On the Generalized Unitarity of Dimensionally Regulated Amplitudes

Angelo Raffaele Fazio

Departamento de Física, Universidad Nacional de Colombia, sede Bogotà

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This talk is based on the paper

A.R. Fazio, P. Mastrolia, E. Mirabella, W.J. Torres,
"On a four-dimensional formulation of dimensionally regulated amplitudes"

hep-ph/14044783

and references therein.

Outline

The NLO computations of Hard processes

A typical $2 \to m$ process at one loop

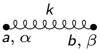
$$\sigma^{NLO} = \int_m d\sigma^B + \int_m \left(d\sigma^V + \int_1 d\sigma^A \right) + \int_{m+1} \left(d\sigma^R - d\sigma^A \right)$$

- $d\sigma^B$ is the Born exclusive cross section ($\bar{\Sigma} A^B A^{B*}$).
- $d\sigma^V$ is the virtual correction ($\bar{\Sigma}\,\Re[A^B A^{V*}]$). It involves loop diagrams to be renormalized in $\bar{MS}$ scheme.
- $d\sigma^R$ is the real corrections, affected (together with $d\sigma^V$) by soft and collinear divergencies.
- $d\sigma^A$ and $\int_1 d\sigma^A$ are unintegrated and integrated counterterms (allowing to compute real emission of massless particles in 4 dimensions).

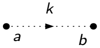
Four Dimensional Feynman Rules for gauge theories regularized bare one-loop diagrams

In Feynman- 't Hooft gauge

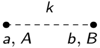
- Gauge propagators in a loop



$$= -i\,\delta^{ab}\,\frac{g^{\alpha\beta}}{k^2-\mu^2} \quad \text{(gluon)},$$



$$= i\,\delta^{ab}\,\frac{1}{k^2-\mu^2} \quad \text{(ghost)},$$



$$= -i\,\delta^{ab}\,\frac{G^{AB}}{k^2-\mu^2} \quad \text{(scalar)},$$

The scalars come from a dimensional reduction of $D = 4 - 2\epsilon$ dimensional gluons vector fields.

In $D = 4 - 2\epsilon$ dimensions we perform the decomposition of the loop momentum $\bar{k}^\alpha$ in a 4D part k^α and in a -2ϵ- dimensional **fixed** vector μ^α

$$\begin{aligned} \bar{k}^\alpha &= k^\alpha + \mu^\alpha \\ \bar{g}^{\alpha\beta} = g^{\alpha\beta} + \tilde{g}^{\alpha\beta} \quad \tilde{g}^{\alpha\beta} \to G^{AB} \;\; \mu^\alpha &\to i\mu Q^A \end{aligned} \tag{2}$$

The tripotent matrix $\tilde{g}^{\alpha\beta}$ cannot have a four dimensional representation, however the orthogonality conditions

$$\tilde{g}^{\mu\rho}\, g_{\rho\nu} = 0\,, \qquad \tilde{g}^{\mu}{}_{\mu} = -2\epsilon \xrightarrow[\epsilon\to 0]{} 0\,, \qquad g^{\mu}{}_{\mu} = 4\,,$$

are satified if

$$G^{AB} G^{BC} = G^{AC}, \qquad G^{AA} = 0, \qquad G^{AB} = G^{BA}.$$

- Fermion propagator in a loop
  Dirac matrices have the following decomposition

$$\bar{\gamma}^{\alpha} = \gamma^{\alpha} + \tilde{\gamma}^{\alpha}$$

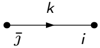
and satisfy in D dimensions the Clifford algebra

$$\{\bar{\gamma}^{\alpha}, \bar{\gamma}^{\beta}\} = \bar{g}^{\alpha\beta}.$$

A possible 4D representation of $\tilde{\gamma}$ matrices is therefore in terms of γ^5

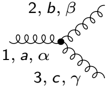
$$\tilde{\gamma}^{\alpha} = \gamma^5 \Gamma^A.$$

By imposing the rule $Q^A \Gamma^A = 1$ in order to recover $\not{\mu}\not{\mu} = -\mu^2$, the fermion propagator suitable for dimensionally regulated amplitudes is

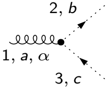


$$= i\delta^{i}_{\bar{\jmath}} \frac{\not{k} + i\mu\gamma^5 + m}{k^2 - m^2 - \mu^2 + i0}.$$

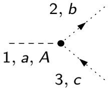
- Vertices



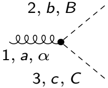
$$= -g\, f^{abc} \left[(k_1 - k_2)^{\gamma} g^{\alpha\beta} + (k_2 - k_3)^{\alpha} g^{\beta\gamma} + (k_3 - k_1)^{\beta} g^{\gamma\alpha} \right] ,$$



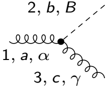
$$= -g\, f^{abc}\, k_2^{\alpha} \,,$$



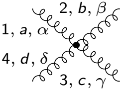
$$= -ig\, f^{abc}\, \mu Q^{A} \,,$$



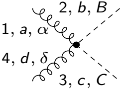
$$= -g\, f^{abc}\, (k_2 - k_3)^{\alpha}\, G^{BC} \,,$$



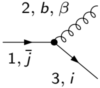
$$= \mp g\, f^{abc}\, (i\mu)\, g^{\gamma\alpha}\, Q^{B} \quad (\tilde{k}_1 = 0, \quad \tilde{k}_3 = \pm\tilde{\ell})$$



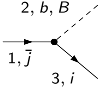
$$= -ig^2 \Big[+ f^{xad} f^{xbc} \left(g^{\alpha\beta} g^{\delta\gamma} - g^{\alpha\gamma} g^{\beta\delta} \right) + f^{xac} f^{xbd} \left(g^{\alpha\beta} g^{\delta\gamma} - g^{\alpha\delta} g^{\beta\gamma} \right) + f^{xab} f^{xdc} \left(g^{\alpha\delta} g^{\beta\gamma} - g^{\alpha\gamma} g^{\beta\delta} \right) \Big] ,$$



$$= 2ig^2 \, g^{\alpha\delta} \left(f^{xab} f^{xcd} + f^{xac} f^{xbd} \right) G^{BC} ,$$



$$= -ig \left(t^b \right)^i_{\bar{\jmath}} \gamma^\beta ,$$



$$= -ig \left(t^b \right)^i_{\bar{\jmath}} \gamma^5 \, \Gamma^B$$

- Selection rules (-2ϵ SRs)
  In the -2ϵ-dimensional vector space the following rules

$$
\begin{aligned}
G^{AB}G^{BC} &= G^{AC}, & G^{AA} &= 0, & G^{AB} &= G^{BA}, \\
\Gamma^A G^{AB} &= \Gamma^B, & \Gamma^A\Gamma^A &= 0, & Q^A\Gamma^A &= 1, \\
Q^A G^{AB} &= Q^B, & Q^A Q^A &= 1
\end{aligned}
$$

  completely define our four dimensional formulation in agreement with the **Four Dimensional Helicity scheme** up to spurious terms as explicitly checked in reproducing the numerator of QCD amplitudes of the following processes

$$
\begin{aligned}
q\,\bar{q} &\to t\,\bar{t}\,, & g\,g &\to t\,\bar{t}\,, & t\,\bar{t} &\to t\,\bar{t}\,, \\
g\,g &\to g\,g\,, & q\,\bar{q} &\to t\,\bar{t}\,g\,, & g\,g &\to t\,\bar{t}\,g\,, \\
q\,\bar{q} &\to t\,\bar{t}\,q'\,\bar{q}'\,.
\end{aligned}
$$

Generalized Internal legs

- Generalized subluminal Dirac equation

$$\left(\not{\ell} + i\mu\gamma^5 + m\right)\ u_\lambda\left(\ell\right) = 0\,,$$
$$\left(\not{\ell} + i\mu\gamma^5 - m\right)\ v_\lambda\left(\ell\right) = 0\,,$$
$$\ell^\mu = \ell^{\flat\mu} + \frac{m^2+\mu^2}{2l\cdot q_\ell} q^\mu{}_\ell; \qquad (\ell^\flat)^2 = 0 = q^2.$$

- Generalized spinors

$$u_+\left(\ell\right) = \left|\ell^\flat\right\rangle + \frac{(m-i\mu)}{\left[\ell^\flat\, q_\ell\right]}\left|q_\ell\right]\,,$$
$$u_-\left(\ell\right) = \left|\ell^\flat\right] + \frac{(m+i\mu)}{\left\langle\ell^\flat\, q_\ell\right\rangle}\left|q_\ell\right\rangle\,,$$
$$v_-\left(\ell\right) = \left|\ell^\flat\right\rangle - \frac{(m-i\mu)}{\left[\ell^\flat\, q_\ell\right]}\left|q_\ell\right]\,,$$
$$v_+\left(\ell\right) = \left|\ell^\flat\right] - \frac{(m+i\mu)}{\left\langle\ell^\flat\, q_\ell\right\rangle}\left|q_\ell\right\rangle\,.$$

- Polarization sum of the generalized fermions

$$\sum_{\lambda=\pm} u_\lambda(\ell)\,\bar{u}_\lambda(\ell) = \not{\ell} + i\mu\gamma^5 + m\,,$$
$$\sum_{\lambda=\pm} v_\lambda(\ell)\,\bar{v}_\lambda(\ell) = \not{\ell} + i\mu\gamma^5 - m\,.$$

- D dimensional Polarization Vectors
  In Arnowitt-Fickler gauge the helicity sum of the transverse polarization vectors is

$$\sum_{i=1}^{D-2} \varepsilon^{\alpha}_{i\,(D)}\left(\bar{\ell},\bar{\eta}\right)\varepsilon^{*\beta}_{i\,(D)}\left(\bar{\ell},\bar{\eta}\right) = -\bar{g}^{\alpha\beta} + \frac{\bar{\ell}^{\alpha}\,\bar{\eta}^{\beta} + \bar{\ell}^{\beta}\,\bar{\eta}^{\alpha}}{\bar{\ell}\cdot\bar{\eta}} - \frac{\bar{\eta}^{2}\,\bar{\ell}^{\alpha}\,\bar{\ell}^{\beta}}{(\bar{\eta}\cdot\bar{\ell})^{2}}$$

$$\bar{\ell}\cdot\bar{\eta} \neq 0$$

  From the gauge invariance in D dimensions the choice of the fixed D-dimensional gauge vector

$$\bar{\eta}^{\alpha} = \mu^{\alpha}$$

  allows the disentanglement

$$\begin{aligned}\sum_{i=1}^{D-2} \varepsilon^{\alpha}_{i\,(D)}\left(\bar{\ell},\bar{\eta}\right)\varepsilon^{*\beta}_{i\,(D)}\left(\bar{\ell},\bar{\eta}\right) &= \left(-g^{\alpha\beta} + \frac{\ell^{\alpha}\ell^{\beta}}{\mu^{2}}\right) \\ &- \left(\tilde{g}^{\alpha\beta} + \frac{\mu^{\alpha}\mu^{\beta}}{\mu^{2}}\right).\end{aligned}$$

- Generalized Polarization Vectors

  By decomposing the massive four 4-momentum ℓ^μ into two massless 4-vectors

$$\ell^\alpha = \ell^{\flat\,\alpha} + \hat{q}_\ell^\alpha$$

  the polarization vectors for a μ-massive vector particle are

$$\varepsilon_+^\alpha(\ell) = -\frac{\left[\ell^\flat\left|\gamma^\alpha\right|\hat{q}_\ell\right\rangle}{\sqrt{2}\mu}\,, \qquad \varepsilon_-^\alpha(\ell) = -\frac{\left\langle\ell^\flat\left|\gamma^\alpha\right|\hat{q}_\ell\right]}{\sqrt{2}\mu}\,,$$

$$\varepsilon_0^\alpha(\ell) = \frac{\ell^{\flat\alpha} - \hat{q}_\ell^\alpha}{\mu}$$

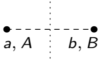
$$\sum_{\lambda=\pm,0} \varepsilon_\lambda^\alpha(\ell)\,\varepsilon_\lambda^{*\beta}(\ell) = -g^{\alpha\beta} + \frac{\ell^\alpha\ell^\beta}{\mu^2}$$

$$\begin{aligned} \varepsilon_\pm^2(\ell) &= 0\,, & \varepsilon_\pm(\ell)\cdot\varepsilon_\mp(\ell) &= -1\,, \\ \varepsilon_0^2(\ell) &= -1\,, & \varepsilon_\pm(\ell)\cdot\varepsilon_0(\ell) &= 0\,, \\ \varepsilon_\lambda(\ell)\cdot\ell &= 0\,. \end{aligned}$$

The numerator of cut propagator of the scalar can be expressed in terms of the (-2ϵ)-SRs:

$$\tilde{g}^{\alpha\beta} + \frac{\mu^\alpha \mu^\beta}{\mu^2} \quad \rightarrow \quad \hat{G}^{AB} \equiv G^{AB} - Q^A Q^B \, .$$

The factor $\hat{G}^{AB}$ can be easily accounted for by defining the cut propagator as



$$= \hat{G}^{AB} \, \delta^{ab} \, .$$

Four point massless one-loop primitive amplitudes A_4

From the reduction theorem a dimensionally regularized A_4 is decomposed in a cut-constructible part and in a rational part ($\mathcal{R}$) expressed in terms of scalar integrals in $D = 4 - 2\epsilon$ dimensions. The coefficients c_i are rational functions of the external momenta and polarizations. In a renormalizable theory, where the rank of an n-point tensor integral is n,

$$A_4 = \frac{1}{(4\pi)^{2-\epsilon}} \Big[c_{1|2|3|4;\,0}\, I_{1|2|3|4} + \big(c_{12|3|4;\,0}\, I_{12|3|4} + c_{1|2|34;\,0}\, I_{1|2|34} + c_{1|23|4;\,0}\, I_{1|23|4} + c_{2|3|41;\,0}\, I_{2|3|41} \big) + \big(c_{12|34;\,0}\, I_{12|34} + c_{23|41;\,0}\, I_{23|41} \big) \Big] + \mathcal{R} + O(\epsilon)\,,$$

$$\mathcal{R} = \frac{1}{(4\pi)^{2-\epsilon}} \Big[c_{1|2|3|4;\,4}\, I_{1|2|3|4}[\mu^4] + \big(c_{12|3|4;\,2}\, I_{12|34}[\mu^2] + c_{1|2|34;\,2}\, I_{1|2|34}[\mu^2] + c_{1|23|4;\,2}\, I_{1|23|4}[\mu^2] + c_{2|3|41;\,2}\, I_{2|3|41}[\mu^2] \big) + \big(c_{12|34;\,2}\, I_{12|34}[\mu^2] + c_{23|41;\,2}\, I_{23|41}[\mu^2] \big) \Big]\,.$$

The integrals over μ^2 can be performed by separating the integration into 4 and $D-4$ dimensional parts

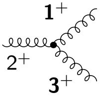
$$\int \frac{d^D\bar{\ell}}{(2\pi)^D} = \int \frac{d^{-\epsilon}(\mu^2)}{(2\pi)^{-2\epsilon}} \int \frac{d^4\ell}{(2\pi)^4}.$$

By using polar coordinates in the -2ϵ dimensional Euclidean vector space, all the integrals in $\mathcal{R}$ can be computed. In particular

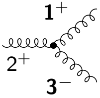
$$\lim_{\epsilon\to 0} I_{1|2|3|4}^{4-2\epsilon}[\mu^4] = \lim_{\epsilon\to 0} \left(-\epsilon(1-\epsilon) I_{1|2|3|4}^{8-2\epsilon}\right) = -\frac{1}{6}.$$

The all helicity-plus four gluons planar amplitude with a gluonic loop

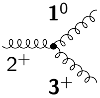
In order to reconstruct the full μ dependence and to obtain by cut construction the rational coefficients of the master decomposition, the following color-ordered trees are needed. With all outgoing momenta



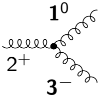
$$= 0\,,$$



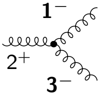
$$= ig\left(\frac{[\mathbf{1}^\flat|2][\hat{q}_{\mathbf{1}}|2]}{\mu} + \frac{\langle r_2|\mathbf{1}|2]}{\langle r_2|2\rangle}\right)\; r_2 - \text{independent},$$



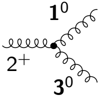
$$= 0\,,$$



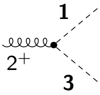
$$= \frac{\sqrt{2} ig\, [\hat{q}_{\mathbf{1}}|2]^2}{\mu} \,,$$



$$= ig \frac{[\hat{q}_{\mathbf{1}}|2]\, [\hat{q}_{\mathbf{3}}|2]\, \langle \mathbf{1}^\flat | \mathbf{3}^\flat \rangle}{\mu^2} \,,$$



$$= -\, ig\, \frac{\langle r_2|\mathbf{1}|2]}{\langle r_2|2\rangle} \Big\{ 1 - \frac{(1+\xi)}{\xi\, \mu^2} \Big[(1+\xi)\, \mu^2 + \xi\, \langle \hat{q}_{\mathbf{1}}|2|\hat{q}_{\mathbf{1}}] \Big] \Big\} \,,$$

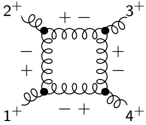


$$= \frac{ig}{\sqrt{2}}\, (\mathbf{3} - \mathbf{1})^\mu\, \varepsilon^+_\mu\, (2, r_2)\, G^{AB}$$
$$= -ig\, \frac{\langle r_2|\mathbf{1}|2]}{\langle r_2|2\rangle} G^{AB}$$

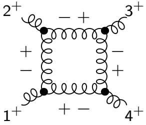
where $\hat{q}_3 = \xi \hat{q}_1$.

The box coefficients are obtained by the following attaching procedure, with the external legs of the trees on the generalized mass-shell

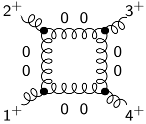
$$C^{[0]}_{1|2|3|4} =$$



$$+$$



$$+$$



,

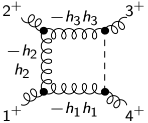
$$c^{[0]}_{1|2|3|4;\,4} = 3g^4 i \frac{[12]\,[34]}{\langle 12\rangle\,\langle 34\rangle}$$

in which the relations

$$\langle \mathbf{j}^\flat|\hat{q}_\mathbf{j}\rangle = [\hat{q}_\mathbf{j}|\mathbf{j}^\flat] = \mu$$

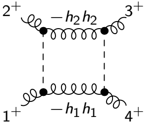
allow to obtain a polinomial numerator in μ.

$$C^{[1]}_{1|2|3|4} = \sum_{h_i=\pm,0} \mathcal{T}_1$$

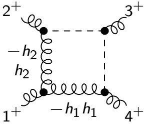


$$+\ \text{c.p.}\,,$$

$$C^{[2]}_{1|2|3|4} = \sum_{h_i=\pm,0} \mathcal{T}_1^2$$

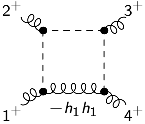


$$+\ \mathcal{T}_2$$



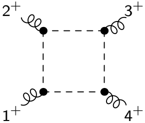
$$+\ \text{c.p.}\,,$$

$$C^{[3]}_{1|2|3|4} = \sum_{h_1=\pm,0} \mathcal{T}_3$$



$$+\ \text{c.p.}\,,$$

,

$$C^{[4]}_{1|2|3|4} = \mathcal{T}_4$$ 

$$\begin{aligned}
\mathcal{T}_1 &= Q^A \hat{G}^{AB} Q^B &&= \ 0\,,\\
\mathcal{T}_2 &= Q^A \hat{G}^{AB} G^{BC} \hat{G}^{CD} Q^D &&= \ 0\,,\\
\mathcal{T}_3 &= Q^A \hat{G}^{AB} G^{BC} \hat{G}^{CD} G^{DE} \hat{G}^{EF} Q^F &&= \ 0\,,\\
\mathcal{T}_4 &= \mathrm{tr}\left(G\,\hat{G}\,G\,\hat{G}\,G\,\hat{G}\,G\,\hat{G}\right) &&= -1\,.
\end{aligned}$$

$$c_{1|2|3|4;\,4} = c^{[0]}_{1|2|3|4;\,4} + c^{[4]}_{1|2|3|4;\,4} = 3g^4 i \frac{[12]\,[34]}{\langle 12\rangle\,\langle 34\rangle} - ig^4 \frac{[12]\,[34]}{\langle 12\rangle\,\langle 34\rangle}\,,$$

$$A_4^{1-\mathrm{loop}}\left(1_g^+, 2_g^+, 3_g^+, 4_g^+\right) = \frac{2ig^4}{16\pi^2} \times \left(-\frac{1}{6}\right) \times \frac{[12]\,[34]}{\langle 12\rangle \langle 34\rangle}\,.$$

External fermion primitive amplitudes

The color ordered tree stripped of the coupling constant

$$A_4^{\text{tree}}(1^+, 2^-, 3^-_{\bar{q}}, 4^+_q) = -i \frac{\langle 13 \rangle^3 \langle 14 \rangle}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle}$$

The leading-color (with color factor $N_c(T^{a_1} T^{a_2})^{i_4}{}_{\bar{\jmath}_3}$) decomposition to a one-loop amplitude with two external massless fermions in QCD with one flavor can be decomposed into **gauge invariant subamplitudes** made of left and right turnings diagrams

$$A_{4;1}^{1-\text{ loop}} = A_4^{\text{L}} - \frac{1}{N_c^2} A_4^{\text{R}} + \frac{1}{N_c} A_4^{\text{L},[1/2]}.$$

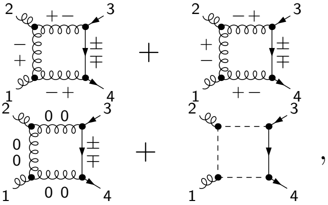
For this specific helicity configuration

$$A_4^{\text{L},[1/2]} = 0.$$

The non-planar subamplitudes (with color factor $\mathrm{Tr}(T^{a_1} T^{a_2})\delta^{i_4}_{\bar{\jmath}_3}$) are expressed in terms of $A_{4;1}$.

LEFT turning amplitude

$$C^{[\mathrm{L}]}_{1|2|3|4} =$$

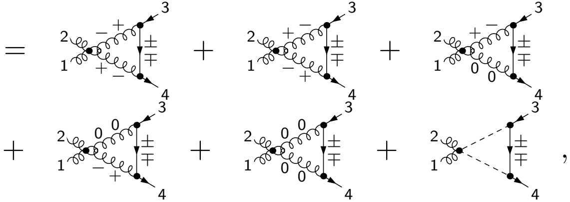


$$c^{[\mathrm{L}]}_{1|2|3|4;\,0} = \frac{1}{2} A_4^{\mathrm{tree}} \left(1 - \frac{s_{14}^3}{s_{13}^3} \right) s_{12} s_{14} \,,$$

$$c^{[\mathrm{L}]}_{1|2|3|4;\,4} = 0 \,.$$

The triple cuts are given by

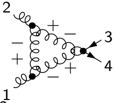
$$C^{[L]}_{12|3|4} =$$



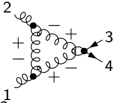
$$c^{[L]}_{12|3|4;\,0} = \frac{1}{2} A_4^{\text{tree}} \left(1 - \frac{s_{14}^3}{s_{13}^3} \right) s_{12}\,,$$

$$c^{[L]}_{12|3|4;\,2} = \frac{1}{2} A_4^{\text{tree}} \left(2 - \frac{s_{12}^2}{s_{13}^2} \right)\;;$$

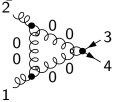
$$C^{[L]}_{1|2|34} =$$



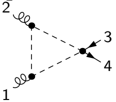
$$+$$



$$+$$



$$+$$

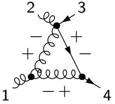


$$,$$

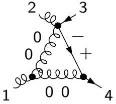
$$c^{[L]}_{1|2|34;\,0} = -\frac{1}{2} A_4^{\text{tree}} \left(1 + \frac{{s_{14}}^3}{{s_{13}}^3} \right) s_{12} \,,$$

$$c^{[L]}_{1|2|34;\,2} = -\frac{1}{2} A_4^{\text{tree}} \frac{s_{12}^2}{s_{13}^2} \,;$$

$$C^{[L]}_{1|23|4} =$$



$$+$$



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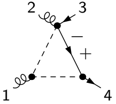
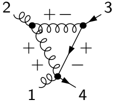


$$,$$

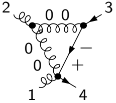
$$c^{[L]}_{1|23|4;\,0} = -\frac{1}{2} A_4^{\text{tree}} \left(1 + \frac{s_{14}^3}{s_{13}^3}\right) s_{14}\,,$$

$$c^{[L]}_{1|23|4;\,2} = -\frac{1}{2} A_4^{\text{tree}} \frac{s_{14} s_{12}}{s_{13}^2}\,;$$

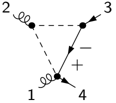
$$C^{[L]}_{2|3|41} =$$



$$+$$



$$+$$

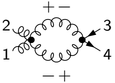


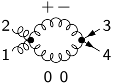
$$,$$

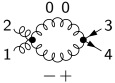
$$c^{[L]}_{2|3|41;\,0} = -\frac{1}{2} A_4^{\text{tree}} \left(1 + \frac{s_{14}^3}{s_{13}^3}\right) s_{14}\,,$$

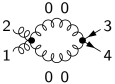
$$c^{[L]}_{2|3|41;\,2} = -\frac{1}{2} A_4^{\text{tree}} \frac{s_{14} s_{12}}{s_{13}^2}\,.$$

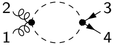
The double cuts read as follows

$$C^{[L]}_{12|34} =$$ 

$+$ 

$+$ 

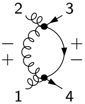
$+$ 

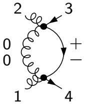
$+$ 

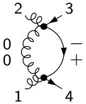
,

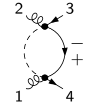
$$c^{[L]}_{12|34;\,0} = A_4^{\text{tree}} \frac{s_{14}}{s_{13}} \left(\frac{s_{14}}{s_{13}} - \frac{1}{2} \right) ,$$

$$c^{[L]}_{12|34;\,2} = 0 \,;$$

$$C^{[L]}_{23|41} =$$ 

$+$ 

$+$ 

$+$ 

,

$$c^{[L]}_{23|41;\,0} = A_4^{\text{tree}} \left(\frac{3}{2} - \frac{s_{14}^2}{s_{13}^2} + \frac{1}{2} \frac{s_{14}}{s_{13}} \right) ,$$

$$c^{[L]}_{23|41;\,2} = 0 \,.$$

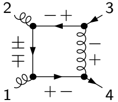
The full one loop unrenormalized **left turning** contribution stripped of the color factor and of the g^4 coupling constant factor is

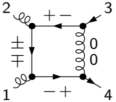
$$A_4^{1-loop}\left(1_g^-,2_g^+,3_{\bar{q}}^-,4_q^+\right) = r_\Gamma A_4^{\mathrm{tree}}\left(-,+;+-\right)\left(-s_{12}\right)^{-\epsilon}\times$$
$$\times\left\{\frac{3}{\epsilon^2}+\frac{3}{2\epsilon}-\frac{2}{\epsilon}\log\left(\frac{s_{14}}{s_{12}}\right)+\frac{7}{2}-\frac{1}{2}\pi^2+\frac{1}{2}\log^2\left(\frac{s_{12}}{s_{14}}\right)\right.$$
$$-\frac{3}{2}\log\left(\frac{s_{14}}{s_{12}}\right)$$
$$\left.+\frac{1}{2}\frac{s_{14}}{s_{13}}\left[\left(1+\left(\frac{s_{14}}{s_{13}}\right)\log\left(\frac{s_{14}}{s_{12}}\right)\right)^2-\log\left(\frac{s_{14}}{s_{12}}\right)+\left(\frac{s_{14}}{s_{13}}\right)^2\pi^2\right]\right\} \tag{15}$$

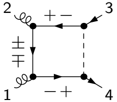
with all the $\frac{1}{\epsilon^2}$ and double *logs* diagnosing the infrared divergencies. The result agrees with the Feynman diagrams computations of Kunszt-Signer-Trocsanyi (KST) (1993) in the FDH scheme.

The right turning amplitude

As a sample of the calculation of the right turning contribution to the amplitude consider the quadruple cut

$$C^{[R]}_{1|2|3|4} =$$ 

$+$ 

$+$ 

,

$$c^{[R]}_{1|2|3|4;\,0} = -\frac{1}{2} A_4^{\rm tree} \frac{s_{12}^3}{s_{13}^3} s_{12} s_{14} \,,$$

$$c^{[R]}_{1|2|3|4;\,4} = 0\,.$$

The total amplitude is in fully agrement with KST (1993).

NLO QCD corrections to Higgs to partons

- For 2 gluons $\rightarrow$ Higgs we use an effective operator with $m_{\rm top} \rightarrow \infty$.

$$L_{\rm int} = \frac{C}{2} H \,{\rm Tr}\, F_{\mu\nu} F^{\mu\nu}$$

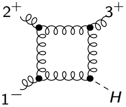
- The leading tree-level color ordered amplitude $0 \rightarrow gggH$

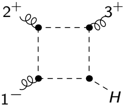
$$A^{\rm tree}_{4,H}(1^- 2^+ 3^+ H) = i \frac{[23]^4}{[12]\,[23]\,[31]}\,.$$

- As an example of application of our regularization scheme to an effective field theory consider at **two loops** in the large $m_{\rm top}$ limit the color ordered primitive amplitude

$$A^{\text{1-loop}}_{4,H}(1^- 2^+ 3^+ H)$$

To see just how the procedure works consider firstly some quadruple cuts:

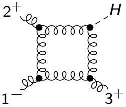
$C_{1|2|3|H} =$ 

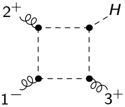
$+$ 

,

$$c_{1|2|3|H;\,0} = -\frac{1}{2} A^{\text{tree}}_{4,H} s_{12} s_{23}\,,$$

$$c_{1|2|3|H;\,4} = 0\,;$$

$C_{1|2|H|3} =$ 

$+$ 

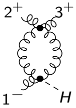
,

$$c_{1|2|H|3;\,0} = -\frac{1}{2} A^{\text{tree}}_{4,H} s_{13} s_{12}\,,$$

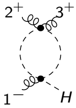
$$c_{1|2|H|3;\,4} = 0\,.$$

......and by omitting for reasons of time many other contributions.....and just considering some among the double cuts

$$C_{23|H1} =$$



$$+$$



$$c_{23|H1;\,0} = 0$$

$$c_{23|H1;\,2} = 4A^{\text{tree}}_{4,H}\frac{s_{12}s_{13}}{s_{23}^3}\,.$$

By collecting all master integrals coefficients we recognize a full agreement with the the full Feynman diagrams calculations in the FDH scheme performed by Schmidt in 1997, the result is

$$
\begin{aligned}
A_4^{1-loop}\left(1^-,2^+,3^+,H\right) &= r_\Gamma A_4^{tree}\times \\
&\left\{\frac{1}{\epsilon^2}\left[(-s_{12})^{-\epsilon}+(-s_{13})^{-\epsilon}+(-s_{23})^{-\epsilon}\right]-\frac{\pi^2}{2}\right. \\
&+\left[2\mathrm{Li}_2\left(1-\frac{s_{12}}{m_H^2}\right)+2\mathrm{Li}_2\left(1-\frac{s_{13}}{m_H^2}\right)+2\mathrm{Li}_2\left(1-\frac{s_{23}}{m_H^2}\right)\right] \\
&+\left[\log\left(\frac{s_{12}}{m_H^2}\right)\log\left(\frac{s_{23}}{m_H^2}\right)+\log\left(\frac{s_{12}}{m_H^2}\right)\log\left(\frac{s_{13}}{m_H^2}\right)\right. \\
&\left.+\log\left(\frac{s_{13}}{m_H^2}\right)\log\left(\frac{s_{23}}{m_H^2}\right)\right] \\
&\left.-\frac{1}{3}\frac{s_{12}s_{13}+s_{12}s_{23}+s_{13}s_{23}}{m_H^2}+1\right\}
\end{aligned}
$$

Conclusions and perspectives

- A four-dimensional formulation (*FDF*) of dimensional regularization of one-loop scattering amplitudes has been applied to generalized unitarity techniques and proved to be robust. At one loop the cut-constructible part and the rational part of scattering amplitudes have been computed by the same on-shell methods.
- The *FDF* Feynman rules will be extended to the recursive methods for generating the integrand of one-loop amplitudes.
- The inclusion of the fermion mass for a one loop amplitude like $0 \to ggt\bar{t}$ at one loop in *FDF* will be analysed.
- More loops and more jets in *FDF* is another goal to achieve.
- An important issue is to apply FDF for real corrections and corresponding subtraction terms of infrared divergencies.