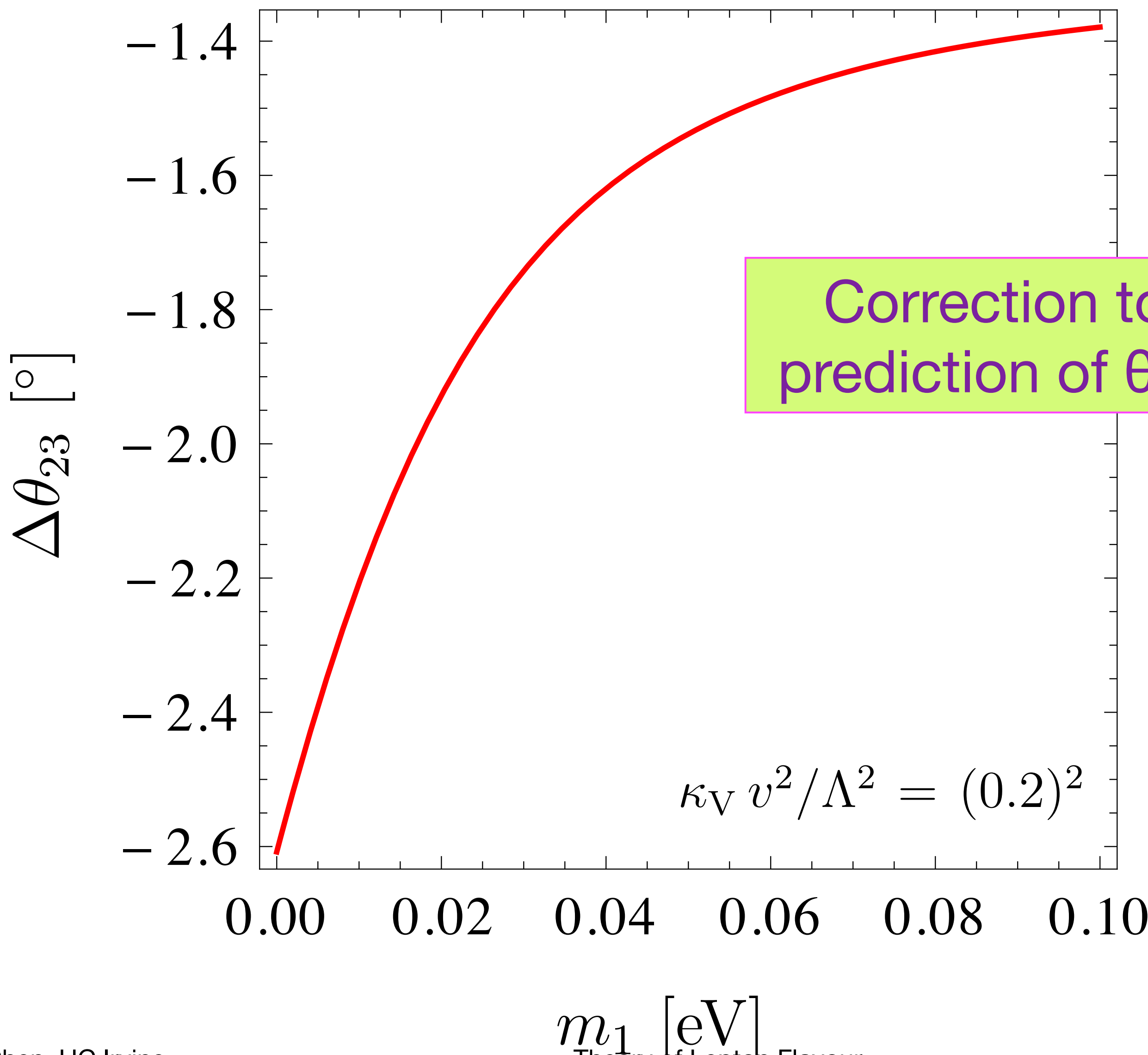
M.-C.C., M. Fallbacher, M. Ratz, C. Staudt (2012)



Correction to TBM prediction of θ_{12} = 35.3°

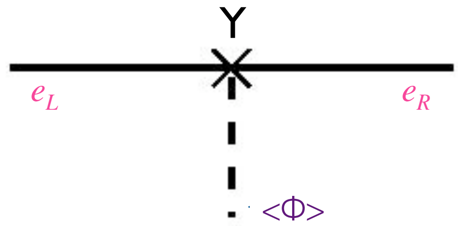
Corresponding Change in θ_{23}

M.-C.C., M. Fallbacher, M. Ratz, C. Staudt (2012)



Correction to TBM prediction of $\theta_{23} = 45°$

A Novel Origin of CP Violation

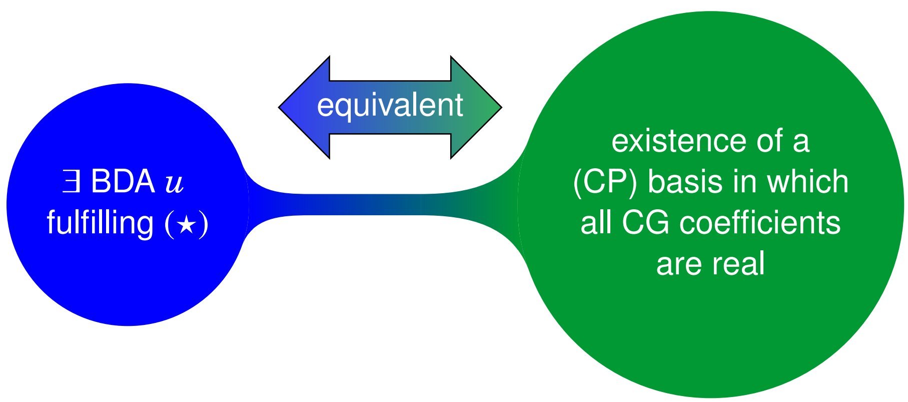


- Conventionally:
  - explicit CP violation: complex Yukawa couplings
  - spontaneous CP violation: complex Higgs VEVs
- Complex CG coefficients in discrete groups ⇒ *explicit* CP violation in quark and lepton sectors (e.g. $\delta \neq 0$)

M.-C.C, K.T. Mahanthappa, Phys. Lett. B681, 444 (2009);
M.-C.C, M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner, Nucl. Phys. B (2014)

- Conditions for a discrete group to admit real CG's   Bickerstaff, Damhus, 1985

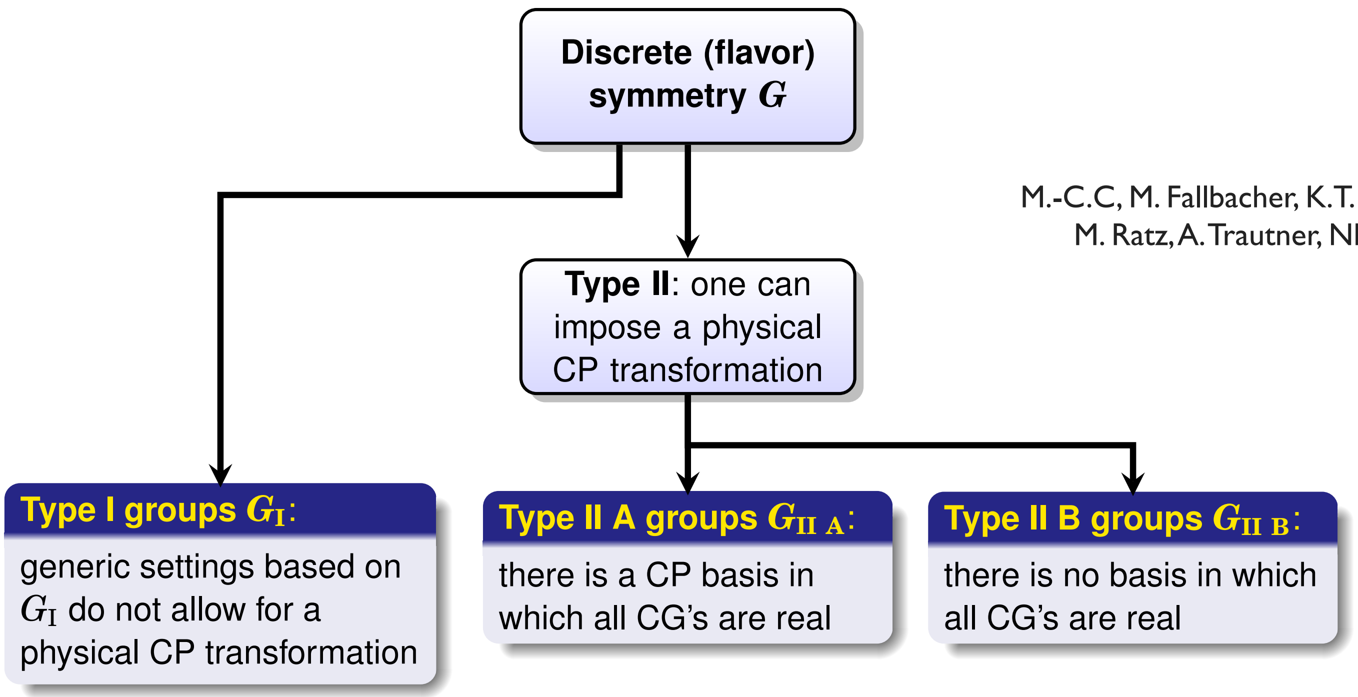
∃ automorphism u, such that $\lambda_k(R) = \lambda_k(u(R))^*$ for all $R \in G$



A Novel Origin of CP Violation

- **more generally, for discrete groups that do not have class-inverting, involutory automorphism, CP is generically broken by complex CG coefficients (Type I Group)**
- **Non-existence of such automorphism ⇔ physical CP violation**

CP Violation from Group Theory!



M.-C.C, M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner, NPB (2014)

Example for a type I group:

$$\Delta(27)$$

- decay asymmetry in a toy model
- prediction of CP violating phase from group theory

Toy Model based on Δ(27)

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

- Field content

fermions

field	S	X	Y	Ψ	Σ
$\Delta(27)$	$\mathbf{1}_0$	$\mathbf{1}_1$	$\mathbf{1}_3$	$\mathbf{3}$	$\mathbf{3}$
U(1)	$q_\Psi - q_\Sigma$	$q_\Psi - q_\Sigma$	0	q_Ψ	q_Σ

$q_\Psi - q_\Sigma \neq 0$

- Interactions

$$\mathscr{L}_{\text{toy}} = f\left[S_{\mathbf{1}_0} \otimes \left(\overline{\Psi}\Sigma\right)_{\mathbf{1}_0}\right]_{\mathbf{1}_0} + g\left[X_{\mathbf{1}_1} \otimes \left(\overline{\Psi}\Sigma\right)_{\mathbf{1}_2}\right]_{\mathbf{1}_0}$$
$$+ h_\Psi\left[Y_{\mathbf{1}_3} \otimes \left(\overline{\Psi}\Psi\right)_{\mathbf{1}_6}\right]_{\mathbf{1}_0} + h_\Sigma\left[Y_{\mathbf{1}_3} \otimes \left(\overline{\Sigma}\Sigma\right)_{\mathbf{1}_6}\right]_{\mathbf{1}_0} + \text{h.c.}$$
$$= F^{ij}\, S\,\overline{\Psi}_i\Sigma_j + G^{ij}\, X\,\overline{\Psi}_i\Sigma_j + H^{ij}_\Psi\, Y\,\overline{\Psi}_i\Psi_j + H^{ij}_\Sigma\, Y\,\overline{\Sigma}_i\Sigma_j + \text{h.c.}$$

$$F = f\,\mathbb{1}_3$$

$$G = g\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

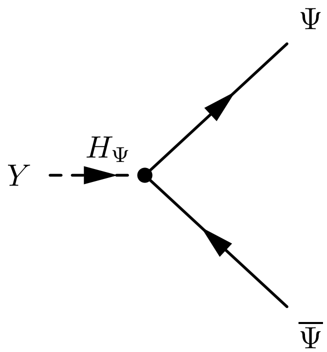
$$H_{\Psi/\Sigma} = h_{\Psi/\Sigma}\begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}$$
with $\omega := e^{2\pi i/3}$

Toy Model based on Δ(27)

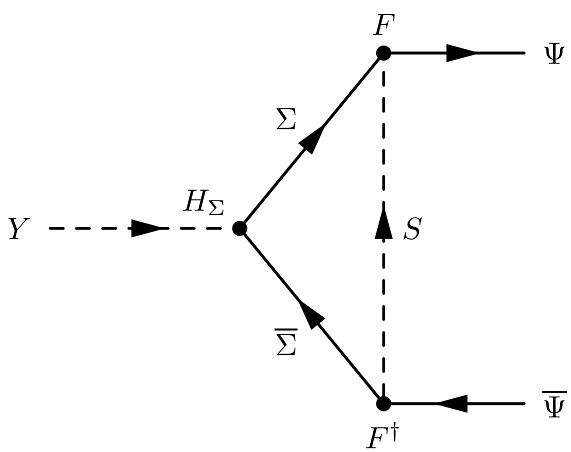
M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

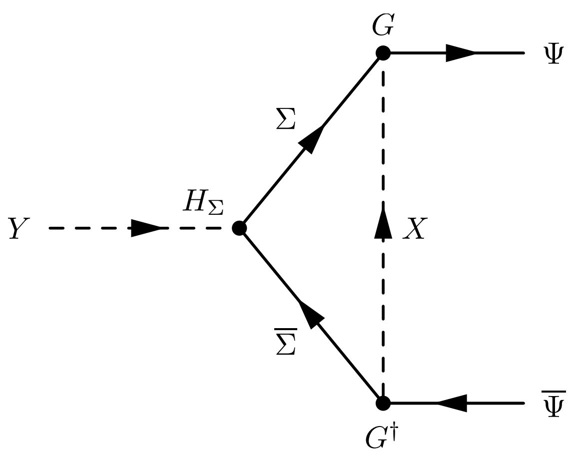
- Particle decay $Y \to \overline{\Psi}\Psi$

interference of



with





Decay Asymmetry

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

- Decay asymmetry

$$\varepsilon_{Y\to\overline{\Psi}\Psi} = |f|^2 \ \mathrm{Im}\left[I_S\right] \ \mathrm{Im}\left[h_\Psi\, h_\Sigma^*\right] + |g|^2 \ \mathrm{Im}\left[I_X\right] \ \mathrm{Im}\left[\omega\, h_\Psi\, h_\Sigma^*\right]$$

  - cancellation requires delicate adjustment of relative phase $\varphi := \arg(h_\Psi\, h_\Sigma^*)$
  - for non-degenerate M_S and M_X: $\mathrm{Im}\left[I_S\right] \neq \mathrm{Im}\left[I_X\right]$
    - phase φ unstable under quantum corrections
  - for $\mathrm{Im}\left[I_S\right] = \mathrm{Im}\left[I_X\right]$ & $|f| = |g|$
    - phase φ stable under quantum corrections
    - relations cannot be ensured by outer automorphism of Δ(27)
    - require symmetry larger than Δ(27)

model based on Δ(27) violates CP!

Spontaneous CP Violation with Calculable CP Phase

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

field	X	Y	Z	Ψ	Σ	ϕ
$\Delta(27)$	$\mathbf{1}_1$	$\mathbf{1}_3$	$\mathbf{1}_8$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}_0$
U(1)	$2q_\Psi$	0	$2q_\Psi$	q_Ψ	$-q_\Psi$	0

- SG$(54,5)$: $\begin{cases} (X,Z) & : \text{doublet} \\ (\Psi,\Sigma^C) & : \text{hexaplet} \\ \phi & : \text{non–trivial 1–dim. representation} \end{cases}$

- non–trivial $\langle\phi\rangle$ breaks SG$(54,5) \to \Delta(27)$

- allowed coupling leads to mass splitting $\mathscr{L}^{\phi}_{\text{toy}} \supset M^2\left(|X|^2+|Z|^2\right)+\left[\frac{\mu}{\sqrt{2}}\langle\phi\rangle\left(|X|^2-|Z|^2\right)+\text{h.c.}\right]$

CG coefficient of SG$(54,5)$

- ➥ CP asymmetry with calculable phases

$$\varepsilon_{Y\to\overline{\Psi}\Psi} \propto |g|^2\,|h_\Psi|^2\,\operatorname{Im}\left[\,\omega\,\right]\left(\operatorname{Im}\left[I_X\right]-\operatorname{Im}\left[I_Z\right]\right)$$

phase predicted by group theory

Group theoretical origin of CP violation!

M.-C.C., K.T. Mahanthappa (2009)

Dirac Neutrino Mass and the μ Term

- Anomaly-free, discrete R-symmetries in MSSM:

M.-C. C., Michael Ratz, Christian Staudt, Patrick Vaudrevange, (2012)

  - absence of perturbative mu term ⇒ constraints on R charges of Hu, Hd

    ➜ non-perturbative mu term ~ TeV automatically arise

$$\mu \sim \langle \mathscr{W} \rangle / M_{\mathrm{P}}^2 \sim m_{3/2}$$

  - absence of perturbative Weinberg operator ⇒ constraints on R charges of leptons

    ➜ non-perturbative, realistic Dirac neutrino mass automatically arise

$$Y_\nu \sim \frac{m_{3/2}}{M_{\mathrm{P}}} \sim \frac{\mu}{M_{\mathrm{P}}}$$

  - solutions automatically forbid dim-4 proton decay, automatically suppress dim-5 proton decay perturbatively in superpotential

Dirac Neutrino Mass and the μ Term

- Search Abelian discrete R symmetries, $\mathbb{Z}_M^R$ that satisfy

  M.-C. C., Michael Ratz, Christian Staudt, Patrick Vaudrevange (2012)

  - anomaly freedom (a la Green-Schwarz)
  - forbidding mu term perturbatively
  - consistent with SU(5)
  - allowing usual Yukawa couplings
  - Weinberg operators forbidden perturbatively

  → classes of models found

- an example: $\mathbb{Z}_8^R$ symmetry
  - at non-perturbative level $\mathscr{W}_{\text{eff}} \sim m_{3/2}\, H_u\, H_d + \frac{m_{3/2}}{M_{\text{P}}}\, L\, H_u\, \bar{\nu} + \frac{m_{3/2}}{M_{\text{P}}^2}\, Q\, Q\, Q\, L$
  - Δ L = 2 operators forbidden ⇒ no neutrinoless double beta decay
  - **ΔL = 4 operators allowed ⇒ new LNV processes**

    M.-C. C., Michael Ratz, Christian Staudt, Patrick Vaudrevange (2012)

- A simultaneous solution possible with discrete generation dependent R symmetries (Abelian or non-Abelian!)

  M.-C.C., M. Ratz, A. Trautner, JHEP 1309 (2013) 096;
  M.-C.C., M.Fallbacher, M. Ratz, G.G. Ross, C. Staudt, V. Takhistov, P. Vaudrevaunge, under preparation

TeV Scale Seesaw and Non-anomalous U(1)

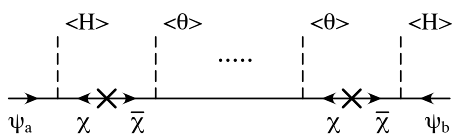
M.-C. C., de Gouvea, Dobrescu (2006)

- SM x $U(1)_{NA}$ + 3 ν_R: charged under $U(1)_{NA}$ symmetry, broken by $\langle\phi\rangle$
- $U(1)_{NA}$ forbids usual dim-4 Dirac operator and dim-5 Majorana operator

$$m_{LL} \sim \frac{HHLL}{M} \to M \sim 10^{14}\ GeV$$

- neutrino masses generated by very high dimensional operators

$$m_{LL} \sim \left(\frac{\langle\phi\rangle}{M}\right)^p \frac{HHLL}{M} \to M \sim TeV, \quad \text{for large } p \qquad \frac{\langle\phi\rangle}{M} \sim \text{not too small} \quad \sim 0.1$$



Λ ~ TeV!

low seesaw scale achieved with all couplings ~ O(1)

- anomaly cancellation: relate generation-dependent fermion charges

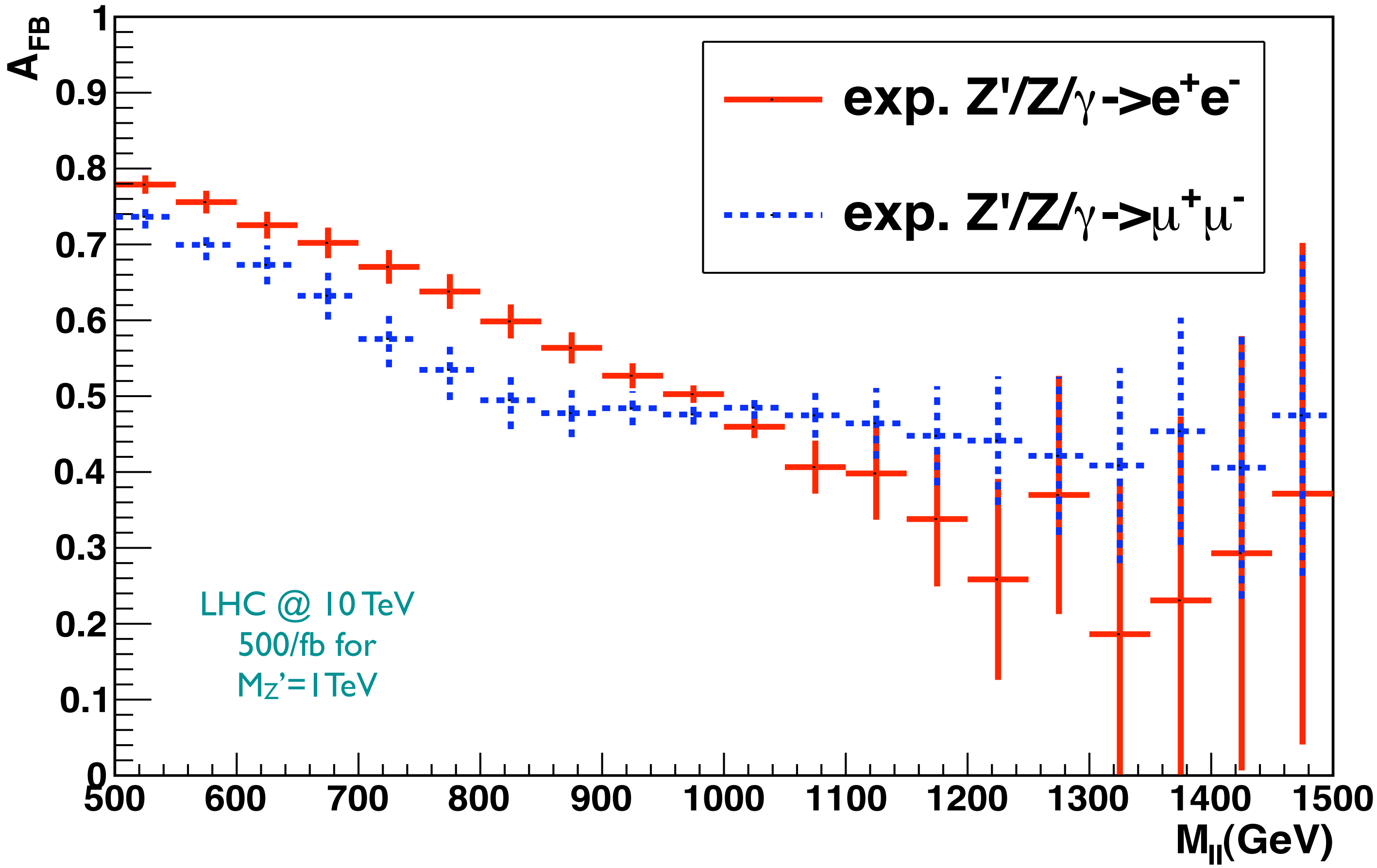
  ⇒ predict mass hierarchy and mixing

  - TeV cutoff possible with 3 RH neutrinos
  - neutrino can either be Dirac or Majorana particles
  - light sterile neutrinos: DM candidate
  - TeV scale Z′: probing flavor sector at LHC

TeV Scale Seesaw and Non-anomalous U(1)

- Establishing “flavorful” nature of Z’: 5 sigma distinction of e and mu channels

M.-C. C., J.-R. Huang (2009)



Prediction for Sparticle Spectrum

- U(1)' family (for quarks and leptons) also dictates sparticle mass spectrum (once SUSY breaking mechanism is specified)
- U(1)' family suppresses mu term
- predict testable (RG invariant) mass sum rules in Anomaly Mediated SUSY Breaking (AMSB) among sparticles at colliders

M.-C. C., J.-R. Huang (2010)

$$\bar{m}^2_{Q_i} + \bar{m}^2_{u^c_i} + \bar{m}^2_{H_u} = (m^2_{Q_i} + m^2_{u^c_i} + m^2_{H_u})_{AMSB} \;\; (i = 1, 2, 3)$$

$$\bar{m}^2_{Q_i} + \bar{m}^2_{d^c_i} + \bar{m}^2_{H_d} = (m^2_{Q_i} + m^2_{d^c_i} + m^2_{H_d})_{AMSB} \;\; (i = 1, 2, 3)$$

$$\bar{m}^2_{L_i} + \bar{m}^2_{e^c_i} + \bar{m}^2_{H_d} = (m^2_{L_i} + m^2_{e^c_i} + m^2_{H_d})_{AMSB} \;\; (i = 1, 2, 3)$$

functions of gauge couplings, Yukawa couplings and gravitino mass ($m_{3/2}$)

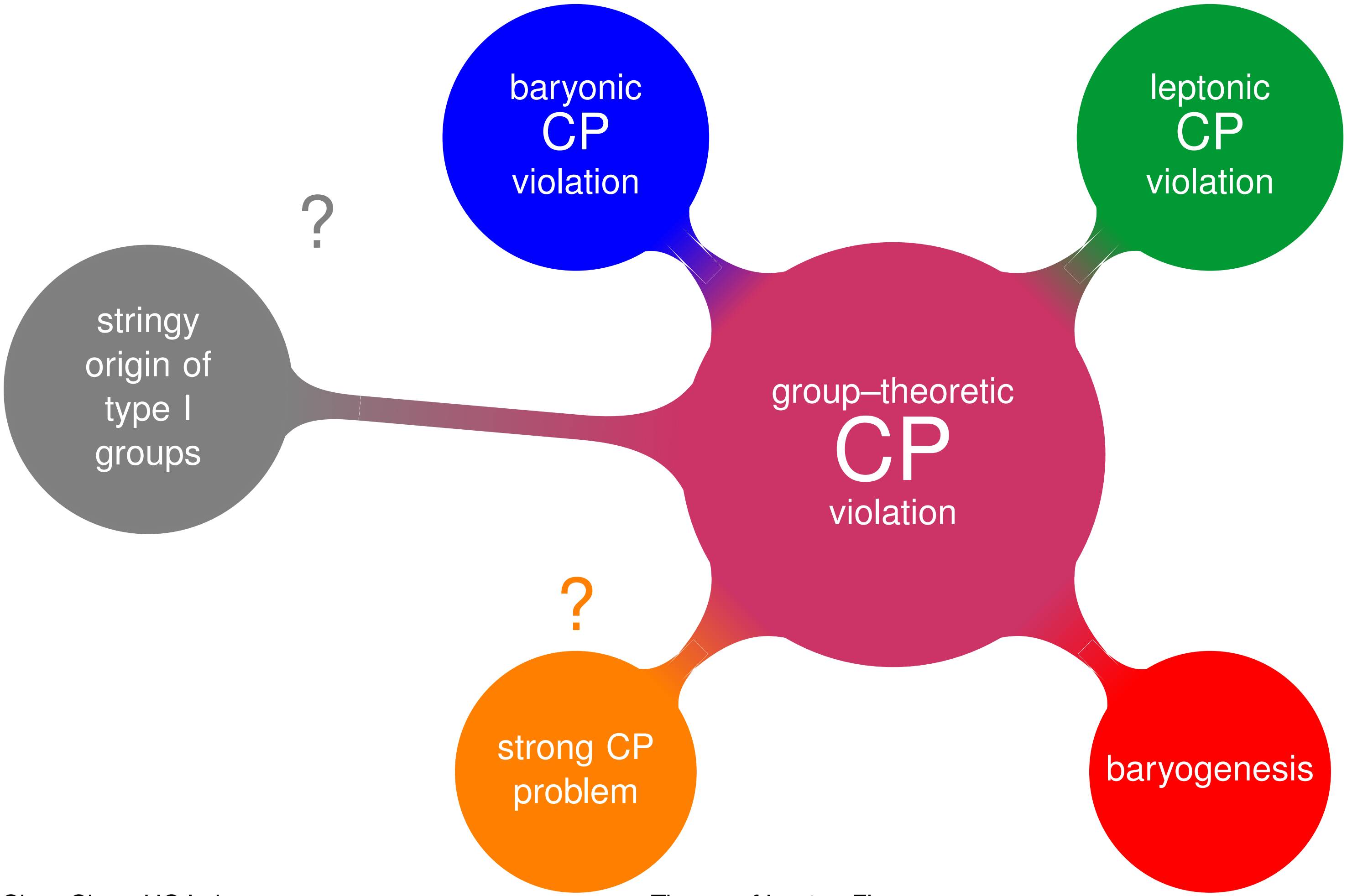
Flavor Physics at the Collider

Summary

- Fundamental origin of fermion mass hierarchy and flavor mixing still not known
- Neutrino masses: evidence of physics beyond the SM
- Symmetries: can provide an understanding of the pattern of fermion masses and mixing
  - Grand unified symmetry + discrete family symmetry ⇒ predictive power
  - Symmetries lead to testable predictions:
    - interesting leading order sum rules between quark and lepton mixing angles
    - lepton flavor violating charged lepton decays and correlations among these processes
    - proton (nucleon) decay
    - correlations among soft SUSY parameters

Conclusion & Outlook

(Type I) Discrete groups afford a new origin of CP violation:



BACK-UP SLIDES

Predictions: a SUSY SU(5) x T´ Model

M.-C.C, K.T. Mahanthappa
Phys. Lett. B652, 34 (2007); Phys. Lett. B681, 444 (2009)

- Charged Fermion Sector: 7 parameters ⇒ 9 masses, 3 angles, 1 phase

$$\theta_c \simeq \left| \sqrt{m_d/m_s} - e^{i\alpha}\sqrt{m_u/m_c} \right| \sim \sqrt{m_d/m_s},$$

SU(5) ⇒ $M_d = (M_e)^T$

⇒ corrections to TBM

related to θ_c

$$\theta_{12}^e \simeq \sqrt{\frac{m_e}{m_\mu}} \simeq \frac{1}{3}\sqrt{\frac{m_d}{m_s}} \sim \frac{1}{3}\theta_c$$

spinorial representations in charged fermion sector ⇒ complex CGs

⇒ CPV in quark and lepton sectors

quark CP phase: γ = 45.6 degrees

Georgi-Jarlskog relations at GUT scale
⇒ $V_{d,L} \neq I$

$$m_d \simeq 3m_e \qquad m_\mu \simeq 3m_s$$

Predictions: a SUSY SU(5) x T´ Model

M.-C.C, K.T. Mahanthappa
Phys. Lett. B652, 34 (2007);
Phys. Lett. B681, 444 (2009)

- Neutrino Sector:

  (2+1 parameters)

$$M_{RR} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} s_0 \Lambda \qquad M_D = \begin{pmatrix} 2\xi_0 + \eta_0 & -\xi_0 & -\xi_0 + \eta_0'' \\ -\xi_0 & 2\xi_0 + \eta_0'' & -\xi_0 + \eta_0 \\ -\xi_0 + \eta_0'' & -\xi_0 + \eta_0 & 2\xi_0 \end{pmatrix} \zeta_0 \zeta_0' v_u$$

- Prediction for MNS matrix: (for $\eta_0'' = 0$)

no adjustable parameters !

$$\tan^2\theta_\odot \simeq \tan^2\theta_{\odot,TBM} + \frac{1}{2}\theta_c \cos\delta$$

neutrino mixing angle | 1/2 | quark mixing angle | leptonic Dirac CPV

$$\theta_{13} \simeq \theta_c / 3\sqrt{2}$$

CGs of SU(5) & T´

⇒ connection between leptogenesis & leptonic CPV at low energy

SuperK best fit: δ = 220 degrees @ Neutrino 2010

- sum rule among absolute masses:

$$m_2^2 - m_1^2 = (\eta_0^4 - (3\xi_0 + \eta_0)^4)\frac{(\zeta_0\zeta_0' v_u)^2}{S_0} > 0$$

$$m_3^2 - m_1^2 = -24\eta_0\xi_0(9\xi_0^2 + \eta_0^2)\frac{(\zeta_0\zeta_0' v_u)^2}{S_0}$$

prediction in 2009 for Dirac CP phase: δ = 227 degrees

normal hierarchy predicted

Kähler Corrections

M.-C.C., M. Fallbacher, M. Ratz, C. Staudt (2012)

- Consider infinitesimal change, x :

$$K \;=\; K_{\text{canonical}} + \Delta K \;=\; L^{\dagger}\,(1 - 2x\,P)\,L$$

- rotate to canonically normalized L':

$$L \;\to\; L' \;=\; (1 - x\,P)\,L$$

⇒ corrections to neutrino mass matrix

$$\begin{aligned}
\mathcal{W}_\nu &= \frac{1}{2}(L\cdot H_u)^T\kappa_\nu(L\cdot H_u) \\
&\simeq \frac{1}{2}[(\mathbb{1}+xP)L'\cdot H_u]^T\kappa_\nu[(\mathbb{1}+xP)L'\cdot H_u] \\
&\simeq \frac{1}{2}(L'\cdot H_u)^T\kappa_\nu L'\cdot H_u + x(L'\cdot H_u)^T(P^T\kappa_\nu+\kappa_\nu P)L'\cdot H_u
\end{aligned}$$

with $\kappa \cdot v_u^2 \;=\; 2m_\nu$

CP Transformation

- Hermiticity of the Lagrangian $$\mathcal{L}(\vec{x},t) = \alpha \mathcal{O}(\vec{x},t) + \alpha^* \mathcal{O}^\dagger(\vec{x},t)$$

- Under (quantum field theory) CP Transformation

$$\mathcal{O}(\vec{x},t) \xrightarrow{\mathcal{CP}} \mathcal{O}^\dagger(-\vec{x},t) \ , \quad \alpha \xrightarrow{\mathcal{CP}} \alpha$$

- The Lagrangian

$$-\mathcal{L}_{\text{Yuk}} \supset \overline{U}_{R,i}(M_u)_{ij}Q_{L,j} + \overline{D}_{R,i}(M_d)_{ij}Q_{L,j} + \overline{E}_{R,i}(M_e)_{ij}\ell_{L,j} + h.c.$$

$$(\overline{U}_R M_u Q_L)^\dagger = (U_R^\dagger \gamma_0 M_u Q_L)^\dagger = \overline{Q}_L M_u^\dagger U_R$$

- CP Violation $\Rightarrow$ Complex Mass Matrices

$$\overline{U}_{R,i}(M_u)_{ij}Q_{L,j} + \overline{Q}_{L,j}(M_u^\dagger)_{ji}U_{R,i} \xrightarrow{\mathcal{CP}} \overline{Q}_{L,j}(M_u)_{ij}U_{R,i} + \overline{U}_{R,i}(M_u)^*_{ij}Q_{L,j}$$

Generalized CP Transformation

- setting w/ discrete symmetry G
- generalized CP transformation
- invariant contraction/coupling in A_4 or T'

Holthausen, Lindner, and Schmidt (2013)

$$\left[\phi_{\mathbf{1}_2} \otimes (x_{\mathbf{3}} \otimes y_{\mathbf{3}})_{\mathbf{1}_1}\right]_{\mathbf{1}_0} \propto \phi\,\left(x_1\,y_1 + \omega^2\,x_2\,y_2 + \omega\,x_3\,y_3\right)$$

$\omega = \mathrm{e}^{2\pi\,\mathrm{i}/3}$

- canonical CP transformation maps A_4/T' invariant contraction to something non–invariant

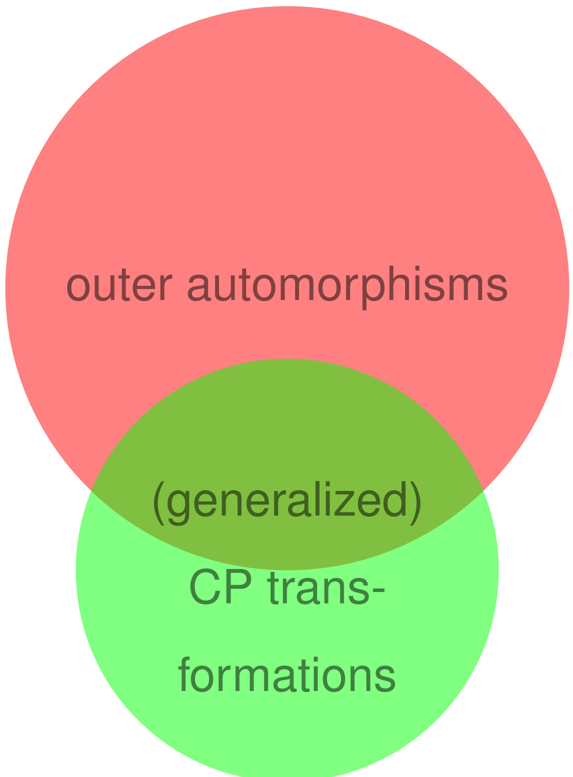
➥ need generalized CP transformation $\widetilde{\mathcal{CP}}$: $\phi \xrightarrow{\widetilde{\mathcal{CP}}} \phi^*$ as usual but

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \xmapsto{\widetilde{\mathcal{CP}}} \begin{pmatrix} x_1^* \\ x_3^* \\ x_2^* \end{pmatrix} \quad \& \quad \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \xmapsto{\widetilde{\mathcal{CP}}} \begin{pmatrix} y_1^* \\ y_3^* \\ y_2^* \end{pmatrix}$$

Physical CP Transformation

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

☞ Not every outer automorphism defines a physical CP transformation!



proper CP transformations:

class–inverting automorphisms of G

How (Not) to Generalize CP

proper CP transformations

- ☞ map field operators to *their own* Hermitean conjugates
- ☞ violation of physical CP is prerequisite for a non–trivial

$$\varepsilon_{i\to f} = \frac{\left|\Gamma\left(i\to f\right)\right|^2 - \left|\Gamma\left(\bar{\imath}\to\bar{f}\right)\right|^2}{\left|\Gamma\left(i\to f\right)\right|^2 + \left|\Gamma\left(\bar{\imath}\to\bar{f}\right)\right|^2}$$

➥ connection to observed ~~CP~~, baryogenesis & . . .

CP–like transformations

- ☞ map some field operators to some other operators
- ☞ such transformations have sometimes been called "generalized CP transformations" in the literature
- ☞ however, imposing CP–like transformations does **not** imply physical CP conservation

➥ **NO** connection to observed ~~CP~~, baryogenesis & . . .

Twisted Frobenius-Schur Indicator

- How can one tell whether or not a given automorphism is a BDA?
- Frobenius-Schur indicator:

$$\mathrm{FS}(\boldsymbol{r}_i) := \frac{1}{|G|}\sum_{g\in G}\chi_{\boldsymbol{r}_i}(g^2) = \frac{1}{|G|}\sum_{g\in G}\mathrm{tr}\left[\rho_{\boldsymbol{r}_i}(g)^2\right]$$

$$\mathrm{FS}(\boldsymbol{r}_i) = \begin{cases} +1, & \text{if } \boldsymbol{r}_i \text{ is a real representation,} \\ 0, & \text{if } \boldsymbol{r}_i \text{ is a complex representation,} \\ -1, & \text{if } \boldsymbol{r}_i \text{ is a pseudo–real representation.} \end{cases}$$

- Twisted Frobenius indicator

Bickerstaff, Damhus (1985); Kawanaka, Matsuyama (1990)

$$\mathrm{FS}_u(\boldsymbol{r}_i) = \frac{1}{|G|}\sum_{g\in G}\left[\rho_{\boldsymbol{r}_i}(g)\right]_{\alpha\beta}\left[\rho_{\boldsymbol{r}_i}(u(g))\right]_{\beta\alpha}$$

$$\mathrm{FS}_u(\boldsymbol{r}_i) = \begin{cases} +1 \quad \forall\, i, & \text{if } u \text{ is a BDA,} \\ +1 \text{ or } -1 \quad \forall\, i, & \text{if } u \text{ is class–inverting and involutory,} \\ \text{different from } \pm 1, & \text{otherwise.} \end{cases}$$

Examples

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

- Type I: all odd order non-Abelian groups

group	$\mathbb{Z}_5 \rtimes \mathbb{Z}_4$	T_7	$\Delta(27)$	$\mathbb{Z}_9 \rtimes \mathbb{Z}_3$
SG	(20,3)	(21,1)	(27,3)	(27,4)

- Type IIA: dihedral and all Abelian groups

group	S_3	Q_8	A_4	$\mathbb{Z}_3 \rtimes \mathbb{Z}_8$	T′	S_4	A_5
SG	(6,1)	(8,4)	(12,3)	(24,1)	(24,3)	(24,12)	(60,5)

- Type IIB

group	$\Sigma(72)$	$((\mathbb{Z}_3 \times \mathbb{Z}_3) \rtimes \mathbb{Z}_4) \rtimes \mathbb{Z}_4$
SG	(72,41)	(144,120)

Decay Asymmetry

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

- Decay asymmetry

$$\epsilon_{Y\to\overline{\Phi}\Phi} = \frac{\Gamma(Y\to\overline{\Phi}\Phi) - \Gamma(Y^*\to\overline{\Phi}\Phi)}{\Gamma(Y\to\overline{\Phi}\Phi) + \Gamma(Y^*\to\overline{\Phi}\Phi)}$$

$$\propto \ \mathrm{Im}\,[I_S]\ \mathrm{Im}\left[\mathrm{tr}\left(F^\dagger\, H_\Psi\, F\, H_\Sigma^\dagger\right)\right] + \mathrm{Im}\,[I_X]\ \mathrm{Im}\left[\mathrm{tr}\left(G^\dagger\, H_\Psi\, G\, H_\Sigma^\dagger\right)\right]$$

$$= \ |f|^2\ \mathrm{Im}\,[I_S]\ \mathrm{Im}\,[h_\Psi\, h_\Sigma^*] + |g|^2\ \mathrm{Im}\,[I_X]\ \mathrm{Im}\,[\omega\, h_\Psi\, h_\Sigma^*]\ .$$

one-loop integral $I_S = I(M_S, M_Y)$

one-loop integral $I_X = I(M_X, M_Y)$

- properties of ε
  - invariant under rephasing of fields
  - independent of phases of f and g
  - basis independent

Some Outer Automorphisms of Δ(27)

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

- sample outer automorphisms of Δ(27)

$$u_1 \;:\; \mathbf{1}_1 \leftrightarrow \mathbf{1}_2 \;,\; \mathbf{1}_4 \leftrightarrow \mathbf{1}_5 \;,\; \mathbf{1}_7 \leftrightarrow \mathbf{1}_8 \;,\; \mathbf{3} \rightarrow U_{u_1}\,\mathbf{3}^*$$
$$u_2 \;:\; \mathbf{1}_1 \leftrightarrow \mathbf{1}_4 \;,\; \mathbf{1}_2 \leftrightarrow \mathbf{1}_8 \;,\; \mathbf{1}_3 \leftrightarrow \mathbf{1}_6 \;,\; \mathbf{3} \rightarrow U_{u_2}\,\mathbf{3}^*$$
$$u_3 \;:\; \mathbf{1}_1 \leftrightarrow \mathbf{1}_8 \;,\; \mathbf{1}_2 \leftrightarrow \mathbf{1}_4 \;,\; \mathbf{1}_5 \leftrightarrow \mathbf{1}_7 \;,\; \mathbf{3} \rightarrow U_{u_3}\,\mathbf{3}^*$$
$$u_4 \;:\; \mathbf{1}_1 \leftrightarrow \mathbf{1}_7 \;,\; \mathbf{1}_2 \leftrightarrow \mathbf{1}_5 \;,\; \mathbf{1}_3 \leftrightarrow \mathbf{1}_6 \;,\; \mathbf{3} \rightarrow U_{u_4}\,\mathbf{3}^*$$
$$u_5 \;:\; \mathbf{1}_i \leftrightarrow \mathbf{1}_i{}^* \;,\; \mathbf{3} \rightarrow U_{u_5}\,\mathbf{3}$$

- twisted Frobenius-Schur indicators

$\boldsymbol{R}$	$\mathbf{1}_0$	$\mathbf{1}_1$	$\mathbf{1}_2$	$\mathbf{1}_3$	$\mathbf{1}_4$	$\mathbf{1}_5$	$\mathbf{1}_6$	$\mathbf{1}_7$	$\mathbf{1}_8$	$\mathbf{3}$	$\overline{\mathbf{3}}$
$\mathrm{FS}_{u_1}(\boldsymbol{R})$	1	1	1	0	0	0	0	0	0	1	1
$\mathrm{FS}_{u_2}(\boldsymbol{R})$	1	0	0	1	0	0	1	0	0	1	1
$\mathrm{FS}_{u_3}(\boldsymbol{R})$	1	0	0	0	0	1	0	1	0	1	1
$\mathrm{FS}_{u_4}(\boldsymbol{R})$	1	0	0	1	0	0	1	0	0	1	1
$\mathrm{FS}_{u_5}(\boldsymbol{R})$	1	1	1	1	1	1	1	1	1	0	0

  - none of the u_i maps all representations to their conjugates
  - however, it is possible to impose CP in (non-generic) models, where only a subset of representations are present, e.g. $\{\boldsymbol{r}_i\} \subset \{\mathbf{1}_0, \mathbf{1}_5, \mathbf{1}_7, \mathbf{3}, \overline{\mathbf{3}}\}$

Mechanisms Naturally Suppress Neutrino Masses with TeV Scale New Physics

Two examples:

- TeV scale U(1)' Family Symmetry for quarks and leptons
  - associated Z' collider phenomenology
- Discrete R-Symmetry in SUSY
  - simultaneous solution to mu problem, proton decay problem, naturally suppressed Dirac neutrino mass

before θ_{13} discovery

Sum Rules: Quark-Lepton Complementarity

Quark Mixing

mixing parameters	best fit	3σ range
θ^q_{23}	2.36°	2.25° - 2.48°
θ^q_{12}	12.88°	12.75° - 13.01°
θ^q_{13}	0.21°	0.17° - 0.25°

Lepton Mixing

mixing parameters	best fit	3σ range
θ^e_{23}	42.8°	35.5° - 53.5°
θ^e_{12}	34.4°	31.5° - 37.6°
θ^e_{13}	5.6°	≤ 12.5°

- QLC-I

$$\theta_C + \theta_{sol} \cong 45^o$$

Raidal, '04; Smirnov, Minakata, '04

(BM) $$\theta^q_{23} + \theta^e_{23} \cong 45^o$$

- QLC-II

$$\tan^2\theta_{sol} \cong \tan^2\theta_{sol,TBM} + (\theta_C / 2) * \cos\delta_e$$

Ferrandis, Pakvasa; King; Dutta, Mimura; M.-C.C., Mahanthappa

(TBM) $$\theta^e_{13} \cong \theta_C / 3\sqrt{2}$$

- testing sum rules: a *more* robust way to distinguish different classes of models

measuring leptonic mixing parameters to the precision of those in quark sector

after θ_{13} discovery

Sum Rules: Quark-Lepton Complementarity

Quark Mixing

mixing parameters	best fit	3σ range
θ^q_{23}	2.36°	2.25° - 2.48°
θ^q_{12}	12.88°	12.75° - 13.01°
θ^q_{13}	0.21°	0.17° - 0.25°

Lepton Mixing

mixing parameters	best fit	3σ range
θ^e_{23}	38.4° ↓	35.1° - 52.6°
θ^e_{12}	33.6°	30.6° - 36.8°
θ^e_{13}	8.9° ↑	7.5° -10.2°

- QLC-I

$\theta_c + \theta_{sol} \cong 45^o$ Raidal, '04; Smirnov, Minakata, '04

(BM) $\theta^q_{23} + \theta^e_{23} \cong 45^o$ ☜ **inconsistent @ 2σ**

- QLC-II

$\tan^2\theta_{sol} \cong \tan^2\theta_{sol,TBM} + (\theta_c / 2) * \cos\delta_e$

Ferrandis, Pakvasa; King; Dutta, Mimura; M.-C.C., Mahanthappa

(TBM) $\theta^e_{13} \cong \theta_c / 3\sqrt{2}$ ☜ **Too small**

- testing sum rules: a *more* robust way to distinguish different classes of models

measuring leptonic mixing parameters to the precision of those in quark sector